Analysis of students' critical thinking skills in solving mathematics problems in terms of students' initial ability

Adek Nilasari Harahap1,6*, Alwen Bentri2, Edwin Musdi3, Yerizon4, Armati5
1,2,3,4,5Universitas Negeri Padang, Indonesia
6*Universitas Graha Nusantara, Indonesia

*Corresponding author: adek.harahap1988@gmail.com

ABSTRACT

This study aimed to analyze the effect of initial mathematics ability on the critical thinking ability of seventh-grade students of SMP Negeri 8 Padangsidimpuan in solving mathematics problems. Using a qualitative descriptive method, this study involved six students from class VII-D who were selected based on the initial ability test, which was categorized into high, medium, and low. The analysis focuses on four aspects of critical thinking: interpretation, analysis, evaluation, and inference. Based on the data obtained through critical thinking tests and interviews, the results showed significant variations in critical thinking ability according to students' initial ability levels. Students with high initial ability showed full mastery of all four critical thinking indicators, while students with moderate ability achieved two indicators, and students with low initial ability did not fulfill the critical thinking indicators. These results indicate that initial math ability significantly affects student's critical thinking skills in the context of mathematical problem-solving. The implications of this study provide important insights for developing educational strategies that target improving critical thinking skills, taking into account students' initial mathematical abilities.

Keywords: Analysis, Critical thinking skill, Student’s initial ability

Kata Kunci: Analisis, Kemampuan berpikir kritis, Kemampuan awal siswa

ABSTRAK

Tujuan penelitian ini adalah untuk menganalisis pengaruh kemampuan awal matematika terhadap kemampuan berpikir kritis siswa kelas VII SMP Negeri 8 Padangsidimpuan dalam menyelesaikan masalah matematika. Menggunakan metode deskriptif kualitatif, penelitian ini melibatkan 6 siswa dari kelas VII-D yang dipilih berdasarkan tes kemampuan awal, yang dikategorikan menjadi tinggi, sedang, dan rendah. Fokus analisis terletak pada empat aspek berpikir kritis: interpretasi, analisis, evaluasi, dan inferensi. Berdasarkan data yang diperoleh melalui tes berpikir kritis dan wawancara, hasil menunjukkan variasi kemampuan berpikir kritis yang signifikan sesuai dengan tingkat kemampuan awal siswa. Siswa dengan kemampuan awal tinggi menunjukkan penguasaan penuh keempat indikator berpikir kritis, sedangkan siswa dengan kemampuan sedang mencapai dua indikator, dan siswa dengan kemampuan awal rendah tidak memenuhi indikator berpikir kritis. Hasil ini mengindikasikan bahwa kemampuan awal matematika berpengaruh signifikan terhadap kemampuan berpikir kritis siswa dalam konteks pemecahan masalah matematika. Implikasi penelitian ini
1. INTRODUCTION

Education in Indonesia prepares students and teachers to face a knowledge and technology-based world in the 21st century, which requires various critical thinking skills [1]. An effective and efficient learning process will be developed with the help of the cognitive abilities of teachers and students [2]. The critical thinking skills in question relate to problem-solving inside and outside the classroom [3]. Mathematics is one of the sciences that significantly impacts the development of science and technology. Mathematics is taught to students at every level of primary and secondary education so that they can use it to reason (in a logical, analytical, organized, critical, creative, and collaborative manner) [4]. Therefore, developing critical thinking skills is a very important and urgent task to be done in the present time, which is full of problems and difficulties [5].

Learning mathematics focuses on mastering concepts and formulas and, more importantly, developing students' critical thinking skills. This ability is crucial in solving complex mathematical problems, where students are expected not only to be able to calculate but also to understand, analyze, and evaluate the process and results of problem-solving [6]. This makes critical thinking skills a component that needs to be considered in the mathematics learning process, especially at the primary and secondary levels [7]. This study aims to explore more deeply how critical thinking skills can be improved through mathematics learning, especially among junior high school students.

Critical thinking is one of the most essential skills of the 21st century [8]. It enables them to solve problems in difficult situations and to communicate effectively and accurately [9]. Developing the ability to think critically is one of the main goals of the education system, which underscores the importance of integrating the teaching of critical thinking into the curriculum, especially in mathematics education [10], [11]. This emphasizes that mathematics education is about mastering concepts and formulas and developing students' critical thinking skills to prepare them for future challenges.

An essential aspect of mathematics is the ability to think critically [12]. Critical thinking does not lead directly to conclusions or take some evidence, demands, or decisions for granted without really thinking about it. Critical thinking clearly demands interpretation and evaluation of observation, communication, and other sources of information. Critical thinking skills, emphasized in Permendiknas as mandatory competencies in the mathematics curriculum, are essential basic skills for students in learning mathematics [13]. However, the current mathematics learning process often limits students to memorizing formulas and concepts without encouraging further exploration of applications and critical problem-solving [14]. The teacher-centred approach and lecture method that dominate in many schools have the potential to hinder the development of student's critical thinking skills [15]. This calls for a change in strategy in mathematics education that better supports students in developing their critical thinking skills [16].

Critical thinking skills are a vital competency in the mathematics curriculum, as mandated in the Permendiknas, and are essential for students learning mathematics. However, learning practices in many schools tend to emphasize memorization of formulas and concepts without understanding the applications and origins of actual
mathematical solutions. This approach, which the lecture method used by teachers often dominates, risks inhibiting the development of students' critical thinking skills. Therefore, more innovative learning strategies are needed to activate and develop students' critical thinking skills in mathematics.

The 2018 PISA assessment shows that 71.9% of Indonesian students could only solve PISA questions below level 2 [17]. This means that more than half of students in Indonesia are only able to solve level 1 questions and cannot answer PISA questions correctly at all. The PISA question consists of 6 levels, where level 1 is the lowest and level 6 is the highest. Students in Indonesia still have low problem-solving skills, critical thinking skills, and creativity, so they are weak in solving non-routine problems related to proof, reasoning, generalization, making conjectures, and determining the relationship between given facts [18].

Research results Syahbana [19] also showed that junior high school students' average mathematical critical thinking ability is still low. The average value of students' mathematical critical thinking skills is 68, which, on a scale of 0-100, this value is only included in the sufficient category. With the lack of a traditional critical thinking system in schools, students are not accustomed to solving problems that require critical thinking. Eventually, the value of critical thinking skills is low. Next, the results of research conducted by Lestari & Roesdiana show that the critical thinking skills possessed by students in mathematics learning show (0%) for students with excellent mathematical critical thinking skills, in the good category as much as (0%), in the sufficient category (0%), in the deficient category (19.44%), and (80.55%) in the very deficient category [20].

Solving mathematical problems requires consideration of various aspects, including the hierarchical memorization of mathematical material and the interrelationship of mathematical concepts, which then give rise to new, more complex concepts [21]. Students often have difficulty solving problems because their initial mathematical skills are still inadequate. Students are expected to be able to relate existing mathematical concepts in their thinking to the issues they face [22]. Students' initial math skills need to be considered not only because they affect their ability to solve math problems [23] but also because they can potentially affect the development of their critical thinking skills. Strong initial mathematics skills can provide the foundation for students to build a deeper understanding and develop a more critical approach to solving mathematics problems. Therefore, students' success in learning mathematics is determined by their initial ability and how well they develop critical thinking skills as they learn. Each student has a different learning pace [24], and the ability that each student has before starting to learn, known as initial ability, becomes an important starting point for their readiness to receive learning from the teacher [25].

In today's mathematics learning, it is becoming increasingly important to understand how students' initial abilities affect their learning. According to Hanun, students' abilities before participating in mathematics learning are a prerequisite for acquiring new or additional learning [26]. This confirms that students' initial abilities are a fundamental criterion for learning activities and learning outcomes, suggesting that initial abilities can be used as a basis for building new knowledge. Adequate initial skills enable students to connect other knowledge to build new knowledge, making it essential for each individual to acquire additional learning and improve their mathematical critical thinking skills.
There are several studies related to mathematical critical thinking ability with initial skills, such as analysis of students' critical thinking in solving problems [27]-[29], analysis of students in solving problems in terms of initial ability [30], the effect of initial ability on critical thinking ability [31], critical thinking ability in solving problems [32]. However, no research uses a sample of two students for each initial ability category. This approach allows for a more comprehensive analysis of how initial ability affects critical thinking skills in the context of mathematics problem-solving.

This study's findings significantly contribute to our understanding of the importance of students' initial abilities in developing critical thinking skills, especially in mathematics learning. Thus, this study fills a gap in the existing literature by providing evidence of the importance of considering students' initial abilities in developing mathematical critical thinking skills and offers a different sampling and analysis method [29]–[32]. Hopefully, this approach can help educators design and implement more adaptive and effective learning strategies so that the mathematics learning process becomes more creative and meaningful for students with different initial abilities. This marks a significant contribution of this research to the field of mathematics education, especially in efforts to improve students' critical thinking skills.

### Contribution to the literature

The contributions of this research are:

- This study fills a gap in the existing literature by being one of the first to specifically analyze students' critical thinking abilities in solving mathematical problems, focusing on the relationship between students' initial abilities and critical thinking skills.
- The findings of this study provide significant contributions to the understanding of the importance of student's initial abilities in shaping critical thinking skills, especially in the context of mathematics learning.
- This research provides an empirical basis for developing more effective learning strategies to enhance students' critical thinking abilities, particularly by considering differences in their initial abilities.

### 2. METHOD

The researchers used a qualitative descriptive approach sourced from Taylor and Bogdan. They explain qualitative research, a data search process to produce descriptive data such as writing or speech and human behaviour that is seen/observed [33]. The subjects in this study were students of class VII-D junior high school, which focused on students' critical thinking skills in solving mathematics problems, which were selected based on the results of the initial ability test. Furthermore, the researchers chose six subjects representing each subject: two students with low initial ability, two with medium initial ability, and two with high initial ability. The selection was made with several considerations, including (1) critical thinking ability, (2) input by the mathematics teacher, and (3) data collection obtained from the subject. The categories of mathematical ability refer to a rating scale defined as follows [34].

<table>
<thead>
<tr>
<th>Students' Mathematical Ability</th>
<th>Value Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Initial Ability</td>
<td>80 ≤ score ≤ 100</td>
</tr>
<tr>
<td>Medium Initial Ability</td>
<td>60 ≤ score ≤ 80</td>
</tr>
<tr>
<td>Low Initial Ability</td>
<td>0 ≤ score ≤ 60</td>
</tr>
</tbody>
</table>
The data collection techniques used in this study were in the form of student initial ability tests, critical thinking ability tests, and interviews conducted directly with students. The critical thinking skills test was used to get the critical thinking skills of the research subjects. The critical thinking test used in this research consists of two descriptive questions or essays with a time limit of 20 minutes. These questions have undergone a validation process involving evaluation by a panel of experts consisting of mathematics educators and educational psychologists who assess the questions' alignment with the Critical Thinking Skills Framework. In addition, pre-tests were conducted with a small sample of students to assess the clarity of instructions and the difficulty of the questions and identify potential biases. The results of these two processes were used to refine the test questions before they were used in the research. Furthermore, interviews were conducted after students completed two critical thinking test questions, which aimed to learn the students' answers after completing the critical thinking skills test. Regarding the data collection techniques used in the interviews, researchers used semi-structured interviews [35].

Data analysis is an activity that classifies/sorts, marks, organizes, and categorizes a unit based on the focus or problem to be answered [36]. Based on Huberman and Miles [35], the process/analysis of qualitative discussions can be done through interactive stages with data reduction, presentation and conclusion drawing. The data analysis technique in this study was carried out in three stages: data reduction, data presentation, and conclusion drawing. The data analysis technique in this study was carried out in three stages: data reduction, data presentation, and conclusion drawing.

![Figure 1. Stages and flow of data analysis techniques interactive model (Redrawn from Huberman and Miles) [37]](image)

3. RESULTS AND DISCUSSION

The initial ability test was conducted in class VII-D, totalling 21 students. The research began by making observations and asking permission from the principal to conduct research at SMP Negeri 8 Padangsidimpuan. Researchers distributed initial ability tests to class VII-D, which were distributed face-to-face. The researcher distributed the second test to 6 subjects where two students each represented each level to measure students' critical thinking skills and afterwards interviewed to get a more in-depth picture when solving algebra and flat-sided figures problems. Students with high initial ability were FRA and SNH; students with medium initial ability were MF and AD; students with low initial ability were MRH and NA. The subjects were selected according to the results of the initial ability test, with each representing high, medium, and low. The following is the coding of the research subjects.
Table 2. Coding of research subjects

<table>
<thead>
<tr>
<th>Subject</th>
<th>Code Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Initial Ability</td>
<td>KT</td>
</tr>
<tr>
<td>Medium Initial Ability</td>
<td>KS</td>
</tr>
<tr>
<td>Low Initial Ability</td>
<td>KR</td>
</tr>
</tbody>
</table>

3.1 Subjects with High Initial Ability

3.1.1 Interpreting Stage

Based on the subject's answer to the question for the interpreting stage. This data is based on the ability to think critically when working on flat-sided figures problems.

From the results seen in Figures 2 and 3 based on the results of the completion of students with high initial ability, it can be seen that both samples can write what is known and what is asked from the problem so that both students fulfill the interpretation indicator. And it is by what is asked in the question.

3.1.2 Analyzing Stage

The following are the answers from the subject's question and answer test on the question items at the analyzing stage. This data is based on the ability to think critically when working on flat-sided figures problems.
According to the answers of high-ability students on item number two in Figures 4 and 5 at the analysis stage, students can explain the relationships and concepts used while working on the problem. Both subjects can make mathematical models appropriately by providing explanations to find solutions to these problems.

### 3.1.3 Evaluating Stage
The following are answers from the question and answer test at the evaluating stage. This data is based on the ability to think critically when working on flat-sided figures problems.

![Figure 6. KT-1 work results](image)

![Figure 7. KT-2 work results](image)

Based on the answers of high-ability students in question number two in Figures 6 and 7 of the evaluation stage, students can use the right strategy to solve the problem, complete with the correct steps in performing calculations to solve the given problem. Students can find the right solution (fulfilling the evaluation indicator) to answer the question with the correct results.

### 3.1.4 Inference Stage
The following are the answers from the subject's question and answer test on the question items at the inference stage. This data is about thinking critically when working on flat-sided figures problems.

![Figure 7. KT-1 work results](image)

![Figure 8. KT-2 work results](image)
According to the answers of high-ability students on item number two of the inference stage, students can make conclusions correctly according to what is asked in the question. The following is presented in the interview excerpt:

P-01 : Okay, explain the steps you used to solve the problem!
KT1-01 : After getting the area of one side of the roof, which is 432 m$^2$, I used the trapezoidal area formula, which is $\frac{1}{2}$ parallel sides × height.

P-02 : Why did you use the trapezoid area formula?
KT1-02 : Because what is known is the roof area of Putri's house, then the roof of Putri's house is trapezoidal.

P-03 : Oh yeah, try to explain the next step!
KT1-03 : What is known is the area of one side of the roof = 432 m$^2$ and the parallel sides = 30 m and 24m, so just put the values into the formula so

$$432 \text{ m}^2 = \frac{1}{2} (30 \text{ m + 24 m}) \times \text{height}$$

$$432 \text{ m}^2 = \frac{1}{2} \times 57 \text{ m} \times \text{height}$$

57 m is divided by 2 to 27 m, so

$$432 \text{ m}^2 = 27 \text{ m} \times \text{height}$$

Height = $\frac{432 \text{ m}^2}{27 \text{ m}}$

Height = 16 m

So the height is 16 m.

Based on the results of the analysis of the written test, it can be seen that subject KT for students FRA and SNH (who have high ability criteria) has fulfilled the critical thinking indicators. The results of the two questions showed that the subject, KT-1, obtained a total score of 25. From this score, subject KT-1 obtained a score of 89.28. In comparison, subject KT-2 obtained a total score of 28 from the test scores. From that score, the subject KT-2 obtained a score of 93.75.

### 3.2 Subjects with Moderate Initial Ability

#### 3.2.1 Interpreting Stage

Based on the subject's answer to the question for the interpreting stage. This data is based on the ability to think critically when working on flat-sided figures problems.

![Figure 9. KS-1 work results](image)

Students with moderate initial ability in student interpretation can correctly write what is known and what is asked in the problem. Therefore, it fulfils the problem's desired requirements by critical thinking ability indicator 1. And have written according to what is desired in the problem.

#### 3.2.2 Analyzing Stage

Samples with moderate critical thinking skills could not connect information from the question to complete the answer because the student could not determine the solution appropriately. This is in line with Maryanih [14], who stated that in learning mathematics, students are often faced with formulas, but mathematics requires a connection between its objects.
3.2.3 Evaluating Stage

The following answers the question-and-answer test on the evaluation stage question. This data is about critical thinking skills when working on flat-sided figures problems.

\[
\text{Luas trapesium} = \frac{1}{2} \times \text{Jumlah sisi sejajar} \times t
\]
\[
= \frac{1}{2} \times (20 \text{ m} + 24 \text{ m}) \times \text{t}
\]
\[
= \frac{1}{2} \times 44 \text{ m} \times \text{t}
\]
\[
= 22 \text{ m} \times \text{t}
\]
\[
\text{t} = \frac{64 \text{ m}^2}{22 \text{ m}}
\]

Figure 10. KS-1 work results

Students with moderate initial ability at the evaluation stage can use the right strategy to solve the problem, but the subject makes calculation mistakes.

3.2.4 Inference Stage

The following are answers to the question-and-answer test on the inference stage items. This data is based on critical thinking skills when working on flat-sided figures problems.

Based on the analysis results conducted on the sample with moderate initial ability, it was found that students with moderate initial ability at the inference stage could make conclusions from the answers obtained but were less precise. It can be seen that the subject KS has fulfilled two critical thinking indicators, namely interpretation and evaluation because, at the analysis stage in question number one, the subject KS could not make a mathematical model correctly. Hence, it obtained a score of 2, and in number two, the subject KS did not make a mathematical model of the problem, so it obtained a score of 0. At the evaluation stage, the subject KS could not make conclusions correctly, so it obtained a score of 2. Based on the results of the two questions, the subject KS obtained a total score of 20. From that score, the subject KS obtained a score of 62.5. From that score, the subject KS scored 62.5. So, it can be concluded that subject KS has moderate critical thinking skills.

3.3 Subjects with Low Initial Ability

The following is the answer to the question-and-answer test. This data is based on critical thinking skills when working on flat-sided figures materials.

Based on the analysis results conducted on the sample with low initial ability, it was found that students with low initial ability at the inference stage could make conclusions from the answers obtained but were less precise. It can be seen that the subject KR has fulfilled two critical thinking indicators, namely interpretation and evaluation because, at the analysis stage in question number one, the subject KR could not make a mathematical model correctly. Hence, it obtained a score of 2, and in number two, the subject KR did not make a mathematical model of the problem, so it obtained a score of 0. At the evaluation stage, the subject KR could not make conclusions correctly, so it obtained a score of 2. Based on the results of the two questions, the subject KR obtained a total score of 20. From that score, the subject KR obtained a score of 62.5. From that score, the subject KR scored 62.5. So, it can be concluded that subject KR has moderate critical thinking skills.
3.3.1 Inference Stage
From the low-ability subject's answer to the question in Figure 12, the interpretation stage shows that the subject cannot correctly write what is known and what is asked in the question.

3.3.2 Analyzing Stage
From the low-ability subject's answer to the question in Figure 12, the analysis stage shows that the subject cannot make a mathematical model correctly.

3.3.3 Evaluating Stage
From the low-ability subject's answer to the question in Figure 12, the subject's evaluation stage did not use the correct/appropriate strategy while working on the problem.

3.3.4 Inferencing Stage
Based on the subject's answer to the question in Figure 12, the subject's inference stage cannot conclude from the answers obtained.
Subject KR did not fulfill the critical thinking indicator because subject KR could not answer the question correctly, and during the interview, subject KR was only able to answer questions at the interpretation stage. Based on the results of the two questions, subject KR obtained a total score of 4 from the acquisition of scores number one, 0, and score number two, 4. Therefore, the KS subject scored 12.5. So, it can be concluded that the KR subject has low critical thinking skills.

3.4 Description of High Critical Thinking Ability
Based on the work of subjects with high initial ability, it can be seen that students can solve the problems given properly and correctly. This can be seen when the subject can correctly write the known questions in the problem; students can also identify the relationship between the concepts used when working on the problem. Then, for the evaluation stage, the subject uses the correct/appropriate strategy while working on the problem and can correctly make conclusions.

As for the exposure to the dialogue of the interview excerpts, it can be seen that the subject with high initial ability, at the interpretation stage, seen from the answers during the test and the interview, the subject can understand what is known and what is asked about the problem, and also the subject can explain the steps in solving the problem given, and the subject can make the right conclusion.

3.5 Description of Medium Critical Thinking Ability
Based on the answers of subjects with moderate initial ability, it can be seen that the subject can understand the problem shown in the problem by writing the known and the questions correctly; at the evaluation stage, the subject uses the right strategy in solving the problem but makes mistakes in calculating, thus making inappropriate conclusions.

3.6 Description of Low Critical Thinking Ability
Based on the work of subjects with low critical thinking skills, when working on a problem, the subject cannot solve it correctly. As seen from the subject's answer, the subject cannot write the known and asked questions correctly, and then the subject also
uses an inappropriate strategy to solve the problem; at the inference stage, the subject cannot come to the right conclusion.

The discussion in this research shows the importance of understanding and recognizing the diversity of student's initial abilities in mathematics learning, especially in developing their critical thinking skills. The results show that students with high initial abilities tend to better master different stages of critical thinking, such as interpretation, analysis, evaluation, and inference. This is consistent with the findings of Zulkarnain, who found that high initial abilities positively correlate with better critical thinking performance [38]. Angraini also emphasized that initial mathematics ability is one factor that influences mathematics learning; the higher a student's initial mathematics ability, the better the student's mathematical critical thinking ability [39]. In this study, students with high initial abilities, as demonstrated by FRA and SNH, can understand and express what is known and required in the problem and connect and analyze this information to solve problems with appropriate strategies and draw accurate conclusions.

In contrast, students with medium and low initial abilities face greater challenges at these stages, highlighting the findings of Lapp, Nyman, and Berry regarding students' difficulties in connecting mathematical concepts [40], as well as those of Dewi and Wijayanti which emphasize the need for a more structured approach to learning for students with low initial abilities [41]. This suggests that the ability to critically analyze and evaluate information depends on the instruction provided and the student's initial ability to access and apply that knowledge.

These findings reaffirm the importance of a differentiated instructional approach and support Al-Shehri's ideas about the importance of tailored instructional strategies to support the development of critical thinking skills in students with varying initial ability levels [42]. Therefore, this research provides information for educators and researchers on designing effective and inclusive teaching strategies to improve student's critical thinking skills in mathematics education. This conclusion highlights the importance of paying attention to prior ability as a key factor in planning educational interventions that can effectively improve students' critical thinking skills.

4. CONCLUSION

The findings of this study indicate that students with higher initial mathematical abilities tend to have better critical thinking skills, as evidenced by higher scores in critical thinking tests. The implications for education are the importance of recognizing and adapting teaching strategies according to students' initial abilities to improve critical thinking skills. These results suggest the need for differentiated teaching approaches and the development of accurate assessment tools for critical thinking in mathematics, emphasizing the importance of further research with a broader sample and additional variables to strengthen the generalization of findings.

5. AUTHOR CONTRIBUTION STATEMENT

ANH contributed to the study and was responsible for overall direction and planning. AB contributed to constructing an idea or hypothesis for research and/or manuscript. EM contributed to conceiving and planning the experiments. Y contributed to the interpretation of the results. A contributed to processing the experimental data and performed the analysis.
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masalah matematika berdasarkan polya pada pokok bahasan persamaan kuadrat,”


