LEARNING OBSTACLE IN THE INTRODUCTION TO NUMBER: A CRITICAL STUDY WITHIN DIDACTICAL DESIGN RESEARCH FRAMEWORK

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Article Info

ABSTRACT

This research unveils a profound exploration of learning obstacles experienced by elementary school students in understanding the concept of numbers, particularly in recognizing numbers zero to ten, including their notations. This study employs a qualitative approach with a phenomenological design. It sheds light on students’ comprehension of the meaning of numbers and numerals, identifying five types of learning obstacles: ontogenic psychological obstacles, ontogenic conceptual obstacles, ontogenic instrumental obstacles, didactical obstacles, and epistemological obstacles. Acting as the primary instrument, the researcher undertakes the entire research process using diagnostic assessments and interview guidelines, from data collection to reduction, presentation, and conclusion. The findings illustrate variations in students’ understanding of the meaning and notation of numbers, with the five learning obstacles manifesting in diverse contexts. This analysis is based on students’ responses to diagnostic assessments and in-depth interviews. The insights gained underscore the necessity for didactic designs that accommodate concrete aspects, emotional engagement of students, and the evaluation of instructional materials to enhance the understanding of numerical concepts. The research implications include recommendations for developing effective didactic designs to address the identified learning obstacles.

Keywords:
Didactical design research
Learning obstacle
Mathematics learning
Number recognition

HAMBATAN BELAJAR DALAM PENGENALAN BILANGAN: STUDI KRITIS DALAM KERANGKA DIDACTICAL DESIGN RESEARCH

Kata Kunci:
Didactical design research
Hambatan belajar
Pembelajaran matematika
Pengenalan bilangan

Penelitian ini mengungkapkan eksplorasi mendalam terkait learning obstacle siswa sekolah dasar dalam memahami konsep bilangan, terutama dalam pengenalan bilangan nol hingga sepuluh beserta notasinya. Dengan pendekatan kualitatif dan desain fenomenologi, penelitian ini menyoroti pemahaman siswa mengenai makna bilangan dan angka, serta mengidentifikasi lima jenis hambatan belajar, yakni ontogenic obstacle psikologis, ontogenic obstacle konseptual, ontogenic obstacle instrumental, didactical obstacle, dan epistemological obstacle. Peneliti, sebagai
1. INTRODUCTION

The didactic triangle is a theoretical framework that describes the relationship between three key elements in the teaching and learning process: teachers, students, and materials [1]. In a mathematical context, the didactic triangle provides insight into how mathematical knowledge is constructed and taught. The teacher is a learning facilitator responsible for guiding students’ understanding of mathematical concepts and processes [2]. Teachers are essential in organizing and presenting knowledge, choosing appropriate learning strategies, and fostering an effective learning environment [3]. Teachers’ pedagogical choices significantly influence student engagement and understanding of mathematical ideas [4].

In the didactic triangle, students occupy a central position as active participants in the mathematics learning process. They are not only passive recipients of information but also actors directly involved in acquiring mathematical knowledge. This process occurs through various experiences, assignments, and interactions with teachers and peers [5]. Students bring their prior knowledge, cognitive abilities, and personal interests to the classroom. The knowledge forms the basis for understanding the mathematical concepts taught. The teacher’s role in this context is not only as a transmitter of information but also as a facilitator whose aim is to promote student involvement. Teachers stimulate students’ critical thinking, create an environment that supports the learning process, and address individual needs to promote understanding and solving mathematical problems [6].

This approach recognizes each student’s uniqueness as a learner by placing students at the centre of the didactic triangle. Teachers serve as guides who help students develop their math skills through motivating and supportive interactions [7]. Thus, this approach reflects a learning paradigm that is more inclusive and responsive to the needs and potential of each individual in the class.

Material in mathematics learning refers to the content or substance of the knowledge taught. It involves mathematical concepts, principles, procedures, and problem-solving strategies. The importance of materials lies in the fact that the teacher must carefully carry out the selection and arrangement of materials to ensure the development of coherent and meaningful learning. Teachers must consider several aspects in preparing material, including the appropriate order of topics, the relationship between different mathematical ideas, and relevance to real-world applications [8]. A well-structured sequence and selecting relevant material can help students build a strong foundation in understanding mathematical concepts [9].
By presenting lesson material effectively, teachers can provide optimal support to students in developing a deep understanding of mathematical concepts and their relationship to the real world. Therefore, the choice and presentation of mathematical material are critical elements in creating an adequate learning experience and helping students achieve a deeper understanding and develop relevant mathematical skills.

The didactic triangle emphasizes the interdependence and dynamic interaction between the three main elements: students, teachers and material [10]. The effectiveness of mathematics learning occurs when all these components can interact harmoniously and flexibly. For example, teachers' pedagogical choices in mathematics learning must consider students' background knowledge, learning styles, and motivation. The chosen instructional method must be adapted to the nature of the mathematical content to encourage active student participation and build solid conceptual understanding. Meanwhile, how teachers actualize the material presented often refers to the established curriculum, reflecting what is known as didactical transposition.

The concept of didactical transposition is closely related to how knowledge is taught by teachers to students. Didactic transposition refers to the process by which mathematical knowledge is transformed and adapted for classroom teaching [11]. It involves translating abstract mathematical concepts and methods into more accessible forms that students can communicate and understand effectively. In mathematics education, subject matter (mathematical knowledge) undergoes a series of transformations as it moves from mathematical research and practice to the classroom [12]. Teachers play an essential role in the didactic transposition process, mediating the transfer of mathematical knowledge to students.

The teacher's job is to bridge the gap between specific mathematical knowledge and students' prior knowledge and abilities. They must consider students' cognitive development, understanding of mathematics, and learning objectives to determine the best way to present lesson material [13]. Teachers usually rely on curricula or textbooks as the primary source for teaching because these materials provide a structured framework and overall reference [14]. However, there are still many problems with actualizing school mathematics textbooks in Indonesia. Knowledge formation is only done by developing perceptual and mental skills in the 2013 curriculum mathematics textbook [15]. The absence of justification for the conclusions expected by the task design indicates a lack of reflective and a priori development. Our analysis of the commonly used Grade 1 mathematics textbooks reveals several issues, such as non-epistemic task sequences and the development of unstructured learning trajectories.

It is important to note that a didactic design crafted without considering the epistemic aspect and the orderly learning trajectory can be categorized as an inappropriate didactic design. Improper or poorly designed school textbooks can significantly hinder students' learning processes when inadequate textbooks can hinder students' progress and ability to acquire knowledge effectively. Learning obstacles can arise due to inaccuracy in the presentation of material in textbooks [15]. Learning obstacles are defined as obstacles or difficulties experienced by students during the learning process. Learning obstacles are conditions where the process of acquiring new knowledge is slow or experiences limitations so that problems arise as a result of these events [1].

Learning obstacles are grouped into three categories, namely ontogenic, epistemological and didactical obstacles that can occur in the learning process [1]. Ontogenic obstacle is a type of difficulty related to a child's readiness to learn. The second type of learning obstacle is an epistemological obstacle. This learning difficulty is more due to the limited context when a concept is first studied. As a result, children often have
problems when faced with different contexts. The third type of learning obstacle, namely the didactical obstacle, is caused by the condition of the didactical design used or the teacher's intervention.

In the current educational landscape, the urgency for researchers to explore the analysis of learning obstacles experienced by students in learning mathematics is very pronounced. Mathematics is the foundation of education, fostering critical thinking, problem-solving, and logical reasoning skills [16]. Recognizing and understanding students' challenges while mastering mathematical concepts is essential to unlocking better educational outcomes [17]. Moreover, a comprehensive understanding of learning obstacles in mathematics is integral to achieving broader educational goals. As mathematics plays a pivotal role in shaping cognitive skills, addressing learning barriers becomes crucial in promoting inclusive education. By researching learning obstacles, we can elucidate the specific challenges faced by students from various backgrounds [18]. This knowledge, in turn, enables the development of targeted interventions, innovative teaching strategies, and necessary curriculum modifications to bridge achievement gaps, particularly among marginalized or underrepresented students. Consequently, this approach empowers educators and policymakers to create equitable educational opportunities, ensuring that every student, regardless of their background, can thrive and succeed in mathematics and beyond.

Apart from that, exploring learning obstacles contributes to the progress of teaching methodology [19]. Identifying common misconceptions, cognitive barriers, and frequent errors that hinder students' understanding of mathematics paves the way for innovative pedagogical approaches [20]. Armed with this understanding, teachers can adapt their teaching techniques, design effective instructional materials, and offer comprehensive support to students who grapple with these obstacles. Research findings not only benefit students but also support teachers' professional development.

Furthermore, analyzing learning obstacles can be used as a reference in curriculum development. It highlights areas where the curriculum may fall short or where specific topics consistently prove challenging for students. This research informs curriculum designers in creating more coherent, engaging, and effective learning experiences [18]. By addressing specific obstacles identified through research, curricula can be adapted to meet student needs, ultimately leading to better outcomes.

In mathematics, one of the primary material domains is numbers. Numbers refer to the basic concepts used to measure, calculate, and express quantitative relationships between various objects or phenomena. Numbers are the basis of all mathematical calculations [21]. Without a good understanding of numbers, performing basic mathematical operations such as addition, subtraction, multiplication, and division is complex. Numbers allow us to represent the quantity or amount of something. For example, in science and economics, numbers express sizes, prices, and other parameters. In solving mathematical problems, knowledge of numbers is very important. Numbers are used to formulate and solve various mathematical problems in pure and applied contexts. The concept of numbers forms the basis of more complex mathematical structures [22]. For example, Algebra, analysis, and topology build on a solid understanding of numbers. Numbers are used widely in various fields such as computer science, physics, economics, and engineering. Algorithms, mathematical modelling, and data analysis often involve the manipulation of numbers. This is the basis that this number domain is something that must be paid attention to so that it can be mastered, especially by students. One way is to ensure that the material regarding the number domain can be structured well and conveyed well to produce complete knowledge. The inadequacy of students' understanding of numbers
and their notations can adversely affect the subsequent development of mathematical knowledge. Therefore, it is crucial to identify the learning obstacles experienced by students as a foundation for pinpointing problematic areas.

Based on several studies that have been carried out, it is revealed that understanding the concept of numbers is often faced with various problems that can hinder the mathematics learning process. One of the main problems encountered is students’ difficulty internalizing abstract concepts of numbers, such as fractions or decimals [23]. The research results also show that some students experience difficulty in relating number concepts to everyday life situations, creating a gap between mathematical theory and its application in practical contexts. In addition, it was also found that the inability to master basic number operations, such as addition, subtraction, multiplication and division, is a severe obstacle to holistic understanding [24].

Studies have been conducted on learning barriers, including: an analysis of students' learning obstacles in fraction material [25], [26], and research related to the use of didactic design: didactic transposition in mathematics education [11], didactic design for adding and subtracting mixed fractions [19], didactic design research to explain teachers' pedagogical competence [20], and diagnostics to aid elementary school students' mathematics learning [24]. No previous research has revealed in detail the problems with number recognition material based on the learning obstacles experienced by students.

This research aims to identify certain learning obstacles students face in understanding a concept or subject. The goal is to precisely detail elements that may make it difficult for students to understand. Understanding learning obstacles helps in designing more effective teaching strategies. Research objectives may include developing learning methods or teaching materials to overcome the identified barriers.

2. METHOD

The research uses a qualitative approach with a phenomenological design, which allows for the identification of the theoretical essence of the phenomenon under study. This research is part of the Didactical Design Research (DDR) framework developed by Suryadi [1] which is based on the interpretive paradigm. The interpretive paradigm emphasizes a deep understanding of the social-cultural context and meaning behind the phenomena under study [27]. Disclosure process learning obstacle students are carried out carefully and structured using various instruments to gain a comprehensive understanding. These instruments include diagnostic assessments and interview guides.

Diagnostic assessment is conducted to assess the competencies and monitor students' learning progress from both cognitive and non-cognitive aspects. In this study, the diagnostic evaluation is a series of test questions crafted based on the researcher's analysis of documents (textbooks). The investigation revealed anomalies in the design format provided, which the researcher suspects could be a trigger for the occurrence of learning obstacles. Therefore, the diagnostic assessment aims to measure the student's understanding and identify potential challenges in the learning process that may arise from discrepancies in the design format.

Diagnostic assessments were given to 30 1st-grade elementary school students who had taken the material being studied, providing a reasonably representative sample for analysis. Next, to gain a deeper understanding, researchers conducted interviews with eight students selected from the diagnostic assessment results. The selection of students is carried out using snowball sampling, where the first four students are taken based on answer characteristics that cover all relevant aspects. Students are added one by one until saturated data is obtained, ensuring the diversity and depth of the information collected.
Data from student answer sheets supported by interview results provide detailed information about student understanding, helping researchers identify emerging patterns of learning obstacles.

Researchers conducted a careful and directed selection process in determining interview subjects. First of all, four students were selected based on the characteristics of their answers, ensuring that they covered the entire spectrum of factors that had been identified in the diagnostic assessment results (as seen in Table 1).

Table 1. Variations in Student’s Answers for Each Question

<table>
<thead>
<tr>
<th>Subject Code</th>
<th>Question 1</th>
<th>Question 2</th>
<th>Question 3</th>
<th>Question 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_1$</td>
<td>$\beta_1$</td>
<td>$\beta_2$</td>
<td>$\beta_3$</td>
</tr>
<tr>
<td>S1</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
</tr>
<tr>
<td>S2</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
</tr>
<tr>
<td>S3</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
<td></td>
<td>$\sqrt{}$</td>
</tr>
<tr>
<td>S4</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>$\sqrt{}$</td>
<td></td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
</tr>
<tr>
<td>S7</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S8</td>
<td>$\sqrt{}$</td>
<td>$\sqrt{}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Information: Code $\alpha_1 - \delta_3$ is a variation of students’ answers

Subject selection is critical in ensuring diverse representation and achieving a holistic understanding of student learning obstacles. Apart from only considering the characteristics of students' answers, researchers also included evaluating students' communication skills as a determining factor. Information related to communication skills was obtained through recommendations from teachers so that interview subjects were selected based not only on their understanding of mathematical concepts but also on their ability to communicate this understanding clearly and openly. The importance of this aspect of communication emphasizes that data collection does not only focus on the substance of the material but also on students' ability to articulate their understanding clearly. Furthermore, to identify learning obstacles the first four students may not represent, the researcher also selected four additional students (as seen in Table 2). This selection was based on the need to gain a diversity of perspectives and understand obstacles to learning more thoroughly. Thus, the interview subject selection process reflects a commitment to embracing diversity and complexity in understanding student learning obstacles, which can provide a more solid foundation for developing more inclusive and effective learning strategies.

Table 2. Implementation of Snowball Sampling

<table>
<thead>
<tr>
<th>Subject</th>
<th>Obstacle Type</th>
</tr>
</thead>
</table>
| 1       | Conceptual Ontogenic Obstacle  
          Epistemological Obstacle  
          Didactical Obstacle |
| 2       | Epistemological Obstacle  
          Didactical Obstacle  
          Instrumental Ontogenic Obstacle |
| 3       | Epistemological Obstacle  
          Conceptual Ontogenic Obstacle  
          Didactical Obstacle |
| 4       | Epistemological Obstacle  
          Didactical Obstacle |
| 5       | Epistemological Obstacle  
          Didactical Obstacle |
Table 2 provides a detailed description of the results of selecting interview subjects. Data can be obtained from the first four subjects regarding three types of learning obstacles: conceptual, ontogenic, epistemological, and didactical. While a picture of instrumental and psychological ontogenic obstacles cannot yet be seen, even though they have been discovered, they do not yet represent saturated data. The 5th interview subject, and so on until the 8th subject, was taken into consideration to complete the missing information. Additional subjects were taken until the researcher ensured that all learning obstacles could be adequately described. This process reflects the seriousness of researchers in ensuring the diversity and completeness of data so that research results can be representative and provide an accurate picture of the learning obstacles experienced by students in the material studied. By stopping at subject 8, the research is expected to have reached a point where all relevant variables have been represented, providing a solid foundation for quality analysis and findings.

The phenomenological research design, illustrated in Figure 1, involves two main stages: data collection and data analysis [28]. During the data collection phase, participants immersed in a social phenomenon are interviewed to capture their subjective experiences and perspectives related to the studied phenomenon. In the data analysis stage, the researcher reads the transcripts to gain a comprehensive understanding and identify 'units of significance' that authentically represent participants' subjective experiences.

Figure 1. The Existential Phenomenological Research Method

3. RESULTS AND DISCUSSION

Research findings related to learning obstacles will be described in five parts, namely psychological ontogenic obstacles, conceptual ontogenic obstacles, instrumental ontogenic obstacles, didactical obstacles, and epistemological obstacles. However, first, we will explain students' understanding of the meaning of numbers and numerals. This understanding is an important basis for discussing learning obstacles experienced by students. By understanding how students absorb the meaning of numbers and numerals, research can go further to identify learning obstacles that arise.
3.1 The Meaning of Numbers and Numerals According to Students

The importance of understanding the concept of numbers from an early age is a key factor in students' mathematical development. This section explains the meaning of Grade 1st Elementary School (SD) students towards numbers, with a main focus on two main dimensions: the meaning of numbers as quantities and the meaning of numbers as sequences. Interviews with students illustrate the diversity of understanding regarding numbers as quantities. Some students associate numbers with physical objects around them, creating direct connections to real objects. For example, one student stated, "Numbers are like the number of fruits on a tree," demonstrating a direct connection between numbers and concrete objects. However, this finding also shows challenges, as expressed by another student who admitted difficulties in connecting numbers with quantities concretely, "I'm still confused; how come there are so many numbers?" One teacher interviewed explained, "Students need concrete experiences to understand numbers as quantities. Physical objects or pictures can help them make stronger connections between numbers and quantities." This statement indicates that, for some students, a learning approach that utilizes concrete and tangible elements can be an effective solution for deepening their understanding of numbers as quantities. Thus, implementing learning methods that combine concrete aspects can be a strategic step in overcoming various levels of students' understanding of numbers.

The findings reveal significant variations in the dimension of the meaning of numbers as sequences. Some students could identify number sequences and demonstrated a good understanding of the relationships between numbers. For example, one student confidently stated, "The order is easy, from small to large or vice versa, I know!" However, these findings are also offset by the difficulties several students face. Some students expressed their lack of clarity in arranging the sequence of numbers logically, as one student admitted, "It's difficult; I'm confused about the order." The findings show that students' understanding of numbers is more dominant in the dimension of numbers as quantities, especially in the case of number sequences.

Students' understanding of number order is limited to understanding that small numbers come before larger numbers or vice versa. In this context, students understand that, for example, after the number 1, there will be the number 2, which is associated with observing the quantity of the object. However, there is confusion when students are asked to relate the meaning of numbers as a sequence to their sitting position in class. A series of questions are asked to dig deeper into students' understanding.

Researcher : In class, where do you sit?
Student : On that bench. (points to the second bench from the front)
Researcher : How many benches is that?
Student : Second bench.
Researcher : Who's the first?
Student : The first is X. (mentions the names of other students)
Researcher : Why is sitting position X called first and your sitting position called second?
Student : Yes, because it is calculated from the front.
Researcher : If the first and the second, which one is more?
Student : More second
Researcher : What's so much?
Student : (Confused, thinking) Don't know, sir.
The results of this interview clearly illustrate that students' understanding of numbers is limited to the concept of object quantity. They relate numbers to sequences based on simple observations but have difficulty applying the concept in more abstract contexts, such as comparing number sequences with seating positions in class. It is important to note that students tend to link their understanding to concrete, everyday experiences. Although they can identify number sequences visually, relating them to abstract concepts such as "more" or "position" causes difficulties.

3.2 Psychological Ontogenic Obstacle

In the initial stage of the interview, the main focus was on identifying the psychological ontogenic obstacles experienced by students in learning numbers, especially in the material regarding the introduction of the numbers zero to ten. Psychological ontogenic obstacles include students' unpreparedness in terms of motivation and interest in the material being studied. In explaining the findings of this research, two student subjects will be appointed as representatives of the identified problems, namely Subject 6 and Subject 8.

One of the main findings in this section is a psychological ontogenic obstacle, which is reflected in the students' lack of motivation and interest in number material. Subject 6 showed significant symptoms of disinterest in this material. In the interview, Subject 6 stated, "I don't like numbers. I prefer the colours and the pictures to the numbers." This statement reflects students' lack of emotional involvement in number material. Students tend to be more interested in visual elements or colours that can stimulate their thinking power. This lack of interest affects students' motivation to learn and understand number material well. In this context, Psychological ontogenic obstacles hinder students' ability to absorb mathematical concepts with enthusiasm. The following is an interview excerpt from Subject 6 regarding this matter:

Researcher : Why don’t you like numbers, Subject 6?
Subject 6 : I do not know. I was bored. Colorful pictures are more exciting.

The interview excerpt shows that Subject 6 cannot explain exactly why he felt bored with the number material. However, researchers saw that students tended more towards visuals and colours, which may be the main factor in their disinterest in number material.

Meanwhile, Subject 8 showed special psychological challenges in facing the number zero. In the interview, Subject 8 revealed, "I'm confused by zero. Zero is empty, but why is it a number? I don't like numbers that have no value." This quote reflects an abstract concept regarding the number zero that is difficult for Subject 8 to understand. This psychological challenge is not only limited to a lack of interest but also involves a sense of confusion and difficulty in assigning value to the number zero. This psychological ontogenic obstacle influences the way students view the concept of numbers, especially the number zero, which he considers "empty" and difficult to value. The following is an interview excerpt from Subject 8 regarding this issue:

Researcher : What confuses you about zero, Subject 8?
Subject 8 : Zero doesn't have a number, but the teacher said it was a number. Strange, right? Zero is empty.
This quote shows that Subject 8 faces difficulty in combining the concept that zero is a number, even though, mathematically, zero is considered a value. These findings indicate that psychological ontogenic obstacles in learning numbers in elementary school students manifest in a lack of motivation, disinterest, and difficulty in understanding abstract concepts such as zero. So far, two research subjects have clearly represented this problem, with each demonstrating different levels of challenge.

3.3 Instrumental Ontogenic Obstacle

Instrumental ontogenic obstacles are highlighted in this research, especially regarding students' unpreparedness in facing important technical aspects of learning mathematics. In these findings, the research focuses on student responses and errors that arise while solving number identification questions. The first problem presents a picture of several types of balls in a box, and students are asked to identify the numbers that can be found in the picture (see Figure 2). Supposedly, only one number can be identified, namely 10. Empirical findings show that some students can answer correctly but cannot provide appropriate justification. Two types of dominant answers emerged, namely, students only wrote the number 10 or students wrote the numbers 1, 2, 3, and 4.

The single answer "10" emerged as the students' dominant response. This indicates students' tendency to default to associating ten with the number of objects or elements without identifying the other numbers present. They got the number 10 based on calculating all the balls in the box. Check out the following interview excerpt:

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Why is the answer 10?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject 2</td>
<td>Because there are 10 in it, sir.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Try to count!</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject 2</td>
<td>one, two, three, four, five, six, seven, eight, nine, ten (while pointing to the balls one by one)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Researcher</th>
<th>What's in it?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject 2</td>
<td>Basketball, soccer, basketball, and takraw football.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Are there any other numbers besides 10?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject 2</td>
<td>There isn't any.</td>
</tr>
</tbody>
</table>

English version:

Counting Objects
Look at the image above!
Count the objects in the box; what numbers do you find?
Write down the notation for all the numbers you find!

Figure 2. Visual of the First Question in the Diagnostic Assessment
Even though Subject 2 correctly answered 10, in the interview excerpt, it can be seen that there was an inability to identify numbers in the context of one unit, namely a ball. Subject 2 still identified the ball based on its type. Thus, it can be said that students experience instrumental ontogenic obstacles.

Meanwhile, instrumental ontogenic obstacles were also seen in the second answer type in Subject 7. During the interview, the researcher confirmed the numbers that could be identified in the first question. Subject 7 glanced at the question and immediately answered, "One, two, three, and four", without needing to count one by one as Subject 2 did. Subject 7 could subitize (distinguish between sets containing different numbers of objects without using an explicit counting process), namely sets of several types of balls containing 1 to 4 objects. However, subitizing became unreliable, making it difficult for Subject 7 to determine the number 10. This means that Subject 7 had neglected an important aspect, namely counting as a whole. In short, although Subject 7 could recognize some numbers, limitations in identifying all numbers were apparent.

Another case found was in the second question, where in this second question, students were asked to describe numbers (see Figure 3). It appears that there are a total of nine pencils, and six of them are coloured. Then, students are asked to solve three types of problems, two of which are open-ended.

It appeared in Subject 7, where he did not understand which numbers had to be deciphered. Subject 7 understood that the numbers that had to be deciphered were written numbers, so in the first problem, he wrote the same answer as in the third problem (see Figure 4).

Based on Figure 4 above, it appears that Subject 7 does not understand that the numbers in the box with two lines must be described. In the interview session, Subject 7 confirmed, "My understanding is that the numbers that must be deciphered are the numbers that are written." This shows that Subject 7 does not understand important
technical aspects of learning. In reality, the method for decomposing numbers is the same as in Figure 4, adapted from the method written in the textbook. In the remaining two questions, the researchers found no instrumental ontogenic obstacles students experienced.

### 3.4 Conceptual Ontogenic obstacle

Conceptual ontogenic obstacles in mathematics learning are a significant challenge. This refers to students’ unpreparedness due to insufficient previous learning experiences. If this foundation is not built well, students tend to have difficulty connecting new concepts with the knowledge they already have. It is important to understand that conceptual ontogenic obstacles can also arise because students fail to understand fundamental ideas. This indicates that learning mathematics is about remembering facts and understanding the concepts that are the basis of the material. Conceptual ontogenic obstacles can also be triggered by a lack of understanding of supporting material requirements. Some students may be unable to relate mathematical concepts to real situations or find it difficult to see the relevance of the material to everyday life. Therefore, mathematics learning must be able to explain the relationship between abstract concepts and their application in practical contexts. Apart from that, the knowledge students gain outside mathematics can also influence their understanding of mathematical material. This creates a cognitive obstacle that must be overcome for students to fully understand mathematics.

In the case of the first question, the researcher did not find any conceptual ontogenic obstacles. In the case of the second question, a conceptual ontogenic obstacle was identified when it was discovered that students could not describe numbers in several versions. In this context, there are two subjects, Subject 7 and Subject 6, who experience similar problems deciphering the number nine. The obstacle faced by Subject 7 can be seen in its limitations in describing the number 9 in only two versions, namely 4 and 5 (see Figure 4). This reflects a lack of understanding that numbers can be broken down into various combinations that produce certain quantities. When confirmed during the interview, "Are there other versions besides 4 and 5?" Subject 7 clearly stated, "There isn't any". Something similar applies to Subject 6, which is also limited to two versions of the decomposition of the number 9, namely 3 and 6.

![Figure 5. Visual of Subject 3's Answer to the Second Question](image)

A more severe conceptual ontogenic obstacle was seen in the second case, especially in Subject 3. This subject showed a deep lack of understanding of the concept of number decomposition. As depicted in Figure 5, Subject 3 failed to understand how to decompose numbers. Figure 5 provides concrete evidence regarding Subject 3's difficulty in overcoming the concept of number decomposition. Subject 3 does not seem to have achieved an understanding of this concept, which is reflected in his inappropriate responses and shows his inability to identify relationships between numbers. It is important to note that in the three cases presented in the answer sheet ($X_1$ and $X_2$), it appears that the answers written are related to Subject 3's understanding of number sequences. At the same time, the case $X_3$ indicates answers that may be spontaneous or not well thought out. This shows
that the conceptual ontogenic obstacle does not only include understanding the relationships between numbers but also skills in describing numbers correctly. However, the results of the further investigation through interviews revealed something different. Pay attention to the following interview excerpt.

Researcher : why is it written 7 and 8? (pointing to Subject 3's answer \((X_3)\))
Subject 3 : The material is the numbers 0 to 10, and before the number 9, there are the numbers 7 and 8
Researcher : This section discusses decomposing numbers, not number sequences. Subject 3 : do you know the difference between describing and sorting?
Subject 3 : No, as far as I know, this is asking to order numbers
Researcher : If you order numbers, why is it 2, 9, 3? (pointing \(X_3\))
Subject 3 : Then I see the whole. There are 7, 8, and 9(pointing \(X_1\)), and there are 4, 5, and 6 (pointing\(X_2\)). So, if you sort it, it means that 2 and 3 don’t exist if 9 already exists.

The interview results confirmed the correctness of the analysis of Subject 3's answers in Figure 5 relating to the understanding used to answer the second question, namely number order. The interview results also corrected that in writing 2 and 3 on \(X_3\), not necessarily without consideration, as can be seen from the results of the answer sheet analysis. Answers to Subject 3 on \(X_3\) still refer to students' understanding of the concept of number order. These results make it clear that Subject 3 is proven to experience a conceptual ontogenic obstacle.

Next, we discuss the conceptual ontogenic obstacle that occurs in the case of the third question. The third problem focuses on comparing two numbers (see Figure 6), where students are asked to count two different groups of objects and then compare them.

Apart from evaluating students' understanding of the concept of comparing two numbers, this section is driven by additional motivation to explore students' understanding of the meaning of the concept of numbers themselves. In the third question, it appears that students who experience a conceptual ontogenic obstacle have something in common: they tend to see that comparing sizes is not about quantity (lots of objects), especially in the context of the second picture, which shows a pencil. In evaluating the pencil drawing in question three, it appears that students tend to be more fixated on the visual or physical characteristics of the drawing, such as the length of the pencil, rather than understanding that the comparison should refer to the quantity or number of pencils.
Researcher : Why are C and D different?
Subject 6 : From the picture in the question, it can be seen that pencil C is longer than pencil D.

Universal agreement among all students that A is the same as B indicates a solid understanding of comparing two numbers. However, an interesting finding emerged when several students, including Subject 1, Subject 3, Subject 6, and Subject 7, expressed the view that C was different from D. Despite this objection, in terms of quantity, it was clear that C and D each had a quantity of 1. Therefore, C should be equivalent to D. Students' difficulty in comparing C and D as two equivalent numbers indicates that they are facing a conceptual ontogenic obstacle. This can explain that the conceptual ontogenic obstacle in comparing numbers occurs due to limitations in students' understanding regarding the abstraction of the concept of numbers as quantities.

In the case of the fourth question, only Subject 3 was identified as experiencing a conceptual ontogenic obstacle. Question number four relates to the order of numbers from largest to smallest or vice versa in the whole number range between zero and ten. However, in the design of this question, the researcher tried to present a sequence of prime, odd and even numbers (see Figure 7). It is important to note that the focus of this fourth question should not be on understanding prime, odd, and even numbers but rather on students' ability to order numbers based on large and small quantities.

In his answer sheet, Subject 3 indicated that all the sequences in the fourth question were considered incorrect number sequences. According to Subject 3, conveyed through the interview results, ordering numbers should not involve jumping. Further, Subject 3 stated that numbers must be ordered sequentially, such as 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, or vice versa. It is important to remember that Subject 3's view about not jumping around in number ordering reflects a conceptual ontogenic obstacle, where understanding the concept of number order has not yet been fully formed. Subject 3 identified that the order written in the fourth question did not follow the understood number order rules. Ultimately, this section provides richer insight into how conceptual ontogenic obstacles can be reflected in students' thinking regarding the concept of number recognition.

### 3.5 Didactical Obstacle

Didactic learning obstacles can arise due to the didactic system, which includes factors such as the sequence and stages of the curriculum, including how the material is presented in the classroom learning context. Didactical obstacles need to be considered primarily based on the sequence of material, both structurally, which involves the relationship between concepts, and functionally, which refers to the continuity of the thinking process. In this case, structural analysis can include evaluating the extent to which
the concepts taught are related and whether the learning sequence creates a coherent understanding for students. By detailing the relationships between concepts, we can identify possible didactic obstacles that arise due to ambiguity or contradiction in the presentation of the material. On the other hand, functional analysis is related to the stages of presenting material. Consideration should be given to whether the presentation of the material is insufficiently detailed or, conversely, too detailed. Understanding that it is less detailed may make it difficult for students to follow and internalize concepts. At the same time, too detailed explanations can make students lose focus or feel overwhelmed by excessive information. Therefore, this section will approach didactic obstacles with two main dimensions, namely the structural dimension and the functional dimension.

Researchers found that the second type of answer to the first question could be identified as a result of didactic obstacles experienced by students. Several students who answered 1, 2, 3, and 4 indicated that their knowledge of answering 1, 2, 3, and 4 was due to the design of the material in the textbook. It is important to note that in textbooks, similar cases have been actualized and lead students to the calculation of objects based on their types (see Figure 8).

In Figure 8, it appears that there is a cupboard containing several types of toys. This is similar to the first problem, where the researcher placed several types of balls in a box. Even though the first question does not explicitly determine the types being counted, students can find numbers based on the type of ball. The results of the following interview reinforce this.

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Why are the numbers 1, 2, 3, 4 found?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject 4</td>
<td>There's one basketball, two baseballs, three takraw balls and four soccer balls.</td>
</tr>
<tr>
<td>Researcher</td>
<td>In the question, you are not asked to count each ball</td>
</tr>
</tbody>
</table>
Subject 4 : Yes, usually in books and what teachers teach is like that.

The visuals in Figure 8 and the results of the interview excerpts are sufficient to be used as evidence of the didactical obstacles experienced by students on this topic.

Next, it relates to the case of Subject 3 in Figure 5, which is related to decomposing numbers. Previously, the analysis showed that Subject 3 experienced a conceptual ontogenic obstacle in facing this problem. However, when further analysis was carried out, it turned out that the knowledge used by Subject 3 to answer the second question turned out to be the result of his construction in understanding number ordering material. For example, it can be observed in Figure 9a, which relates to the material of number ordering. In explaining the sequence of numbers, the didactic design is arranged to involve lines connecting several objects. Therefore, when Subject 3 faced the second problem case, similar to the material design in Figure 9a, his train of thought automatically led to the sequence of numbers, not the decomposition of numbers (see Figure 9). This analysis illustrates that Subject 3’s knowledge construction in number ordering material indirectly influenced his approach to the second problem. Thus, it can be concluded that Subject 3 experienced a didactical obstacle.

Didactic obstacles in the case of the second question also arise when students face difficulties in decomposing numbers involving zeros. Although the didactic design has established the knowledge that a number can be described in several versions, as seen in
Figure 10, no explanation explicitly addresses the involvement of zero in decomposing the number. Further analysis may detail that a lack of emphasis on the role of zero in decomposing numbers may create didactical obstacles for students. In Figure 10, it appears that the book's compiler wants to show students that five can be decomposed into several versions. Other tasks related to decomposing numbers are also designed with a similar design. Thus, it is proven that the student's inability to use zero to describe a number is a manifestation of a didactical obstacle.

Next is related to the comparison of two numbers in the third question. There was an interesting finding related to students' inability to compare two numbers, which turned out to be students using their understanding of the size of objects visually. Researchers identified that this obstacle does not solely come from students' limited understanding but is also influenced by the didactic design used to compare two numbers. The textbooks used by students always display objects of the same size (see Figure 11), which is crucial in analyzing students' learning obstacles. The design designer's focus in presenting material comparing two numbers is on the number of objects being compared without realizing that it turns out students are not only looking at the number of groups of objects but also the size of the objects being compared. Suppose students are constantly exposed to the understanding that number comparisons are based on the size of objects. In that case, this can form false beliefs and become an obstacle to understanding comparing two numbers. Thus, this phenomenon can be identified as a didactic obstacle students experienced in comparing two numbers.

3.6 Epistemological Obstacle

Epistemological learning obstacles can arise due to limitations in the context of didactic design, which refers to obstacles in understanding and developing epistemology or how students view sources of knowledge and concept formation. Inadequate didactic design, especially when the learning context does not include multiple perspectives or relevant sources of knowledge, can form a narrow or limited view in students. It is important to note that these obstacles to epistemological learning are not an individual student's problem but result from a learning environment that does not support the
development of more complex epistemological understanding. In this research, epistemological obstacles were only identified in the case of the first question.

In the first question, several students could identify the numbers 1, 2, 3, and 4 based on the type of ball illustrated. Even though it may seem like there is no problem at first because their answers follow the information in the textbook, this kind of understanding has the potential to give rise to deeper epistemic problems, especially related to the set material described in several previous sections. It should be noted that students' knowledge in this context may be static and limited to the information provided in textbooks without considering broader conceptual elements or links to deeper mathematical concepts. When students can identify numbers that refer to the type of ball, then students should also identify the number zero. Because in the first question, there is no notation at all that leads to the conclusion that the balls are separated based on their type. Students' inability to identify these limitations as needing attention indicates that they are experiencing epistemological obstacles. In this context, it needs to be emphasized that these obstacles occur not because individual students themselves cause them but because of the incompleteness of the concepts presented in the didactic design.

Based on the research results on students' understanding of the concept of numbers, the findings show signs of learning obstacles related to understanding the concept of numbers. These symptoms include confusion, frustration, and the inability of students to apply their knowledge regarding number concepts in the current context. This research reflects that students' mental conditions and understanding are not yet fully mature enough to accept more complex number concepts. This is the same as what Nurani said: students' understanding of numbers has not been fully mastered, which causes students to be hampered in understanding further number material [25]. Furthermore, some students do not seem to understand the meaning of numbers, both as quantity and as a sequence, following the essence of the concept of numbers, as mathematicians explain. These findings also highlight the unpreparedness of some students regarding key aspects of number concepts, such as the difference between numbers and numerals, decomposing numbers, comparing numbers, and ordering numbers. Then, inconsistencies in students' answers often appear, indicating that students have difficulty justifying their understanding. This results in difficulties in facing the challenges of high-level number concept questions. Even though there are high conceptual demands, these difficulties are not only intellectual but also impact students' psychological aspects. Students' lack of emotional involvement in learning material about numbers also influences their motivation to understand this concept. This confirms the existence of ontogenic obstacles, including instrumental, conceptual, and psychological aspects of students' understanding of numbers. Therefore, a learning approach that considers these obstacles is needed so that students can overcome difficulties and develop a deeper understanding of number concepts.

As discussed previously, students' lack of maturity in understanding the concept of numbers has implications for the static and limited nature of the meaning of their concept of numbers. Students encounter difficulties in applying the meaning of their number concepts when facing various number concept problems in different contexts and forms. Students' understanding has not yet reached a level where they can grasp broader conceptual elements or see connections to deeper mathematical concepts. This can also be identified as a form of inconsistency in the meaning of the concept of number, as explained previously. Students' limitations in linking context and form in meaning the concept of number results in the formation of limited student knowledge regarding the concept of number. This situation indicates that the process of constructing the meaning of numbers in students has not been able to convey holistic and in-depth concepts. This view is
consistent with the opinion of Daulay, who emphasizes that students' understanding of a mathematical concept, in this case, the concept of numbers, requires a comprehensive series of learning processes [29]. This condition increasingly emphasizes the existence of epistemological obstacles that students face in understanding number concepts, as theorized by Brousseau [30].

Learning obstacles are a consequence of inappropriate didactic design [10]. This underscores the critical need for an educational design that aligns with student's learning needs and experiences. Identifying learning obstacles emphasizes the importance of crafting didactic approaches that are academically sound and sensitive to the diverse ways students comprehend and engage with educational content. Recognizing these obstacles becomes a call to action for educators and curriculum designers to develop pedagogical strategies that are responsive and tailored to address the varied challenges students may encounter during the learning process. Acknowledging learning obstacles is a compelling argument for a well-suited didactic design promoting effective and inclusive educational experiences.

Jean Piaget contributed the concepts of assimilation and accommodation in children's cognitive development, stating that knowledge is built in children's minds through these two processes [31]. Assimilation occurs when individuals encounter new information or experiences and attempt to integrate them into existing cognitive structures. This process can be observed in students during learning, where their cognitive structures are built from information obtained from textbooks or teacher intervention. However, in the context of learning number concepts, this assimilation seems to have been unsuccessful. As discussed previously, knowledge gained through textbooks containing many misconceptions can negatively impact students' knowledge construction. This is similar to the case found by Rohmah, namely that the textbook did not present the concept of numbers very clearly, and there were several errors [26]. This knowledge becomes a new structure that is used to build new knowledge. When students bring their misconceptions when facing new situations or information, they cannot accommodate previous knowledge with the new information. From the perspective of didactic situation theory developed by Brousseau, the lack of optimization of action situations and formulations in students seems to be a challenge in learning number concepts [30]. The students' learning process regarding numbers does not yet present correct action situations, so students formulate the wrong understanding. This brings us to the concept of didactical obstacles to numbers, especially related to the stages of presenting number material [30].

Gap analysis involves assessing existing studies on similar subjects conducted by various researchers. Based on several studies that have been carried out, it is revealed that understanding the concept of numbers is often faced with various problems that can hinder the mathematics learning process. One of the main problems encountered is students' difficulty internalizing abstract concepts of numbers, such as fractions or decimals [23]. The research results also show that some students experience difficulty in relating number concepts to everyday life situations, creating a gap between mathematical theory and its application in practical contexts. In addition, it was also found that the inability to master basic number operations, such as addition, subtraction, multiplication and division, is a serious obstacle to holistic understanding [24]. In this context, several studies have explored related topics. However, none have specifically focused on the in-depth analysis of learning obstacles undertaken by the researcher. The novelty of this study lies in its distinct approach to examining and dissecting learning obstacles within the given subject matter. While previous research may have covered related aspects, the specific emphasis on the comprehensive analysis of learning obstacles represents a unique and innovative
contribution. This distinction underscores the novelty of the current study and its potential to fill a crucial gap in the existing literature by providing valuable insights into the intricacies of learning obstacles within the specified domain.

The limitations of this research stem from the researcher's constraints in fully executing the Didactical Design Research (DDR) framework. This study is primarily an interpretative exploration within the DDR framework, focusing on understanding and interpreting the existing context. However, it does not extend into the critical phase of DDR, where the findings about learning obstacles are utilized to develop and implement specific didactic designs. The critical aspect of DDR involves a more proactive approach, incorporating the identified learning obstacles into creating instructional designs. Unfortunately, this study could not progress beyond the interpretative stage due to limitations such as time constraints or resource availability.

Consequently, the research lacks the comprehensive development of didactic designs based on insights from analyzing learning obstacles. While the interpretative phase provides valuable insights into understanding the context, the absence of a subsequent critical phase limits the practical applications of the study. Future research endeavours could overcome these limitations by conducting a more comprehensive DDR, including both interpretative and critical phases, to offer a more robust contribution to educational research.

4. CONCLUSION

The results of this research illustrate findings related to obstacles in learning number concepts among elementary school students. The analysis involves students' understanding of the meaning of numbers and numerals and identifying five types of learning obstacles: psychological, conceptual, instrumental, didactical, and epistemological. First, students' understanding of the meaning of numbers and numerals shows variations. Some students relate numbers to physical objects, showing deep understanding. In contrast, others have difficulty relating them concretely, including the emergence of problems in distinguishing numbers as a quantity and numbers as a sequence. Then, psychological ontogenic obstacles arise, especially in students who are less interested and unmotivated in studying mathematics or number material. Instrumental ontogenic obstacles are reflected in students' difficulty solving number identification problems.

Furthermore, conceptual ontogenic obstacles occur when students are unprepared to link new concepts with the knowledge they already have. Didactical obstacles, related to obstacles in the didactic system, arise from the design of learning materials. The results show that students' answers to the first question can be influenced by the design of the material in the textbook, which places more emphasis on calculations based on the type of object, causing students to experience epistemological obstacles. This research provides an in-depth understanding of elementary school students' obstacles in learning number concepts. The implications of these findings include the need to design appropriate didactic designs to overcome these obstacles. Recommendations for future research include further exploring effective learning designs and developing materials that support a comprehensive understanding of number concepts.

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