Al-Jabar: Jurnal Pendidikan Matematika
Volume 13, Number 2, 2022, Pages 231 – 249
http://ejournal.radenintan.ac.id/index.php/al-jabar/index

Exploration of high school students' reasoning in solving trigonometric function problems

Marufi Marufi[1]*, Muhammad Ilyas[1], Muhammad Ikram[1], Rosidah Rosidah[2], Phimlikid Kaewhanam[3]

[1] Department of Mathematics Education, Cokroaminoto Palopo University, Indonesia.
[2] Department of Postgraduate Mathematics Education, Universitas Negeri Makassar, Indonesia.
[3] Public Administration, Kalasin University, Thailand.
✉

Article Information
Submitted Aug 08, 2022
Revised Oct 19, 2022
Accepted Oct 29, 2022

Keywords
Reasoning;
Logical Thinking Skills;
Inductive;
Deductive.

Abstract

Reasoning has been extensively studied by many experts. However, Research on student reasoning in trigonometric problem solving, particularly those related to logical thinking skills is still sorely needed. This study aimed to explore students' reasoning in solving trigonometric function problems regarding logical thinking skills. The research was conducted using a qualitative approach. The research subjects involved high school students in Palopo, Indonesia. Based on the logical ability test results, three subjects were selected, namely students with high, medium, and low logical abilities. Research instruments in mathematical problem-solving tasks and interview guidelines are valid and reliable. Data collection was carried out through task-based interviews and think-aloud. The results of the study: (1) the reasoning subjects with high and moderate logical abilities in solving trigonometric function problems are the same in every type of question, always starting with inductive reasoning and then doing deductive reasoning (2) the reasoning of subjects with high and medium logical abilities is different in solving trigonometric function problems in the initial identification. Subjects with low logical ability showed no mental activity in solving trigonometric function problems. The research finding is that the subject has a high logical ability and is solving trigonometric function problems first by inductive reasoning and then deductive reasoning. In general, it is concluded that students with high and moderate logical abilities use inductive and deductive thinking patterns interchangeably in solving trigonometric function problems.

INTRODUCTION

The expected Mathematical Learning Objectives of the Curriculum in Indonesia focus on aspects of problem solving, reasoning, communication, as well as connection. The recent curriculum applied in Indonesia continues to be developed as the goal is to encourage the Programme International for Student Assessment (PISA) results of Indonesian students. (Elia et al., 2009). It should be noted that the assessment of mathematical literacy based on the PISA is intended to determine the students' reasoning in using concepts, procedures, facts and mathematical sets when describing, explaining and predicting phenomena (Fiallo & Gutiérrez, 2017). By this, it can be said that the curriculum, both in Indonesia and the PISA, have a focus on reasoning ability to boost critical thinking skills for the purpose of solving a problem from phenomena that occur around us with mathematical literacy assistance.

However, it cannot be denied that the results of previous studies reveal that these aspects have not been achieved based on the results of the PISA survey, due to the students'

How to cite: Marufi, M., Ilyas, M., Ikram, M., Rosidah, R., & Kaewhanam, P. (2022). The exploration of high school students' reasoning in solving trigonometric function problems. *Al-Jabar: Jurnal Pendidikan Matematika*, *13*(2), 231-249.
e-ISSN: 2540-7562.
Published by: Mathematics Education Department, UIN Raden Intan Lampung.

low ability in mathematics (Kamber et al., 2014; Syarifuddin & Rott, 2019). In addition, teachers concentrated too much on procedural, mechanical, teacher-centered things, and students are trained to solve manyproblems without a deep understanding (Lithner et al., 2015). As a result, most students only study to face the exam and frequently forgot about what they have learnt . What's more, students have been stuck on the assumption that mathematics is dominated by problem solving (Siyepu, 2015). Therefore, when the existing problems have been solved, it can be categorized that their proficiency is good . The influence is dreadful for students' thinking skills used as an instrument to measure students' attainment.

To measure the student's achievement, it can be traced through its reasoning process. In this regard, reasoning is an important mental activity for students such as proofing, solving problems that require mathematical reasoning, drawing generalizations or conjectures, and finding relationships among given facts (Ellis et al., 2017). Nevertheless, the majority of problems encountered by students with regard to reasoning have been identified by several previous studies, one of which is that when students solve contextual problems, they find it difficult to communicate ideas into mathematics language (Post et al., 2010). Furthermore, students' failure to reason is caused by their failure to understand mathematical concepts such as trigonometry (Moore, 2014b). Based on our preliminary study, students thought that trigonometry is a complex concept. They did not have a deep understanding about it because of the number of formulas that should be memorized or the less meaningful material in the learning process. Besides that, it was also found that students cannot connect the concept of trigonometry with the concepts of quadratic equations, limits, derivatives, matrices, and others (Çekmez, 2020).

Trigonometry is a branch of mathematics dealing with the sides and angles in a triangle. However, trigonometric material is frequently viewed as difficult and confusing material due to its relation in many disciplines, namely algebra, geometry, and graphics. In the curriculum of the intermediate level, students have studied trigonometry from algebraic functions. Moreover, at the university level, they have been introduced to trigonometric functions involving derivatives and anti-derivatives, exponents and logarithms, hyperbolic functions, and series and sequences. Hence, trigonometry is often used to build new ideas and concepts (Siyepu, 2013, 2015).

Numerous studies have revealed that many students are unable to develop their ideas in trigonometry, particularly the informal use of algebraic notation. Explains that most students in calculus classes show low performance in operating trigonometric expressions, e.g. when operating intermediate multiplication of $\sin x \times \sin x$. Siyepu (2013, 2015) reported that students tend to overgeneralize the principles of $f(a * b) = f(a) * f(b)$ for all cases, so they make mistakes while writing $\sin(a + b) = \sin a + \sin b$. Kamber & Takaci (2018) found that students had difficulty in understanding the periodic function of trigonometry when they faced inequality problems.

There are two causes for this error. First, the ineffectiveness of trigonometric learning in classroom (Tallman & Frank, 2020). Second, the curriculum does not suggest how students relate different representations of the trigonometry concept and reduce the use of textbooks emphasizing memorization and procedures (Fiallo & Gutiérrez, 2017). It is a priority to minimize students' error by designing learning that stimulates students to comprehend the concept of trigonometry (Mesa & Herbst, 2011). Efforts to overcome problems in teaching trigonometry resulted in some researchers proposing the needs for teachers and prospective teachers to develop quantitative and covariational reasoning (Moore, 2014b, 2014a; Moore et al., 2013), or reversible reasoning (Ikram et al., 2020) in the teaching of trigonometry. Furthermore, Tallman & Frank (2020) argued that teachers need to have disposition skill to support students' reasoning by emphasizing the coherence of the basic concepts of trigonometry.

Another fact is that only a few students successfully involve reasoning in solving trigonometric problems. In this case, the researcher wondered about how the reasoning process that students use when working on trigonometric problems. Therefore, this study has three contributions to the field of mathematics education: (1) It provides a framework for studying students' reasoning in solving trigonometry problems; (2) studies on reasoning can be used to design effective classroom learning; and (3) the results of this investigation have direct implications on the development of trigonometry curriculum in the classroom. Therefore, in this article, the researcher proposes to answer the following research questions: "*How is students' reasoning in solving trigonometric problems?*"

METHODS

The data obtained from this research came from the study with a qualitative approach. Therefore, the research used task-based interviews and think aloud as data collection methods. In a task-based interview, one participant met with an interviewer who introduced the task to the participant in a pre-planned way, using audio or video to capture verbal expressions of the participant which would later be analyzed, and sometimes it also recorded the participant's mental activity while solving a problem (Mejía-Ramos &Weber, 2020). Therefore, task-based interviews were semi-structured in the sense that the interviewer has a number of pre-planned questions to ask the participant during the interview. Furthermore, in this section, the researcher described the context of the research conducted, participants, the methods used to collect data, the design of the instruments that were used, and methods used to analyze data.

Participants and data Collection

Three students joining in this study were selected based on different logical thinking skills (high, average, and low). To collect data, it was conducted by way of task-based interviews and think aloud, where participants were given paper and pens to do some tasks while explaining their mental activity in solving trigonometry problems. Further in-depth interviews and observations were also carried out to explore the reasons why they reached the conclusions that they have made and other possible solutions that could be done. The ability of the interviewees was studied through the interpretation or representation that the participant provided in answering the interviewer's questions. The validity of data is an important concept in qualitative research. Examination of the validity of data aims to reduce the bias during the time of data collection. The validity of data in qualitative research includes tests of credibility, transferability, dependability, and confirmation (Miles et al., 2014).

Instrument

Three tasks were developed from the pre-assessment test, where all of them have gone through validation process by two experts in the field of mathematics education, both the validity of the form, content, and construct. All three problems had also gone through field tests, and the results showed that all three items could be used as references to explore the reasoning that students did during solving problems. Although, previously, some statements of the task were not understood and did not give rise to reasoning, so the researcher made modifications (e.g., Changing the type of the question into an open problem). All three tasks were used in think aloud sessions. The task is presented by Table 1.

Table 1. Tasks for Think Aloud

Task 1	Sketch the graph of the function $f(x) = -\frac{1}{2}\sin 2x$ and $g(x) = 2\sin 2x$ in the interval of $0 \leq x \leq 2\pi$.
Task 2	Let $\alpha, \beta,$ dan γ are the angles of a triangle. If $\sin\alpha = \frac{1}{2}$ and $\sin\beta = \frac{1}{3}$ where α is acute dan β is obtuse. Determine $\sin\gamma$ and $\cos\gamma$ and investigate whether γ is a an obtuse or an acute angle.
Task 3	Let $\alpha, \beta,$ dan γ are the angles of ABC triangle, then prove that: $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha \tan\beta \tan\gamma$ Clue: Use the sum of all three interior angles in a triangle equals to 180°

Analysis

All interview data was collected, and audio and video recordings were transcribed. Copies of student work were combined with each transcript. All forms of data were analyzed systematically and independently encoded according to the think aloud protocol that had been developed as an analysis tool. For data analysis, the researcher employed thematic analysis methods developed by Braun & Clarke (2006). Data analysis procedures involved open, axial, and selective coding processes for qualitative data (Creswell, 2012; Miles et al., 2014) and continuously involved a fixed comparison analysis between each new category and an emerging category. The researcher transcribed the think aloud data, interview transcripts and student answers. To analyze the data, transcripts of think aloud data and interviews were reduced to fragments containing student explanations (main ideas). The encoded data was sorted and read over and over again to obtain the answer to research questions.

The validity of the data was ensured by determining that that the collection of data was accurate and complete by managing tasks in written form and producing word-by-word transcripts of each think aloud and interview. In addition, validation of the coding and re-coding process of several components of reversible reasoning was conducted through discussions with one professor lecturer and two doctoral lecturers in the field of Mathematics education. In the results section, it was then discussed regarding the findings based on the emerging themes by looking for similarities and differences with previous research findings.

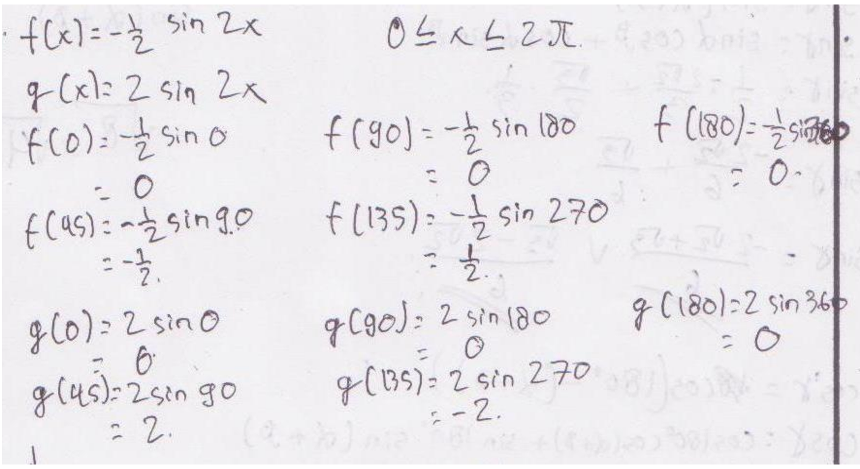
RESULTS AND DISCUSSION

This study explores students' reasoning in solving trigonometric function problems based on logical ability. The research results focused on subjects with high logical ability (ST) and moderate logical ability (SS). Meanwhile, this study's subjects with low logical ability (SR) were not studied in detail.

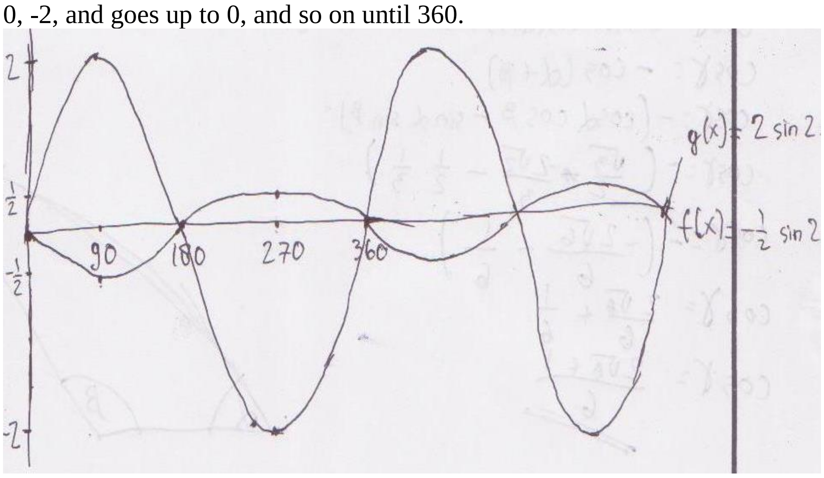
A. Students with High Logical Thinking Skills (ST)

Based on the results of data analysis, it was found that the subject with high logical ability processes information to determine the purpose of the problem, understand the situation, and respond to the questions asked. This study's results align with research conducted by (Ikram et al., 2020; Poon, 2012). Students perform mental activities by processing data about a given problem and generating initial perceptions. The data of subjects with high logical abilities are presented in table 2 below.

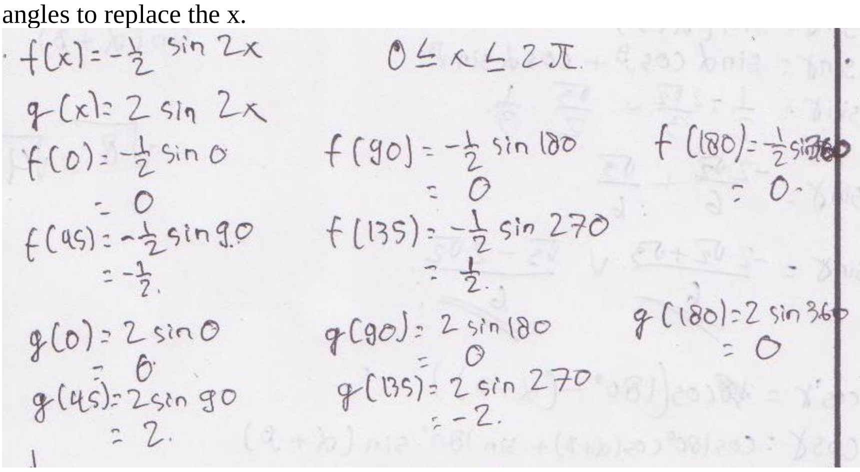
Table 2. Subjects with High Logical Ability

Code	Q/A	Details
ST1-007	Q	Tell me what's on your mind when you read question no 1?
ST1-007	A	Question no 1 asks me to sketch the graphs of function $f(x) = -\frac{1}{2}\sin 2x$ and of function $g(x) = 2\sin 2x$With an interval of between 0 and x and between 0 and 2π . The first thing that came to mind is that π is probably an angle, angle π=180. I must use angles to sketch graphs from functions $f(x)$ and $g(x)$. Then I attempted to recall that if trigonometry exists, specifically sin, every function contains sin. As a result, I may undoubtedly use unique angles such as 0, 90, 180, and 360 degrees.
⋮		
ST1-015	Q	Tell me what you will do to solve the problem?
ST1-015	A	I replace the x with one of the unique angles, 0, 90, 180, 270, 360, and so on for $f(x) = -\frac{1}{2}\sin 2x$ .To find the angle, I divide the number by 2: $\frac{0}{2} = 0$, for example $f(0) = -\frac{1}{2}\sin 0$, $\sin 0$ must be 0, $-\frac{1}{2}$ is multiplied by 0 to get 0. If I use 90 as the unique angle, then $\frac{90}{2}$ equals 45. I replace the x with 45 and do the multiplication $-\frac{1}{2}\sin 90$. $\sin 90$ is 1. So, $-\frac{1}{2}$ times one equals $-\frac{1}{2}$ and so on until 360. I did the same way on $f(x)$ by replacing x in $f(x)$ with 0, 45 because $g(x) = 2\sin 2x$, $\sin 2x$ in function $f(x)$ Use 0, 45, 90, and so on for the unique angles to replace the x. 
⋮		
ST1-018	Q	How did you create the graph?

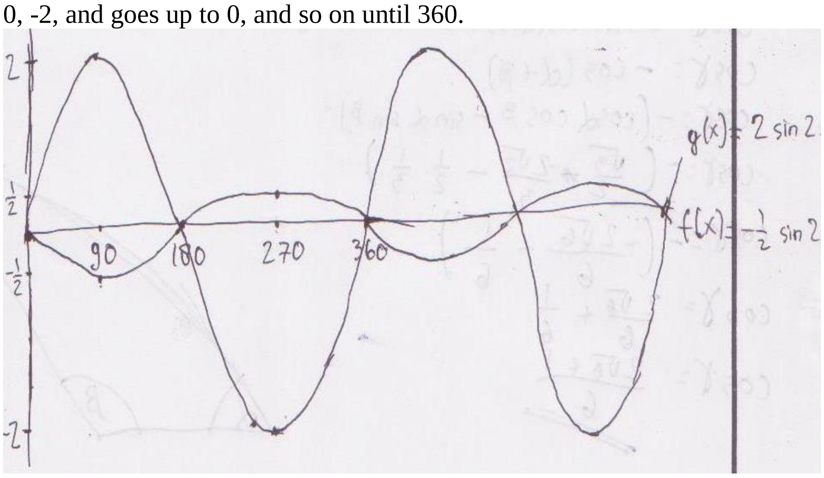
ST1-018	A	I obtained the graph from the function. For example, graph $f(x)$ starts from the bottom to 0 and $-\frac{1}{2}$And declines to 0 and $\frac{1}{2}$, then it returns to 0 and 360. For function $g(x)$, the graph starts from 0, reaches 90 at two as the peak, declines to 0, -2, and goes up to 0, and so on until 360.



Code	Q/A	Details
ST1-007	Q	Tell me what's on your mind when you read question no 1?
ST1-007	A	Question no 1 asks me to sketch the graphs of function $f(x) = -\frac{1}{2}\sin 2x$ and of function $g(x) = 2\sin 2x$With an interval of between 0 and x and between 0 and 2π . The first thing that came to mind is that π is probably an angle, angle π=180. I must use angles to sketch graphs from functions $f(x)$ and $g(x)$. Then I attempted to recall that if trigonometry exists, specifically sin, every function contains sin. As a result, I may undoubtedly use unique angles such as 0, 90, 180, and 360 degrees.
⋮		
ST1-015	Q	Tell me what you will do to solve the problem?
ST1-015	A	I replace the x with one of the unique angles, 0, 90, 180, 270, 360, and so on for $f(x) = -\frac{1}{2}\sin 2x$ .To find the angle, I divide the number by 2: $\frac{0}{2}$= 0, for example $f(0) = -\frac{1}{2}\sin 0$, $\sin 0$ must be 0, $-\frac{1}{2}$ is multiplied by 0 to get 0. If I use 90 as the unique angle, then $\frac{90}{2}$ equals 45. I replace the x with 45 and do the multiplication $-\frac{1}{2}$ $\sin 90$. $\sin 90$ is 1. So, $-\frac{1}{2}$ times one equals $-\frac{1}{2}$ and so on until 360. I did the same way on $f(x)$ by replacing x in $f(x)$ with 0, 45 because $g(x) = 2\sin 2x$, $\sin 2x$ in function $f(x)$ Use 0, 45, 90, and so on for the unique angles to replace the x.



Code	Q/A	Details
⋮		
ST1-018	Q	How did you create the graph?

ST1-018	A	I obtained the graph from the function. For example, graph $f(x)$ starts from the bottom to 0 and $-\frac{1}{2}$And declines to 0 and $\frac{1}{2}$, then it returns to 0 and 360. For function $g(x)$, the graph starts from 0, reaches 90 at two as the peak, declines to 0, -2, and goes up to 0, and so on until 360. 

Source: Author's Processed Data 2022

The data in Table 2 shows that before sketching a graph, ST tries first to make assumptions about certain angles from the graph of trigonometric functions. ST selects the maximum and minimum values and utilizes certain x values, especially 0, 90, 180, 270, and 360. The results of this study align with research conducted that students' reasoning is influenced by problem situations they have faced before. ST subjects sketched graphs of trigonometric functions by utilizing the maximum or minimum function values as an initial reference in sketching graphs. The ST subject then proved the first conjecture by identifying some of the ideas needed to define the function's graph. ST can determine the method used to solve the problem (García et al., 2011; Mcgowen & Tall, 2013; Rohimah & Prabawanto, 2019). ST students then utilize procedural skills to design graphs of functions by determining the values of trigonometric functions and using maximum and minimum values. Thus, the ST subject is involved in mental activity by establishing initial assumptions and applying the function's maximum and minimum values before drawing a trigonometric function's graph (Nabie et al., 2018).

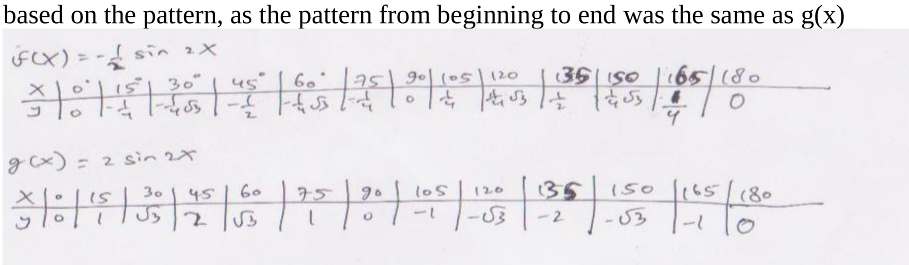
ST subjects imagine changing the value of a certain angle (variable x) or 0, 90, 180, 270, 360 adjusted by trigonometric functions to get x according to trigonometric functions. This study's results align with research that the subject builds initial guesses through assumptions and utilizes certain algebraic values to produce solutions. Thus, ST subjects use procedural skills to process data regarding graphs of trigonometric functions (Lithner, 2008; Poon, 2012). ST involves mental operations in the form of deductive reasoning to determine the value of trigonometric functions. Before sketching the function, ST examines the relationship between the function and (Martín-Fernández et al., 2019). The involvement of mental operations by deductive thinking also plays a role for the subject during the completion process to produce conclusions (Jeannotte & Kieran, 2017). Thus, ST subjects process information based on facts and perform mental activities by concluding the relationship between function and (Choy & Dindyal, 2018).

B. Student Low Logical Thinking Skills (SR)

Based on the results of the data analysis, it was found that the subject had a moderate logical ability, used visual skills to identify the problems at hand, remembered questions related to prior knowledge, and connected previous knowledge with the questions presented. This study's results align with research (Poon, 2012; Rohimah & Prabawanto, 2019) that students process information by taking previously acquired knowledge from long-term memory and relating it to the problems at hand. The data of logically capable subjects are presented in table 3 below.

Table 3. Subjects with Moderate Logical Ability

Code	Q/A	Details
SS1-006	Q	Tell me what's on your mind when you read question no 1?
SS1-006	A	First, it's written here "in an interval" (x is bigger than 0 and smaller than 2π). I insert the values into f(x)= $-\frac{1}{2}\ sin\ 2x$ and $g(x) = 2\ sin\ 2x$ to sketch a graph.
⋮		
SS1-010	Q	Tell me what steps you took to solve the problem?
SS1-010	A	It's written here f(x)= $-\frac{1}{2}\ sin\ 2x$ there is also $g(x) = 2\ sin\ 2x$ then the interval is (x is bigger than 0, with x is smaller than 360) $0 \leq x \leq 2\pi$, then I replaced the values with unique angles and put them into the table, for example, 0 is $-\frac{1}{2}\ sin\ 2x$ times 0 equals $sin\ 0$. $sin\ 0$ is 0, so 0 is multiplied by $-\frac{1}{2}$ equals 0, and so on until g(x). I drew a conclusion for the drawing based on it, and then I created the drawing based on the pattern, as the pattern from beginning to end was the same as g(x)



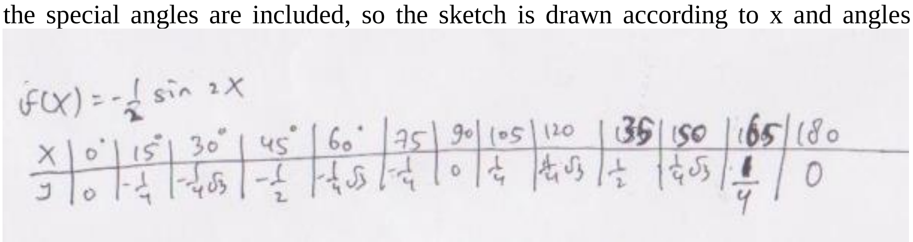
$f(x) = -\frac{1}{2} \sin 2x$

x	0°	15°	30°	45°	60°	75	90	105	120	135	150	165	180
y	0	$-\frac{1}{4}$	$-\frac{1}{4}\sqrt{3}$	$-\frac{1}{2}$	$-\frac{1}{4}\sqrt{3}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}\sqrt{3}$	$\frac{1}{2}$	$\frac{1}{4}\sqrt{3}$	$\frac{1}{4}$	0

$g(x) = 2 \sin 2x$

x	0	15	30	45	60	75	90	105	120	135	150	165	180
y	0	1	$\sqrt{3}$	2	$\sqrt{3}$	1	0	-1	$-\sqrt{3}$	-2	$-\sqrt{3}$	-1	0

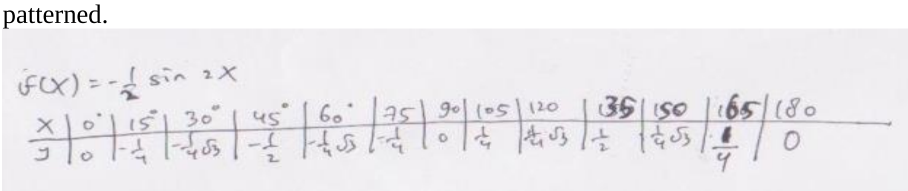
Code	Q/A	Details
⋮		
SS1-018	Q	Take a look at the picture why of each of the functions $\frac{1}{2}$ equals 2 . Why does the picture look like that?
SS1-018	A	Why $\frac{1}{2}$ equals $-\frac{1}{2}2$ equals -2? The values were obtained from a table in which all the special angles are included, so the sketch is drawn according to x and angles



$f(x) = -\frac{1}{2} \sin 2x$

x	0°	15°	30°	45°	60°	75	90	105	120	135	150	165	180
y	0	$-\frac{1}{4}$	$-\frac{1}{4}\sqrt{3}$	$-\frac{1}{2}$	$-\frac{1}{4}\sqrt{3}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}\sqrt{3}$	$\frac{1}{2}$	$\frac{1}{4}\sqrt{3}$	$\frac{1}{4}$	0

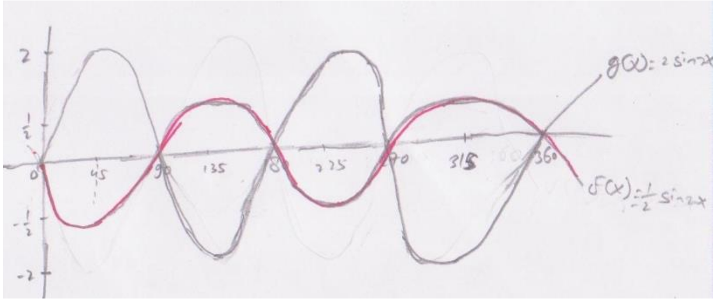
Code	Q/A	Details
⋮		
SS1-036	Q	So why did you write x=0? Why did you use 15, 30, 45, 60, and 75?
SS1-036	A	To get the closest interval, the graphs don't go too far as they are structured and patterned.



$f(x) = -\frac{1}{2} \sin 2x$

x	0°	15°	30°	45°	60°	75	90	105	120	135	150	165	180
y	0	$-\frac{1}{4}$	$-\frac{1}{4}\sqrt{3}$	$-\frac{1}{2}$	$-\frac{1}{4}\sqrt{3}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}\sqrt{3}$	$\frac{1}{2}$	$\frac{1}{4}\sqrt{3}$	$\frac{1}{4}$	0

Code	Q/A	Details
SS1-038	Q	How did you figure out that sin 0 equals 0, or sin 15 times 2 equals sin 30 equals $\frac{1}{2}$ to

		obtain $-\frac{1}{4}$. How did you obtain the values?
SS1-038	A	Because it's the standard that sin 0 is equal to 0, sin 30 is equal to $\frac{1}{2}$Sin 45 is equal to $\frac{1}{2}\sqrt{2}$And sin 60 is equal to $\frac{1}{2}\sqrt{3}$. If it's reversed, x 15, so $-\frac{1}{2}\sin 2x$ is equal to 15, which is used to replace x, so two times 15 equals 30. Sin 30 is equal to $\frac{1}{2}$ 

Source: Author's Processed Data 2022

Table 3 shows that SS students set their initial guess by trial and error before making a graph using the data tabulation technique. The results of this study are in line with the results of research conducted (Ikram et al., 2020) that the tendency of the subject's way of thinking is that every settlement process involves a trial and error strategy (Elia et al., 2009). Thus, the SS subject engages in mental activity based on the observed data and concludes by trial and error. SS generates a value for the x function by identifying the pattern and changing the x coefficient according to a certain angle from the first quadrant to the second quadrant. The subject tested several appropriate problem-solving strategies and involved a back-and-forth thought flow (Hohensee, 2016; Martin & Towers, 2016) or working back and forth. This shows that SS engages in mental activity by processing information and then using procedural abilities to identify the x value corresponding to a certain angle. SS students infer patterns from tables and graphs of trigonometric functions. SS subjects prepare solutions using inductive reasoning to graph functions using various generalizations and analogies (Sangwin & Jones, 2017).

Next, SS creates a pattern by entering the x value corresponding to a certain angle. In line with research (Rivera, FD, & Becker, 2019) that students identify these patterns stimulate the subject in the completion process to identify patterns that are adapted to the problem at hand (e.g., changing the coefficient of x according to a certain angle in the first quadrant to the second quadrant). Thus the subject has logical abilities and is involved in mental activities such as making inferences to graph trigonometric functions (Rivera, F. D., & Becker, 2019). SS also found a relationship between function and function. SS students complete the function using the same method, namely using a table strategy, using a table to find the x coefficient. Then find the pattern of the results obtained from the table. Therefore, the whole process of solving the subject indicates that problem-solving involves an inductive process (Jeannotte & Kieran, 2017). This shows that SS processes information based on facts and performs mental activities by concluding the relationship between functions. SS shows that the taken x value only reaches the second quadrant, after which SS determines the next value using the pattern. This indicates that the SS arrived at conclusions about the function's graph through mental activity and observation before drawing the graph. Thus it can be concluded that the subject of SS uses deductive and inductive reasoning in carrying out problem-solving plans and applying inductive thinking through generalizations and comparisons (Jeannotte & Kieran, 2017).

Based on the results of data analysis, it was found that each selected subject with low logical ability did not show any mental activity in reasoning every given problem, especially when given problems related to trigonometric functions. In line with Walkington's research, students with moderate and low abilities use the method used to solve a problem and tend to give less accurate answers because of the difficulty in finding mathematical problems (Walkington et al., 2018). Thus, the subject of SR has difficulty recognizing and understanding symbols and mistakes made in solving each given problem, namely a lack of understanding of symbols, place values, calculations, and using the wrong process. Several studies (Ikram et al., 2020; Lithner, 2017) show that subjects with a low logical ability (SR) who are selected lack of understanding of symbols shown in the inability of subjects with a low logical ability (SR), especially trigonometry, namely (1) not understand the meaning of the symbol in describing the graph of a function, (2) do not understand the meaning of the symbols and at the interval $0 \leq x \leq 2\pi$ in the problem of describing the graph of a function. Then the lack of understanding of calculation is shown in the inability of subjects with a low logical ability (SR) to perform simple arithmetic operations, especially when adding two fractions. Disturbances cause this, for example, 1) disturbances in spatial relations, (2) abnormal visual perception, (3) visual-motor associations, (4) perseveration, (5) difficulty recognizing and understanding symbols, (6) disturbances in appreciation of the body, (7) difficulties in language and reading (Ikram et al., 2020; Lithner, 2017).

C. The Difference between the ST and SS Responses

ST and SS students use analogy-inductive reasoning to solve trigonometric function problems. ST and SS made initial guesses and concluded that the procedures used to generate graphs of certain trigonometric functions could also be used to graph other functions (Wassie & Zergaw, 2018). ST and SS use different inductive reasoning techniques when planning to graph trigonometric functions. ST students use inductive-generalization reasoning to solve a graph of a trigonometric function, first determine the approach to solving the task and then determine the pattern to obtain the value of x using the maximum and minimum values of the function. In contrast, SS uses analogical reasoning and generalization, where generalization involves determining the approach to be followed and then identifying patterns in finding the value of x by manipulating the coefficient of the x variable. By analogy, SS makes conjectures about how to graph trigonometric functions (Misrom et al., 2020).

Regarding completing the task of drawing trigonometric graphs, ST and SS perform generalized inductive and syllogistic-deductive reasoning. The reasoning process expressed by ST and SS in generalization-inductive reasoning and syllogistic-deductive reasoning is different. In inductive-generalization reasoning, ST proves initial conjectures through procedural abilities and establishes the relationship between functions f(x) and g(x), then draws conclusions using the value of x that corresponds to a certain angle (0, 90, 180, 270, 360). Meanwhile, SS justifies its initial assumptions through procedural skills by determining the relationship between the functions f(x) and g(x), drawing conclusions based on the information obtained from the table, and using the coefficient x from the table to find patterns

(Hwang et al., 2020). Using syllogistic-deductive reasoning, ST determines how to graph trigonometric functions by showing that the special angles used in the function's graph are the angles that give the maximum and minimum values of the function. As a result, it is concluded that ST, before generating graphs of trigonometric functions, uses angles corresponding to the maximum and minimum values of the function.

Meanwhile, SS concludes by stating that if the value of x entered into a function matches the angles, a pattern will be generated to determine the result of the function. Thus, it can be said that SS uses the table technique when describing the graphs of trigonometric functions (Studies, 2022). The difference between the two subjects in responding to the task of drawing graphs of trigonometric functions can be seen in Table 4.

Table 4. Differences between the Two Subjects

ST	SS	Code
	Identifying the Problem	
ST revealed that the steps required to draw the graph are predictable.	SS made initial assumptions about drawing graphs of trigonometric functions by trial and error at special angles	DB-1
ST made initial assumptions about the unique angles used to draw a trigonometric function graph.		DS-1
	Planning the Solution	
ST tested the initial assumptions by utilizing the maximum and minimum values of the function as well as the values of x (0, 90, 180, 270, 360)	SS proved the initial assumptions by trial and error and displayed the data in a table before drawing the graph.	DS-2
	SS estimated the values of function x by determining a pattern by manipulating the coefficients of x that correspond to the unique angles at the first and second quadrants.	DB-2
	Executing the Problem-Solving Plan	
ST calculated the values of the trigonometric functions by utilizing their maximum and minimum values.	SS estimated the values of function x by determining a pattern by manipulating the coefficients of x that correspond to the unique angles at the first and second quadrants.	DB-3
ST estimated the values of x by changing the values of unique angles (x-variables) or adjusting 0, 90, 180, 270, and 360 to the trigonometric function.	SS estimated a pattern to determine the result of a function by inserting the values of x that correspond to the unique angles.	DS-3
ST examined the relationship between functions $f(x)$ and $g(x)$ and used the same strategy to solve both functions, namely using the values of x that correspond to the unique angles (0, 90, 180, 270, 360)	SS examined the relationship between functions f(x) and g(x) and used the same strategy to solve both functions, namely using a table, utilizing the coefficients of x, and determining a pattern of the function based on the table.	DS-4
	In presenting the table, SS revealed that the values of x were taken only from the second	DB-4

ST	SS	Code
	quadrant. Then, with his logical ability, SS attempted to find a pattern to determine subsequent values.	
	Reviewing the Solution	
ST reviewed his answer without referring to his notes (memos) and was convinced of the solution.	SS reviewed his answer by referring to his notes (memos) and was convinced of the solution.	DB-5

Source: Author's Processed Data 2022

The ideas generated by students vary, depending on their experience and knowledge. If it is associated with research results, ST and SS have different ideas or views in mathematics, especially in reasoning a problem. However, it cannot be denied that the reasoning process will be the same, and the difference is what each subject is reasoning as well as alternatives and strategies in solving problems, especially trigonometry. If it is associated with constructivism theory (Norton & D'Ambrosio, 2008), each individual builds his knowledge (constructs) his knowledge. Another result is that ST and SS reasoning is shown by making mathematical conjectures in starting or solving a trigonometric function problem. ST and SS found planning patterns for completion through mental activities and observations. If it is associated with the initial stage in solving the problem, ST and SS make initial observations and continue to emerge conclusions based on memory. The chronology of reasoning is mental activity starting from sensory observation or empirical observation (Adu-Gyamfi & Bossé, 2014). This process in mind produces many meanings and propositions at once. Based on similar sensory observations. This process is called reasoning because it is based on some known or considered true propositions and then used to conclude a new proposition that was previously unknown (Appel et al., 2020).

Deductive reasoning between ST and SS is seen at the stage of carrying out a settlement plan by calculating activities based on certain rules or formulas, drawing logical conclusions based on syllogistic rules, and compiling proof. In line with Rivera & Becker (2016) that activities classified as deductive reasoning, namely 1) carry out calculations based on certain rules or formulas; 2) draw logical conclusions based on rules, inference, check the validity of arguments, prove and construct valid arguments; 3) compiling direct proof, indirect proof and proof by mathematical induction. While at the re-examination stage, ST and SS subjects carefully and thoroughly evaluate the steps in completing (Syarifuddin et al., 2020). SS subjects retry the steps one by one carefully. In this case, the subject can distinguish between conclusions based on the truth of the solutions obtained. While the ST subject only repeats each step through imagery. However, what the two subjects did was not classified as a reasoning activity but only used procedural skills in re-examining the written steps (Norton & D'Ambrosio, 2008).

Based on this description, students' tendency to solve a problem, especially trigonometric functions, always begins with inductive reasoning, using deductive reasoning to solve the problem. This is in line with the results of previous research that students construct mathematical knowledge using an inductive mindset. Learning activities can start by presenting some examples of observed facts, listing traits that appear, and estimating possible results. Then students can be directed to make generalizations deductively (Čadež & Kolar, 2018). Furthermore, students can be asked to prove the generalizations obtained deductively if possible. In general, in solving trigonometric function problems, students use inductive and deductive thinking interchangeably (Syarifuddin et al., 2020).

The questions used in our study were (1) a question describing a graph, (2) a question using mathematical properties, (3) a proof question. The question of describing graphs was used to reveal how the student's reasoning in describing the graph of functions as well as what strategies were used in describing the graph of those functions. The question of using the trigonometry properties was used to investigate the participants' reasoning in relating information and the concepts associated in finding the relationships of each concept in solving the problem as well as provide several counterexamples and asking students to investigate whether the examples given fit the definition or problem given. Next, the proof question provided information that in the activity of proving, there is a reasoning activity, which is the process of mental activity which connects facts or evidence towards a conclusion. Through proof, students are required to be more careful in choosing what concepts are used to prove (Weber, 2008). In addition, the only thing that can guarantee the truth of a mathematical statement is deductive reasoning (Job & Schneider, 2014). Proof obtained through reason is deductively intended to establish the certainty of mathematical knowledge, but that certainty is not absolute. Therefore, truth can be obtained through proof, while proof requires reasoning.

In this study, the interviewees were a student who have high logical thinking skills (ST), a student with an average logical thinking skill (SS), and a student with low logical thinking ability (SR). The participants with low skills (SR) did not show the presence of mental activity in reasoning all trigonometry problems. In this case, SR did not provide information about the mental activity expressed in problem solving analytically. The participant also cannot determine the first step in problem solving and they could not understand the problem. Students with low logical thinking skills had difficulty in learning mathematics. It is in line with Lithner's opinion (2017) which said that the characteristics of learning difficulty in mathematics are (1) the presence of disturbances in the room, (2) abnormalization of visual perception, (3) visual-motor association, (4) perseveration, (5) difficulty in recognizing and understanding symbols, (6) impaired body perception, and (7) difficulty in linguistic and reading.

The participants with low logical ability (SR) had difficulty in recognizing and understanding symbols, as well as the errors made in solving each of them. A lack of understanding of symbols is evident in their incompetence, particularly in trigonometry, i.e. (1) did not understand the meaning of symbols π in describing function graphs, and (2) did not understand the meaning of symbols $\leq$ and $\geq$ at intervals $0 \leq x \leq 2\pi$ on problems describing function graphs. Then, a lack of understanding of calculation is evident in their inability in performing simple calculation operations, especially the addition of two fractions. These findings are supported by the results of Milda's research (2012) showing that there were many student errors including errors in interpreting concepts that are known and asked from the question to the form of sketches, errors in concepts of trigonometric rules, and errors in determining the results of calculations.

The ideas students generated were diverse, depending on their individual experiences and knowledge. If it is associated with the results of research, ST and SS have different ideas in mathematics, especially in reasoning a problem. However, it cannot be denied that the reasoning process conducted will be the same and the difference is what each participant analyzes as well as the alternatives and strategies in the process of solving problems, especially in trigonometry. The theory of constructivism (Norton & D'Ambrosio, 2008) saying that each individual builds his own knowledge supports their differences. Another result is that the understanding of ST and SS is shown by using guesses (make mathematical conjecture) in initiating to solve a problem. They solved trigonometry problems as well as finding patterns of planning completion through mental activity and observation. According to the preliminary stages in solving the problem, ST and SS made initial observations and continued to reach conclusions based on their memory. Chronology of the occurrence of reasoning is shown in mental activities starting from sensory observation or empirical observation (Adu-Gyamfi & Bossé, 2014). The process in the mind generates a number of understandings and propositions at once. Based on the observations of the same types, the process is called reasoning. The reason is based on several propositions known or considered true then used to infer a new proposition that is previously unknown.

Deductive reasoning between ST and SS is seen at the stage of conducting the plan by performing calculation activities based on particular rules or formulas, drawing logical conclusions based on rules, syllogisms, and drafting proofs. In line with the study by Rivera & Becker (2016), that activities are classified as deductive reasoning, namely 1) carrying out calculations based on certain rules or formulas; 2) drawing logical conclusions based on rules, inference, checking the validity of arguments, and proving and compiling valid arguments; and 3) compiling direct proofs, indirect proofs and proofs with mathematical induction. While at the stage of re-examining, ST and SS conducted an evaluation of the steps in completing carefully and thoroughly. SS re-tried the steps one by one carefully. In this case, SS was already able to distinguish between conclusions based on the truth of the solution obtained, while the ST participant simply repeats itself through every step through the game. However, what the two participants did was not classified as a reasoning activity but only used procedural capabilities in re-examining the written steps.

The study by Nike (2015) supports our findings. They said that students with normal and superior intelligence quotient (IQ) mostly used deductive and inductive reasoning in solving trigonometric problems. Moreover, research by Basir, Ubaidah, & Aminudin (2018) confirmed that students with low reasoning skill could not show the reasoning process or could not begin the initial stage of structuring also reported that the student who have a low skill could not even understand the problems.

Therefore, based on the description, the participants (ST and SS) have a tendency, especially in trigonometry problems, to begin by doing inductive reasoning and then using the deductive reasoning in solving the problem. It is in line with the results of previous research (Čadež & Kolar, 2018) that students construct mathematical knowledge by using an inductive mindset. For example, learning activities can begin by presenting some observed examples or facts, making a list of emerging principles, estimating outcomes, and then students can be directed to construct generalization deductively. Furthermore, if possible, students can be asked to prove the generalization obtained deductively. In general, students use an inductive-deductive mindset in solving problems. In those activities, they sometimes use only inductive or deductive mindset, but many problems involves both inductive and deductive mindsets interchangeably.

Reasoning and problem solving are two interconnected mathematic concepts. Based on the results of this study, through the stages of problem solving expressed by Polya (Kang, 2015) at each stage, students' reasoning can be revealed in solving problems, especially trigonometry. In this case, trigonometry is the focus of the problem faced by each student. It is in the form of the student's inability to relate information associated with trigonometry. In terms of students' error and inability in solving trigonometric problems, it can be drawn that one of the causes of students' error and inadequacy in solving trigonometry problems is teachers' skills in managing classroom learning or the factor related to students who pay less attention or inability reason and solve any given problems (Vamvakoussi, 2017). In Indonesia curriculum, the focus of mathematics learning is problem solving, yet they are stuck with the meaning of problem solving itself. As if mathematics is the problems that are always sought solutions to solve the problem. When the condition of the problem has been fulfilled or solved many students will imitate the same problem again if given a problem that is in accordance with the type of the problem. It shows that problem solving is an individual's effort to use his knowledge, skills, and understanding to solve a problem. Each student constructs his own knowledge in solving a problem.

To increase the students' reasoning level, one of the ways is to help students prepare as early as possible at the school by considering that mathematics is one of the largest contributors in shaping their logical thinking and in providing a large portion for students to sharpen their reasoning level. Teachers could begin by creating or giving a meaningful question or problem which involves a reasoning process, not just a simple question that requires the utilization of a certain formula.

Implication

The results of this study have implications for the theory of students' thinking processes in solving trigonometric problems based on logical thinking skills. Both subjects (ST and SS) provided descriptions regarding how they think in solving problems (e.g., graphs, algebra, and proof). This can be a reference in the learning process, where the problem becomes a reference for students to understand the concept of trigonometry in depth

Limitation

From the research findings, we are limited to subjects with low logical ability. This indicates that further investigation is needed. Especially about how the failures and difficulties experienced by these students in solving trigonometric problems. This can open new insights for further research to explore the case. Next, we find aspects that affect students' success in solving problems, for example, cognitive style, gender, or learning style. This is important to be integrated with future research.

Suggestiont

Based on these limitations, we suggest two things for future researchers. First, most students who fail to solve trigonometric problems are caused by a lack of students' logistical abilities.

Thus, the next researcher to examine in detail the causes of student failure in solving trigonometry problems, in students who think low. Second, the student's success factor is caused by other influencing aspects. This can help further regarding other influences that drive student success in solving problems

CONCLUSIONS

What the student with high logical thinking ability and the student with average logical thinking ability has in common in solving trigonometric problems are that they perform inductive reasoning at the stage of understanding the problem and planning a solution, then using deductive reasoning at the stage of carrying out the plan. The most striking difference is that the student with low logical thinking skill did not show reasoning process at all compared to the other two students.

Based on the results obtained in this study, there are some suggestions. The study participants are sometimes inconsistent in revealing what is analyzed when solving problems. Thus, it is recommended to use exploratory research with a qualitative approach to uncover the reasoning process of students who are inconsistent in solving problems. For relevant research, in order to re-examine the process of reasoning more fully, it is necessary to verify by: 1) connecting some trigonometric materials associated with materials such as quadratic equations, quadratic functions, sequences and series, three dimensions and others, 2) completing indicators of reasoning associated with logical words, 3) revising interview guidelines that directly reveal students' reasoning in detailed and structured problem solving, and 4) revising the proficiency test Logical thinking that directly categorizes prospective research participants based on their logical thinking skills

AUTHOR CONTRIBUTIONS STATEMENT

All the authors have made substantive contributions to the article and assume full responsibility for its content. All those who have made substantive contributions to the article have been named as authors.

REFERENCES

Adu-Gyamfi, K., & Bossé, M. J. (2014). Processes and reasoning in representations of linear functions. *International Journal of Science and Mathematics Education*, *12*(1), 167–192.

Appel, G., Grewal, L., Hadi, R., & Stephen, A. T. (2020). The future of social media in marketing. *Journal of the Academy of Marketing Science*, *48*(1), 79–95.

Braun, V., & Clarke, V. (2006). *Using thematic analysis in psychology: Qualitative reaserch in psychology* (Vol. 3). University of the West of England.

Čadež, T. H., & Kolar, V. M. (2018). How fifth-grade pupils reason about fractions: A reliance on part-whole subconstructs. *Educational Studies in Mathematics*, *99*(3), 335–357.

Çekmez, E. (2020). What generalizations do students achieve with respect to trigonometric functions in the transition from angles in degrees to real numbers?. *Journal of*

Mathematical Behavior, *58*(1), 100778.

Choy, B. H., & Dindyal, J. (2018). An approach to teach with variations: Using typical problems. *Avances de Investigacion En Educacion Matematica*, *13*, 21–38.

Creswell, J. W. (2012). Educational research: Planning, conducting, and evaluating quantitative and qualitative research. In *Educational Research* (Vol. 4).

Elia, I., van den Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. *ZDM - International Journal on Mathematics Education*, *41*(5), 605–618.

Ellis, A. B., Ozgur, Z., Vinsonhaler, R., Dogan, M. F., Carolan, T., Lockwood, E., Lynch, A., Sabouri, P., Knuth, E., & Zaslavsky, O. (2017). Student thinking with examples: The criteria-affordances-purposes-strategies framework. *Journal of Mathematical Behavior*, *53*(3), 263-283.

Fiallo, J., & Gutiérrez, A. (2017). Analysis of the cognitive unity or rupture between conjecture and proof when learning to prove on a grade 10 trigonometry course. *Educational Studies in Mathematics*, *96*(2), 145–167.

García, M., Llinares, S., & Sánchez-Matamoros, G. (2011). Characterizing thematized derivative schema by the underlying emergent structures. *International Journal of Science and Mathematics Education*, *9*(5), 1023–1045.

Hohensee, C. (2016). Teachers' awareness of the relationship between prior knowledge and new learning. *Journal for Research in Mathematics Education*, *47*(1), 17–27.

Hwang, W. Y., Zhao, L., Shadiev, R., Lin, L. K., Shih, T. K., & Chen, H. R. (2020). Exploring the effects of ubiquitous geometry learning in real situations. *Educational Technology Research and Development*, *68*(3), 1121–1147.

Ikram, M., Purwanto, Parta, I. N., & Susanto, H. (2020). Exploring the potential role of reversible reasoning: Cognitive research on inverse function problems in mathematics. *Journal for the Education of Gifted Young Scientists*, *8*(1), 591–611.

Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics*, *96*(1), 1-16.

Job, P., & Schneider, M. (2014). Empirical positivism, an epistemological obstacle in the learning of calculus. *ZDM - International Journal on Mathematics Education*, *46*(4), 635–646.

Kamber, D., & Takaci, D. (2018). On problematic aspects in learning trigonometry. *International Journal of Mathematical Education in Science and Technology*, *49*(2), 161–175.

Lithner, J. (2017). Principles for designing mathematical tasks that enhance imitative and creative reasoning. *ZDM - Mathematics Education*, *49*(6), 937–949.

Martín-Fernández, E., Ruiz-Hidalgo, J. F., & Rico, L. (2019). Meaning and understanding of school mathematical concepts by secondary students: The study of sine and cosine. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(12), 1-16.

Martin, L. C., & Towers, J. (2016). Folding back , thickening and mathematical met-befores. *Journal of Mathematical Behavior*, *43*, 89–97.

Mcgowen, M. A., & Tall, D. O. (2013). Flexible thinking and met-befores : Impact on learning mathematics. *Journal of Mathematical Behavior*, *32*(3), 527–537.

Mejía-Ramos, J. P., & Weber, K. (2020). Using task-based interviews to generate hypotheses about mathematical practice: Mathematics education research on mathematicians' use of examples in proof-related activities. *ZDM - Mathematics Education*, *52*(6), 1099–1112.

Mesa, V., & Herbst, P. (2011). Designing representations of trigonometry instruction to study the rationality of community college teaching. *ZDM - International Journal on Mathematics Education*, *43*(1), 41–52.

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative data analysis: A methods sourcebook* (Third Edit). SAGE Publications, Inc.

Misrom, N. B., Muhammad, A., Abdullah, A., Osman, S., Hamzah, M., & Fauzan, A. (2020). Enhancing students' higher-order thinking skills (HOTS) through an inductive reasoning strategy using geogebra. *International Journal of Emerging Technologies in Learning (iJET)*, *15*(3), 156-179.

Moore, K. C. (2014a). *Coherence, quantitative reasoning, and the trigonometry of students*. Pressbook.

Moore, K. C. (2014b). Quantitative reasoning and the sine function: The case of zac. *Journal for Research in Mathematics Education*, *45*(1), 102–138.

Moore, K. C., Paoletti, T., & Musgrave, S. (2013). Covariational reasoning and invariance among coordinate systems. *Journal of Mathematical Behavior*, *32*(3), 461–473.

Nabie, M. J., Akayuure, P., Ibrahim-Bariham, U. A., & Sofo, S. (2018). Trigonometric concepts: Pre-service teachers' perceptions and knowledge. *Journal on Mathematics Education*, *9*(2), 169–182.

Norton, A., & D'Ambrosio, B. S. (2008). ZPC and ZPD: Zones of teaching and learning. *Journal for Research in Mathematics Education*, *39*(3), 220–246.

Poon, K. K. (2012). Tour of a simple trigonometry problem. *International Journal of Mathematical Education in Science and Technology*, *43*(4), 449–461.

Post, T. R., Medhanie, A., Harwell, M., Norman, K. W., Dupuis, D. N., Muchlinski, T., Andersen, E., & Monson, D. (2010). The impact of prior mathematics achievement on the relationship between high school mathematics curricula and postsecondary mathematics performance, course-taking, and persistence. *Journal for Research in Mathematics Education*, *41*(3), 274–308.

Rivera, F. D., & Becker, J. R. (2016). Middle school students' patterning performance on semi-free generalization tasks. *Journal of Mathematical Behavior*, *43*, 53–69.

Rohimah, S. M., & Prabawanto, S. (2019). Student's difficulty identification in completing the problem of equation and trigonometry identities. *International Journal of Trends in Mathematics Education Research*, *2*(1), 34.

Sangwin, C. J., & Jones, I. (2017). Asymmetry in student achievement on multiple-choice and constructed-response items in reversible mathematics processes. *Educational Studies in Mathematics*, *94*(2), 205–222.

Siyepu, S. W. (2013). An exploration of students ' errors in derivatives in a university of technology. *Journal of Mathematical Behavior*, *32*(3), 577–592.

Siyepu, S. W. (2015). Analysis of errors in derivatives of trigonometric functions. *International Journal of STEM Education*, *2*(1), 16.

Studies, E. (2022). Upper secondary students ' mathematical reasoning on a sinusoidal function, *Educational Studies in Mathematics*, *99*(3), 277-291.

Syarifuddin, S., Nusantara, T., Qohar, A., & Muksar, M. (2020). Students' thinking processes connecting quantities in solving covariation mathematical problems in high school students of Indonesia. *Participatory Educational Research*, *7*(3), 59–78.

Tallman, M. A., & Frank, K. M. (2020). Angle measure, quantitative reasoning, and instructional coherence: an examination of the role of mathematical ways of thinking as a component of teachers' knowledge base. *Journal of Mathematics Teacher Education*, *23*(1), 69–95.

Vamvakoussi, X. (2017). Using analogies to facilitate conceptual change in mathematics learning. *ZDM - Mathematics Education*, *49*(4), 497–507.

Walkington, C., Clinton, V., & Shivraj, P. (2018). How readability factors are differentially associated with performance for students of different backgrounds when solving mathematics word problems. *American Educational Research Journal*, *55*(2), 362–414.

Wassie, Y. A., & Zergaw, G. A. (2018). Capabilities and contributions of the dynamic math software, geogebra---a review. *North American GeoGebra Journal*, *7*(1), 68–86.

Weber, K. (2008). How mathematicians determine if an argument is a valid proof. *Journal for Research in Mathematics Education*, *39*(4), 431–459.