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The Sufficient Conditions for Skew Generalized Power Series Module $M[[S,\omega]]$ to be $T[[S,\omega]]$-Noetherian $R[[S,\omega]]$-module

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Abstract

In this paper, we investigate the sufficient conditions for T[[S,ω]] to be a multiplicative subset of skew generalized power series ring R[[S,ω]], where R is a ring, $T \subseteq R$ a multiplicative set, $(S,\leq)$ a strictly ordered monoid, and $\omega : S \to End(R)$ a monoid homomorphism. Furthermore, we obtain sufficient conditions for skew generalized power series module M[[S,ω]] to be a T[[S,ω]]-Noetherian R[[S,ω]]-module, where M is an R-module.

Keywords: Monoid homomorphism; multiplicative set; skew generalized power series; strictly ordered monoid; T-Noetherian

INTRODUCTION

Mazurek and Ziembowski (2008) introduce the structure of Skew Generalized Power Series Rings (SGPSR) $R[[S,\omega]]$, where R is a ring, $T \subseteq R$ a multiplicatively closed set, $(S, \leq)$ a strictly ordered monoid and $\omega : S \to \text{End}(R)$ a monoid homomorphism. This ring is a generalization of the Generalized Power Series Rings (GPSR) $R[[S]]$, the result of the construction of Ribenboim (1990). SGPSR $R[[S,\omega]]$ is also a generalization of the semigroup ring $R[S]$, the Laurent ring $R[[X,X^{-1}]]$, the formal power series ring $R[[X]]$, the Laurent polynomial ring $R[X,X^{-1}]$, and the polynomial ring $R[X]$, by taking trivially ω, and some monoid S and partial order ≤. On the other hand, Varadarajan (2001) studies the structure of the Generalized Power Series Module (GPSM) $M[[S]]$ which is a module over GPSR $R[[S]]$, where M is a module over ring R. Furthermore, Varadarajan also determines the necessary and sufficient conditions for $M[[S]]$ to be a Noetherian module over $R[[S]]$.

The concept of Noetherian rings and modules can be seen in (Adkins & Weintraub, 2012), one of which is the sufficient conditions for the polynomial ring $R[X]$ to be a Noetherian ring. In the module structure, Varadarajan (1982) determines the sufficient conditions for the $R[X]$-module $M[X]$, $R[X, X^{-1}]$-module $M[X, X^{-1}]$, and $R[[X]]$-module $M[[X]]$ to be Noetherian. For the case of the noncommutative ring, the concept of Noetherian rings and modules can be seen in (Goodearl & Warfield, 2004) as well as (Lam, 2001). Furthermore, Gilmer (1984) shows that the semigroup ring $R[S]$ is Noetherian if and only if R is Noetherian and S is finitely generated. As a generalization of semigroup ring $R[S]$, Ribenboim (1992) determines the necessary and sufficient conditions for GPSR $R[[S]]$ to be a Noetherian ring.

As a generalization of the concept of Noetherian ring R and Noetherian modules over R, Anderson, and Dumitrescu (2002) introduce the concept of T-Noetherian rings and modules, where T is a multiplicative subset of ring R. For the noncommutative case, some properties of T-Noetherian rings and modules investigated by Baeck et al. (2016). Anderson and Dumitrescu also give sufficient conditions for $R[X]$ and $R[[X]]$ to be T-Noetherian rings, where $T \subseteq R$ is an anti-Archimedean multiplicative set and an anti-Archimedean multiplicative set containing nonzero divisors, respectively. On the other hand, Faisol et al. (2019 (1)) determine the sufficient conditions for $R[X]$-module $M[X]$ and $R[[X]]$-module $M[[X]]$ to be $T[X]$-Noetherian module and $T[[X]]$-Noetherian (respectively), where $T[X]$ and $T[[X]]$ are multiplicative subsets of $R[X]$ and $R[[X]]$ (respectively).

Furthermore, Zhongkui (2007) gives the necessary and sufficient conditions for GPSR $R[[S]]$ to be a T-Noetherian ring, while Faisol et al. (2019 (2)) determine the sufficient conditions for $R[[S]]$-module $M[[S]]$ to be $T[[S]]$-Noetherian, where $T[[S]]$ is a multiplicative subset of $R[[S]]$. These sufficient conditions are obtained by applying some of the properties that have been studied in Faisol et al. (2018).

Padashnik et al. (2016) give the necessary and sufficient conditions for GPSR $R[[S, \omega]]$ to be T-Noetherian. On the other hand, Faisol et al. (2016) determine the necessary conditions for Skew Generalized Power Series Module (SGPSM) $M[[S,\omega]]$ to be a T-Noetherian $R[[S,\omega]]$-module. But, sufficient conditions for $M[[S,\omega]]$ to be T-Noetherian have not been investigating to date. This fact gives us the motivation to investigate the sufficient conditions for SGPSM $M[[S,\omega]]$ to be a T-Noetherian $R[[S,\omega]]$-module.

The main results of this paper are the sufficient conditions for $M[[S,\omega]]$ to be T-Noetherian $R[[S,\omega]]$-module. Furthermore, we also obtain sufficient conditions for $M[[S,\omega]]$ to be a $T[[S,\omega]]$-Noetherian $R[[S,\omega]]$-module, by first determining the sufficient condition for $T[[S,\omega]]$ to be a multiplicative subset of $R[[S,\omega]]$.

THE RESEARCH METHODS

This research, based on the study of literature books and scientific journals, specifically relating to the concept of Noetherian rings and modules, T-Noetherian rings and modules, and SGPSR $R[[S,\omega]]$.

In the first stage, we investigate the impact of monoid homomorphism ω on the structure of SGPSR $R[[S,\omega]]$. These results are then used to determine the sufficient conditions for SGPSM $M[[S,\omega]]$ to be a finitely generated module over $R[[S,\omega]]$.

In the second step, we determine the sufficient conditions for $M[[S,\omega]]$ to be a Noetherian $R[[S,\omega]]$-module and a T-Noetherian $R[[S,\omega]]$-module. This is obtained by first determining the sufficient conditions of $M[[S,\omega]]$ is a finitely generated module.

In the final step, we give sufficient conditions for $M[[S,\omega]]$ to be a $T[[S,\omega]]$-Noetherian module. This is obtained by first determining the sufficient conditions of the set $T[[S,\omega]]$ is a multiplicative set in $R[[S,\omega]]$.

THE RESULTS OF THE RESEARCH AND THE DISCUSSION

In this section, we give sufficient conditions for SGPSM $M[[S,\omega]]$ to be Noetherian, T-Noetherian, and a $T[[S,\omega]]$-Noetherian $R[[S,\omega]]$-module. Before that, we review the construction of SGPSR $R[[S,\omega]]$, which is introduced by Mazurek and Ziembowski (2008). Furthermore, we give the structure of SGPSM $M[[S,\omega]]$ by following GPSM $M[[S]]$ construction method by Varadarajan (2001).

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Regarding ordered sets, strictly ordered monoids, Artinian and narrow sets, we will be following the terminology in (Ribenboim, 1990). $(S,\leq)$ is said to be a strictly ordered monoid if for any $s, s', u \in S$, $s < s'$ implies $su < s'u$ and $us < us'$. The sequence (s_n) in $(S,\leq)$ is said to be strictly ascending sequence if $s_n < s_{n+1}$ for $n = 1, 2, \ldots$, and it is said to be strictly descending sequence if $s_n > s_{n+1}$ for $n = 1, 2, \ldots$. A partially ordered set $(S,\leq)$ is said to be Artinian if every strictly descending sequence of elements of S are finite, and it is said to be *narrow* if S does not contain an infinite subset consisting of pairwise incomparable elements.

Now, we recall the construction of SGPSR $R[[S,\omega]]$. Let $(S,\leq)$ be a strictly ordered monoid, R a ring with identity element, and $\omega : S \rightarrow \text{End}(R)$ a monoid homomorphism. For any $s \in S$, $\omega(s)$ denoted by ω_s. In other word, ω_s is a ring homomorphism of R. If $1 \in S$ is an identity element, then $\omega_1 = id_R$ is an identity element of End(R). Let $R^S = \{f \mid f: S \rightarrow R\}$ and $R[[S,\omega]] = \{f \in R^S \mid \text{supp}(f) \text{ Artinian and narrow}\}$, where $\text{supp}(f) = \{s \in S \mid f(s) \neq 0\}$.

Since the set $\chi_s(f,g) = \{(x,y) \in \text{supp}(f) \times \text{supp}(g) | xy = s\}$ is finite, we can define the convolution multiplication on $R[[S,\omega]]$ by:

$$(fg)(s) = \sum_{(x,y)\in\chi_s(f,g)} f(x)\omega_x(g(y)), \qquad (1)$$

for every $s \in S$ and $f, g \in R[[S,\omega]]$. Under pointwise addition and convolution multiplication (1), $R[[S,\omega]]$ becomes a ring, which is called Skew Generalized Power Series Rings (SGPSR).

For any $r \in R$ and $s, x \in S$, we define the maps c_r, e_s: $S \rightarrow R$ by:

$$c_r(x) = \begin{cases} 1\,; \text{ if } x = 1 \\ 0\,; \text{ if } x \neq 1 \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

and

$$e_s(x) = \begin{cases} 1\,; \text{ if } x = s \\ 0\,; \text{ if } x \neq s \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Based on equations (2) and (3), $r \mapsto c_r$ is a ring embedding from R to $R[[S, \omega]]$, and $s \mapsto e_s$ is a monoid embedding from S to $R[[S, \omega]]$.

Next, we give the structure of SGPSM $M[[S,\omega]]$ over $R[[S,\omega]]$. Let $(S,\leq)$ be a strictly ordered monoid, R a commutative ring with identity element, M an R-module, and $\omega : S \rightarrow \text{End}(R)$ a monoid homomorphism. Let $M^S = \{f \mid f : S \rightarrow M\}$ and $M[[S,\omega]] = \{\alpha \in M^S | \text{supp}(\alpha)$ Artin and narrow$\}$, where $\text{supp}(\alpha) = \{s \in S \mid \alpha(s) \neq 0\}$.

Since the set $\chi_s(\alpha, f) = \{(x,y) \in \text{supp}(\alpha) \times \text{supp}(f) | xy = s\}$ is finite, we can define the scalar multiplication by:

$$(\alpha f)(s) = \sum_{(x,y)\in\chi_s(\alpha,f)} \alpha(x)\omega_x(f(y)), \qquad (4)$$

For every $s \in S$, $\alpha \in M[[S,\omega]]$ and $f \in R[[S,\omega]]$.

Under pointwise addition and scalar multiplication (4), $M[[S,\omega]]$ becomes a module over $R[[S,\omega]]$. This module is called Skew Generalized Power Series Module (SGPSM).

For any $m \in M$ and $s \in S$, we define a map $d_m^s: S \rightarrow M$ by:

$$d_m^s(x) = \begin{cases} m\,; \text{ if } x = s \\ 0\,; \text{ if } x \neq s \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

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Based on equation (5), it is clear that $m \mapsto d_m^0$ is a module embedding of M into M[[S,ω]].

For any ring R and $n \geq 1$, $R \oplus R \oplus \cdots \oplus R$ (n factor) is denoted by $\oplus_{i=1}^{n} R$. Necessary and sufficient conditions for R-module M to be a finitely generated module are given by following lemma.

Lemma 1. (Faisol, Surodjo, & Wahyuni, 2019(1)) *Let M be a module over a ring R. M is finitely generated if and only if it is isomorphic to a quotient of* $\oplus_{i=1}^{n} R$*, for some n*>0.

For any subset N of R-module M, let $N[[S,\omega]] = \{\alpha \in M[[S,\omega]] \mid \alpha(s) \in N; \forall\, s \in S\}$. This following lemma shows that N[[S,ω]] is an R[[S,ω]]-submodule of M[[S,ω]].

Lemma 2. *Let R be a ring, M an R-module,* (S,≤) *a strictly ordered monoid, and* $\omega : S \to \text{End}(R)$ *a monoid homomorphism. If N is an R-submodule of M, then N*[[S,ω]] *is an R*[[S,ω]]*-submodul of M*[[S,ω]].

Proof: For any $\alpha, \beta \in N[[S,\omega]]$ and $f, g \in R[[S,\omega]]$, we will show that $\alpha f + \beta g \in N[[S,\omega]]$. In other word, it is enough to show that $(\alpha f + \beta g)(s) \in N$, for all $s \in S$. For any $\alpha \in N[[S,\omega]]$, $f \in R[[S,\omega]]$, and $s \in S$, $(\alpha f)(s) = \sum_{uv=s} \alpha(u)\omega_u\big(f(v)\big)$. Since N is an R-submodule of M, we obtain $\alpha(u)\omega_u\big(f(v)\big) \in N$ for every $uv = s \in S$. Therefore, $(\alpha f)(s) \in N$ for every $s \in S$. In a similar way, for any $\beta \in N[[S,\omega]]$, $g \in R[[S,\omega]]$, and $s \in S$, we have $(\beta g)(s) \in N$. Hence, $(\alpha f + \beta g)(s) \in N$ for every $s \in S$. So, it is prove that $N[[S,\omega]]$ is an R[[S,ω]]-submodul of M[[S,ω]]. ■

In Lemma 2. above, if we take ω trivially, i.e. ω(s) = ω_1 for all $s \in S$, then we get Lemma 3.2. in (Faisol, Surodjo, & Wahyuni, 2019(2)). The following proposition shows that SGPSM with coefisien in module quotient of M/N is isomorphic to a module quotient of $M[[S,\omega]]/N[[S,\omega]]$.

Proposition 3. *Let R be a ring, M an R-module,* (S,≤) *a strictly ordered monoid, and* $\omega : S \to$ End(R) *a monoid homomorphism. If N is an R-submodule of M, then* $(M/N)[[S,\omega]] \cong M[[S,\omega]]/N[[S,\omega]]$.

Proof: For any R-submodule N of M, we define p_N as a natural projection. Next, we define a map $\varphi: M[[S,\omega]] \to (M/N)[[S,\omega]]$ by:

$$\alpha \longmapsto \overline{\alpha} = p_N \circ \alpha,$$

for every $\alpha \in M[[S,\omega]]$. It is clear that supp($\overline{\alpha}$) is Artinian and narrow, which is imply $\overline{\alpha} \in (M/N)[[S,\omega]]$. For any $\overline{\alpha} \in (M/N)[[S,\omega]]$, there is $\alpha \in M[[S,\omega]]$ such that $\varphi(a) = \overline{\alpha}$. Hence, φ is surjective. Therefore, $Im(\varphi) = (M/N)[[S,\omega]]$. Furthermore, if $\varphi(\alpha) = \overline{0}$ for any $\alpha \in M[[S,\omega]]$, then $\overline{\alpha} = p_N \circ \alpha = \overline{0}$. Therefore, $\alpha(s) \in N$ for all $s \in S$. So, $Ker(\varphi) = N[[S,\omega]]$. Then based on the Isomorphism Fundamental Theore, we obtain $(M/N)[[S,\omega]] \cong M[[S,\omega]]/N[[S,\omega]]$. ■

In Proposition 3. above, if we take ω trivially, then we get Proposition 3.3. in (Faisol, Surodjo, & Wahyuni, 2019(2)). If $\omega^{(1)}: S \to \text{End}(R_1)$ and $\omega^{(2)}: S \to \text{End}(R_2)$ are monoid

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homomorphisms, then $\omega^{(1)} \oplus \omega^{(2)}: S \rightarrow \text{End}(R_1 \oplus R_2)$ is also a monoid homomorphism with definition $\left(\omega^{(1)} \oplus \omega^{(2)}\right)_s(r_1, r_2) = \left(\omega_s^{(1)}(r_1), \omega_s^{(2)}(r_2)\right)$, for all $s \in S$ and $(r_1, r_2) \in R_1 \oplus R_2$ (Faisol, Surodjo, & Wahyuni, 2018).

The following Proposition shows that SGPSR with coefisien in $\bigoplus_{i=1}^{n} R$ is isomorphic to direct sum of SGPSR $R_1[[S, \omega^{(1)}]] \oplus \cdots \oplus R_n[[S, \omega^{(n)}]]$ (n factor). In the other side, the following proposition is a generalization of Proposition 2.7. in (Faisol, Surodjo, & Wahyuni, 2018).

Proposition 4. *Let R_i be a ring, $(S,\leq)$ a strictly ordered monoid, and $\omega^{(i)}: S \rightarrow \text{End}(R_i)$ a monoid homomorphism, with i =1,2,…n, then*

$$\left(\bigoplus_{i=1}^{n} R_i\right)[[S, \bigoplus_{i=1}^{n} \omega^{(i)}]] \cong \bigoplus_{i=1}^{n} \left(R_i[[S, \omega^{(i)}]]\right).$$

A direct consequence of Proposition 4. is Proposition 3.4. in (Faisol, Surodjo, & Wahyuni, 2019(2)) by taking ω trivially. Next, sufficient conditions for $R[[S, \omega]]$-modul $M[[S, \omega]]$ to be a finitely generated module are given by the following proposition.

Proposition 5. *Let R be a ring, M an R-module, $(S,\leq)$ a strictly ordered monoid, and $\omega : S \rightarrow \text{End}(R)$ a monoid homomorphism. If R-module M is finitely generated, then $M[[S,\omega]]$ is a finitely generated module over $R[[S,\omega]]$.*

Proof: Based on Lemma 1., it is enough to show that $M[[S, \omega]] \cong \left(\bigoplus_{i=1}^{n} (R[[S, \omega]])\right)/N$, for some submodule N of $\bigoplus_{i=1}^{n} R[[S, \omega]]$. Since M is finitely generated, based on Lemma 1., $M \cong (\bigoplus_{i=1}^{n} R)/K$, for some submodule K of $\bigoplus_{i=1}^{n} R$. Hence, based on Lemma 2., $K[[S, \omega]]$ is submodule of $(\bigoplus_{i=1}^{n} R)[[S, \bigoplus_{i=1}^{n} \omega]]$. Next, based on Proposition 4., we obtain $K[[S, \omega]]$ is submodule of $\bigoplus_{i=1}^{n} (R[[S, \omega]])$. Therefore, we can choose $N = K[[S, \omega]]$. Based on Proposition 3., we have

$$\left((\bigoplus_{i=1}^{n} R)/K\right)[[S, \omega]] \cong (\bigoplus_{i=1}^{n} R)[[S, \bigoplus_{i=1}^{n} \omega]]/K[[S, \omega]].$$

Moreover, by using Proposition 4., we get $\left((\bigoplus_{i=1}^{n} R)/K\right)[[S, \omega]] \cong (\bigoplus_{i=1}^{n} R)[[S, \bigoplus_{i=1}^{n} \omega]]/K[[S, \omega]] \cong \left(\bigoplus_{i=1}^{n} (R[[S, \omega]])\right)/K[[S, \omega]]$.

In the other word, $M[[S, \omega]] \cong \left(\bigoplus_{i=1}^{n} (R[[S, \omega]])\right)/N$, with $N = K[[S, \omega]]$ is submodule of $M[[S, \omega]]$ over $R[[S, \omega]]$. So, it is proving that $M[[S, \omega]]$ is a finitely generated module over $R[[S, \omega]]$. ■

The following theorem gives the sufficient conditions for SGPSM $M[[S,\omega]]$ to be a Noetherian $R[[S,\omega]]$-module.

Theorem 6. *Let R be a ring, M an R-module, $(S,\leq)$ a positive strictly ordered monoid, and $\omega : S \rightarrow \text{End}(R)$ a monoid homomorphism such that ω_s is an automorphism of R with $\omega_s\omega_u = \omega_u\omega_s$*

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for every $s,u \in S$*. If* R *is Noetherian,* M *and* S *are finitely generated, then* $M[[S,\omega]]$ *is a Noetherian* $R[[S,\omega]]$*-module.*

Proof: Since R is a Noetherian ring, $(S,\leq)$ a positive strictly ordered monoid, S finitely generated, and ω_s an automorphism of R such that $\omega_s\omega_u = \omega_u\omega_s$, for every $s,u \in S$, based on Theorem 3.2. (Padashnik, Moussavi, & Mousavi, 2016) $R[[S,\omega]]$ is a Noetherian ring. Furthermore, since M is a finitely generated module over R, based on Proposition 5. $M[[S,\omega]]$ is a finitely generated $R[[S,\omega]]$-module. Therefore, $M[[S,\omega]]$ is a Noetherian module over $R[[S,\omega]]$. ■

Next, for any subset T of a ring R, we define

$$T[[S,\omega]] = \{f \in R[[S,\omega]] | f(s) \in T; \forall s \in \text{supp}(f)\}.$$

It is clear that, $T[[S,\omega]] \subseteq R[[S,\omega]]$. The sufficient conditions for $T[[S,\omega]]$ to be multiplicatively closed subset of SGPSM $R[[S,\omega]]$ are given by the following lemma.

Lemma 7. *Let* R *be a ring,* $T \subseteq R$ *a multiplicative set,* $(S,\leq)$ *a strictly ordered monoid, and* $\omega : S \to \text{End}(R)$ *a monoid homomorphism. If* T *is closed under addition, then* $T[[S,\omega]]$ *is a multiplicative subset of* $R[[S,\omega]]$.

Proof: For any $f, g \in T[[S,\omega]]$, we will show that $fg \in T[[S,\omega]]$. Based on convolution multiplication on (1), for any $s \in \text{supp}(fg)$ we obtain $(fg)(s) = \sum_{xy=s} f(x)\omega_x\big(g(y)\big)$. Since $T \subseteq R$ is closed under multiplication, we get $f(x)\omega_x\big(g(y)\big) \in T$ for every $\text{xy} = \text{s} \in \text{supp}(fg)$. Next, since T is closed under additona, we have $\sum_{xy=s} f(x)\omega_x\big(g(y)\big) \in T$ for every $s \in \text{supp}(fg)$. In the other word, $fg \in T[[S,\omega]]$. So, it is proving that $T[[S,\omega]]$ is a multiplicative subset of $R[[S,\omega]]$. ■

Based on equation (2), it is easy to shoew that R is isomorphis to a subring $\{c_r \mid r \in R\}$ of $R[[S,\omega]]$. Hence, we get the following lemma.

Lemma 8. *If* T *is a multiplicative subset of a ring* R*, then* $C(T) = \{c_t \mid t \in T\}$ *is a multiplicative subset of* SGPSR $R[[S,\omega]]$, and $T \cong C(T) \subseteq T[[S,\omega]]$.

A multiplicative set $T \subseteq R$ is called anti-Archimedean if $\bigcap_{n\geq 1} t^n R \cap T \neq \emptyset$, for all $t \in T$ (Anderson, & Dumitrescu, 2002). The sufficient conditions for $M[[S,\omega]]$ to be a $T[[S,\omega]]$-Noetherian $R[[S,\omega]]$-module are given by the following theorem

Teorema 9. *Let* R *be a duo ring,* M *an* R*-module,* $T \subseteq R$ *is a denominator* ω_s*-anti-Archimedean consicting nonzero divisor,* $(S,\leq)$ *a commutative positive strictly ordered monoid, and* $\omega : S \to \text{End}(R)$ *is a monoid homomorphism such that* ω_s *is a monomorphism of* R*, with* $\omega_s\omega_u = \omega_u\omega_s$ *for every* $s,u \in S$*. If* R *is Noetherian,* $R[[S,\omega]]$ *is duo ring, and both* M *and* S *are finitely generated, then* $M[[S,\omega]]$ *is a* $T[[S,\omega]]$*-Noetherian module over* $R[[S,\omega]]$.

Proof: Since R is a duo ring and also Noetherian, $(S,\leq)$ a commutative positive strictly ordered monoid, $\omega : S \to \text{End}(R)$ a monoid homomorphism such that ω_s is a monomorphism of R, with

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$\omega_s\omega_u = \omega_u\omega_s$ for every $s,u \in S$, and $T \subseteq R$ a denominator ω_s-anti-Archimedean consicting nonzero divisor, based on Theorem 4.8. (Padashnik, Moussavi, & Mousavi, 2016) $R[[S,\omega]]$ is a T-Noetherian ring. Based on Lemma 8., $T \subseteq T[[S,\omega]]$. Therefore, based on *Remark* 2.11.(2) in (Baeck, Lee, & Lim, 2016), $R[[S,\omega]]$ is $T[[S,\omega]]$-Noetherian ring. Next, since M is a finitely generated module over R, based on Proposition 5. $M[[S,\omega]]$ is a finitely generated $R[[S,\omega]]$-module. Then, based on Lemma 2.14.(4) in (Baeck, Lee, & Lim, 2016), it is proving that $M[[S,\omega]]$ is $T[[S,\omega]]$-Noetherian. ■

CONCLUSION AND SUGGESTION

SGPSM $M[[S,\omega]]$ is a $T[[S,\omega]]$-Noetherian module over $R[[S,\omega]]$, if we give the following conditions: (1) Duo ring R that also Noetherian; (2) Commutative positive strictly ordered monoid $(S,\leq)$ that also finitely generated; (3) Monoid homomorphism $\omega : S \rightarrow \mathrm{End}(R)$ with ω_s is a monomorphism of R such that $\omega_s\omega_u = \omega_u\omega_s$ for all $s,u \in S$; (4) Semiring denominator ω_s-anti-Archimedean $T \subseteq R$ consisting nonzero divisors.

In this paper, determination of sufficient conditions for SGPSM $M[[S,w]]$ to be a $T[[S,w]]$-Noetherian module over $R[[S,w]]$ is done by applying the properties of a finitely generated module over T-Noetherian ring is a T-Noetherian module. In other words, the results of this paper depend on the sufficient conditions of $M[[S,w]]$ is a finitely generated $R[[S,w]]$-module. The use of module M as a T-Noetherian module over R as one of the sufficient conditions to show M [[S, □]] T-Noether is an open problem that can be studied further. Also, there are still opportunities to study the sufficient conditions of M[[S,□]] is T-Noetherian by applying the relation between the concept of an almost Noetherian module, an almost finitely generated module, and T-Noetherian module that has been studied by Faisol et al. (2019 (3)).

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