Do students' errors still occur in mathematical word problem-solving?: A newman error analysis

Marni Zulyanty¹, Ainun Mardia¹
✉ marnizulyanty@uinjambi.ac.id

Abstract
Mathematical word problems can be utilized to improve students' mathematical problem-solving skills. However, students' error still occurs in mathematical word problem-solving. This research aimed to trace and reveal students' errors in problem-solving using the Newman Error Analysis stages. This research is descriptive qualitative research. The research subjects were moderate-ability students of State Madrasah Tsanawiyah (MTs) in Jambi. Mathematical word problem worksheets and interview templates were used as instruments in this research. Students with the moderate ability category were given worksheets on algebraic and the Pythagorean Theorem operation. The students were also interviewed to get more information about the errors they experienced. This research found that the students' errors during word problem-solving had implications for the incorrect answer. Students' errors occurred at the comprehension, transformation, process skill, and encoding stages of the Newman Error Analysis stages. Indeed, the Newman Error Analysis stage is a cycle that means errors at the first stage are more likely to cause errors in the next stages and lead to an incorrect answer. Furthermore, error at the comprehension stage is the most crucial error in mathematical problem-solving.

INTRODUCTION
Mathematical word problems are a modification of questions widely used in mathematics learning. Mathematical word problems are different from other tasks in mathematics because the operations used are not explicitly described. The importance of mathematical word problems is seen from the fact that encouraging students to succeed in mathematics can be done by developing students' abilities in solving mathematical word problems (Gutstein, 2007). Mathematical word problems are very effective in improving math problem-solving skills. Mathematical problem-solving is fundamental and a standard part of the mathematics learning process. Mathematical problem-solving can make students create new ideas and find new techniques and procedures (Fadhilah & Alfiana, 2017). Math word problems are appropriate for measuring mathematical problem-solving abilities because they require different strategies and thinking that involve creative, logical, and directed thinking processes. Besides, word problems are considered appropriate because a problem solver needs more self-development through solving word problems (Lesh & English, 2005; Pongsakdi et al., 2020). After all, solving them requires goals and possible solutions for each process. Mathematical problem-solving abilities can also represent a complex level and mental activity from the aspects of an individual's psychomotor, cognitive, and affective domain (Fitriani et al., 2018; Montague et
Moreover, problem-solving abilities can reflect understanding and the ability to apply mathematical ideas in everyday life (Foreman-Murray & Fuchs, 2018). Thus, students who learn mathematics should be able to think mathematically in any learning, including solving problems (Tanujaya et al., 2017).

Based on PISA 2012 (OECD, 2014), Indonesia was ranked 64th out of 65 countries with a score of 375. In 2015 (OECD, 2016), Indonesia ranked 64 out of 72 countries with a score of 386. Indonesia's ranking on the PISA results illustrates that Indonesian students' mathematical problem-solving abilities are low. Besides, it was found that the mathematics scores of students at school for all levels were still low compared to the scores for other fields (Putra et al., 2017; Widodo et al., 2017). This result leads to a warning to get students used to mathematical problem-solving. Besides, the results of Tanujaya et al. (2017) show that most school students are at a low level in terms of students' ability to solve mathematical problems. The low ability is caused by many errors experienced by students during the search for solutions. The error during the process triggers the wrong solution, which leads to the students' common mathematical problem-solving ability.

Therefore, before getting students used to solving mathematical word problems, an analysis should be carried out to examine the errors experienced by students. This analysis is required to correct the errors, and students can get used to solving math word problem-solving. Even though the mistakes in solving these mathematical word problems can also be a clue to help students wisely in their thinking processes, it requires a more in-depth investigation of how students translate mathematical theory into practice, especially in solving math story problems (Goulet-Lyle et al., 2020). Also, Sari et al. (2018) suggested that the investigation of student errors can improve students' cognitive structure. Errors in problem-solving can be analyzed using the Newman Error Analysis stage. The Newman Error Analysis stage looks at the thinking process when solving mathematical word problems. The Newman Error Analysis stage can see the structure of the problem and facilitate metacognition awareness and mathematical problem-solving abilities (Reid O'Connor & Norton, 2020). Besides, according to White (2010), Newman Error Analysis is considered excellent and appropriate for analyzing errors because of its credibility. The Newman Error Analysis stage consists of 5 stages: reading, comprehension, transformation, process skills, and encoding.

Several studies on students' ability to solve mathematical word problems have been conducted, including research by Acosta-tello (2010) on students' abilities concerning the reliability of mathematical word problems. His research showed that mathematical word problems could be understood and solved if presented in the right language (Jha, 2012). Study showed that students with low grades have language deficiencies and a lack of conceptual comprehension in solving the problem. Apart from that, the relationship between mathematics word problem solver and mathematics word problem, as Garderen (2004) stated in their research, shows that an effective mathematical problem solver can achieve the purpose of the problem.

Students' errors have been detected, but the problems on algebraic operations and the Pythagorean Theorem have not been specified. This error tracing will be carried out by identifying the errors experienced by students and conducting a more profound analysis with the help of interviews. From this analysis, information and facts related to errors that occur in each stage of the Newman Error Analysis will be obtained.
METHODS
This study applied a descriptive qualitative approach to achieve the research objectives. Researchers are treated as the main instrument that will collect, process, and interpret data (Creswell, 2018). The supporting instruments are the mathematical word problem worksheets and interview templates. Word problem worksheets are made on algebraic operations and the Pythagorean Theorem. The word problems are designed following the essential competencies of each material. However, the interview template is prepared based on the Newman Error Analysis stage. As supporting instruments, the word problem worksheets and interview templates are validated by Mathematics Education experts before being used in the research. Therefore, the instrument is revised and returned to the validator until it is valid and can be used for research.

State Madrasah Tsanawiyah (MTs) students in Jambi with moderate ability are the subject of this research. The State Madrasah Tsanawiyah in Jambi city includes State Madrasah Tsanawiyah 1 Jambi, State Madrasah Tsanawiyah 2 Jambi, State Madrasah Tsanawiyah 3 Jambi, State Madrasah Tsanawiyah 4 Jambi, State Madrasah Tsanawiyah 5 Jambi, State Madrasah Tsanawiyah 6 Jambi. During the research, students with a moderate ability category were given mathematical word problems and time to solve them. After students completed the word problems, each student was interviewed according to the interview templates. The interview conducted was semi-structured. The total subjects in this research were 36 students representing six different schools. The determination of the category of moderate ability is based on the student's mathematical ability obtained directly from their school mathematics teachers.

In this research, there are several stages of data analysis (Creswell, 2018), namely: preparing and collecting data, developing and coding data, making a description of each code, presenting and reporting the results of the study, interpreting the results and validating the accuracy of the results. To check the validity of data, this research use triangulation. The types of triangulation are source triangulation and technique triangulation.

RESULTS AND DISCUSSION
Result of Working Word Problem Worksheet
The results of solving the word problem worksheet are classified into three assessment criteria, namely: True (T), False (F), and No Answer (NA). The results of the working word problem worksheet on the subject matter of algebraic operations and the Pythagorean Theorem are presented in Table 1.

<table>
<thead>
<tr>
<th>The Subject Matter</th>
<th>True (T)</th>
<th>%</th>
<th>False (F)</th>
<th>%</th>
<th>No Answer (NA)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic Operations</td>
<td>11</td>
<td>30</td>
<td>23</td>
<td>64</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>The Pythagorean Theorem</td>
<td>7</td>
<td>20</td>
<td>26</td>
<td>72</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 1. The Results of Working Word Problem Worksheet
Table 1 shows that students in the moderate ability category still experience errors in solving word problems about algebraic operations and the Pythagorean Theorem. Also, the percentage of errors that occur is considered high.

**Result of Research on Moderate Ability Students**

Students as the subject of this research consist of 36 students with moderate ability. Of the 36 students, 64% experience errors when solving algebraic word problems, and 72% experience errors when solving word problems about the Pythagorean Theorem. For the next step, only students who experience an error in solving mathematical word problems will be analyzed more deeply. In contrast, students who do not answer are not analyzed because there are no written answers. The errors occur both in all steps and in several steps of work. The error experienced by the moderate ability group in solving word problems on algebraic operations and the Pythagorean Theorem can be seen in Table 2.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Algebraic Operations</th>
<th>The Pythagorean Theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Comprehension</td>
<td>31</td>
<td>Interpretation</td>
</tr>
<tr>
<td>Transformation</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Addition Operations</td>
<td>Subtraction Operations</td>
</tr>
<tr>
<td></td>
<td>Multiples of 8</td>
<td>Multiplication Operations</td>
</tr>
<tr>
<td></td>
<td>Division by 2</td>
<td>Division Operations</td>
</tr>
<tr>
<td></td>
<td>Addition 8</td>
<td>Multiplication and Division Operations</td>
</tr>
<tr>
<td>Process Skill</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Algebraic Operations in Modelling</td>
<td>Algebraic Operations in Modelling</td>
</tr>
<tr>
<td>Encoding</td>
<td>69</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Interpretation</td>
<td>Interpretation</td>
</tr>
</tbody>
</table>

The examples of students' errors from a worksheet in an algebraic word problem can be seen in Figure 1. In Figure 1, it can be seen that various students' errors when answering algebraic word problems, including student answering using the exponential with basis 2. And the other student's errors when answering using addition operation without knowing the relation in three numbers.
The examples of students' errors from a worksheet in the Pythagorean Theorem word problem can be seen in Figure 2. In Figure 2, it can be seen that various student's errors when answering the Pythagorean Theorem word problem, including answering using the Pythagorean Theorem with an error claiming the length of the side of the triangle and error in Pythagorean Theorem because without squaring every length. And the other student’s errors when answering using addition, subtraction, and multiplication operations without connecting with the previous Pythagorean Theorem.

**Translated Version**

**Figure 1.** An example of a student’s errors in an algebraic word problem

Solution: Know: A child keeps three numbers secret

Asked: Find the sum of the three numbers

Answer: $2^a + 2^b + 2^c = 28 + 36 + 44$

= 108

= 2(a + b + c)

= 2(28 + 36 + 44)

= 2(108)

= 216

Solution: Know: The three numbers that are kept secret are the sum of each of the two numbers in a row, namely 28, 36, 44

Asked: Find the sum of the three numbers

Answer: $28 + 36 + 44$

= 64 + 44

= 108

Solution: Know: Stair length = 8 meter

Wall height = 4 meter

Stairfoot = 2 meters from the wall

Asked: Determine the length of the part of the ladder protruding above the wall

Answer: $\sqrt{4^2 + 2^2}$

= $\sqrt{16 + 4}$

= $\sqrt{20}$ = 4.5

= $8 - 2 = 6$ meter

Solution: Know: Long Stair 8 m

A High wall of 4 meter

The foot of the ladder is 2 meters from the wall

Asked: Determine the length of the part of the ladder protruding above the wall

Answer: $8m - 4m + 2m$

= $4m + 2m$

= $6m$

Solution: Know: $p = 8m$

$t = 4m$

$l = 2m$

Asked: length of stairs

Answer: $8 ÷ 4 = 2 \times 2$

= $4 m$
Figure 2. The Example Of Student’s Errors in The Pythagorean Theorem word problem

**Errors in Solving Algebraic Word Problem**

Twenty-three students (64%) out of 36 students in a moderate ability group experience errors when solving an algebraic word problem. Based on Table 2, several types of errors occur at each stage of the Newman Error Analysis. Of the five stages of Newman Error Analysis, students with moderate ability do not experience any errors at the reading stage. However, errors occur at four other stages. Further analysis based on the interview result related to the Newman Error Analysis stage can be seen in Table 3 below.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Types of Errors</th>
<th>Analysis based on the results of the interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Comprehension</td>
<td>Interpretation</td>
<td>The students misunderstand the problem, which says, “... the sum of the two numbers respectively is 28, 36, 44” as information that 28, 36, and 44 are the values of each number asked.</td>
</tr>
<tr>
<td>Transformation</td>
<td>Addition Operations</td>
<td>Students assume that to calculate the sum of the three numbers, they only need to add up the known numbers.</td>
</tr>
<tr>
<td></td>
<td>Multiples of</td>
<td>Students assume that 28, 36, and 44 as the multiples of 8 so that the sum of the three will also be a multiple of 8</td>
</tr>
<tr>
<td></td>
<td>Division by 2</td>
<td>Students think it would be easier to divide 28, 36, and 44 by 2 to determine the sum of the three numbers.</td>
</tr>
<tr>
<td></td>
<td>Addition 8</td>
<td>Students assume that the number series 28, 36, and 44 is the sum of adding 8 to each number as 28 + 8 = 36 and 36 + 8 = 44. Thus, the sum of the three numbers is 44 + 8 = 52.</td>
</tr>
<tr>
<td>Process Skill</td>
<td>Algebraic Operations in Modelling</td>
<td>Students found that the value of ( y = 10 ) turned false when they solve for the value of another variable with equation ( z = -8 - y ). It is changed to ( z = 8y ).</td>
</tr>
<tr>
<td>Encoding</td>
<td>Interpretation</td>
<td>The students also believe drawing and writing a conclusion from the value obtained is unnecessary because the result has been known.</td>
</tr>
</tbody>
</table>

**Errors in Solving Pythagorean Theorem Word Problem**

26 (72%) of the 36 students from the moderate ability group experience errors when solving algebraic word problems. Based on Table 2, several types of errors occur at each stage of the Newman Error Analysis. Of the five stages of the Newman Error Analysis, students with moderate ability avoided errors at the reading and comprehension stages and experienced errors at the other stages. Further analysis based on the results of interviews with the Newman Error Analysis stage can be seen in Table 4 below.
Table 4. Errors analysis based on the results of the interview

<table>
<thead>
<tr>
<th>Stage</th>
<th>Types of Errors</th>
<th>Analysis based on the results of the interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Comprehension</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Transformation</td>
<td>Subtraction Operations</td>
<td>Since the ladder length is 8(m) and the base is 2(m) away from the base of the building, students misunderstood to subtract the length of the ladder and the distance between the ladder base and the building. The calculation is 8(m) - 2(m) = 6(m). After that, they subtract the result and the length of the ladder. It is known that 6(m) - 4(m) = 2(m)</td>
</tr>
<tr>
<td>Multiplication</td>
<td>Operations</td>
<td>The students find the length of the overhang part of the ladder with the formulation (l \times h \times w = 8 \times 4 \times 2 = 64 \text{ m}), where (l) is the length, (h) is the height, and (w) is the width or distance.</td>
</tr>
<tr>
<td>Division Operations</td>
<td></td>
<td>The students find the length of the overhang part of the ladder as 8 (\div 4 \div 2 = 1 \text{ m})</td>
</tr>
<tr>
<td>Multiplication and</td>
<td>Division Operations</td>
<td>Students then divide the length of the ladder by the height of the wall and then multiply by the distance of the stairs. The calculation is 8 (\div 4 = 2 \times 2 = 4)</td>
</tr>
</tbody>
</table>
| Process Skill          | Algebraic Operations in Modelling | ➢ Students calculate the length of the overhang part of the ladder by subtracting the value found with the Pythagorean calculation  
\(m = \sqrt{20}\). As a result of the operation, students found that the length of the overhanging ladder is \(\sqrt{20} - 8 \text{ m}\)  
➢ Students assume that \(\sqrt{4 + 2} = \sqrt{8} = 16\)                                                                                   |
| Encoding               | Interpretation          | Students decide it is not essential to draw and write a conclusion from the value obtained because the answer to the word problem has already been solved.                                      |

Based on the explanation above, the students' types of errors and perceptions towards the word problem solving are identified. However, this error leads to an incorrect answer. Of the five stages of Newman Error Analysis, students' error occurs at the comprehension stage, the problem transformation stage, the process skill stage, and the encoding stage. Simply put, the only stage where the students did not experience errors is the reading stage. Therefore, this fact is very concerning because these errors have implications for incorrect problem-solving.

Students' errors at the comprehension stage are caused by students' inability to understand the core information and determine the problem's goal. At the same time, the comprehension stage is an essential factor in solving mathematical word problems, which is not
only a process of numerical manipulation and symbols but also the process of interpreting and understanding deeper mathematical word problems (Duru & Koklu, 2011; Singh et al., 2010). Generally, the comprehension stage is drawing or sketching a problem from a given story problem (Xin, 2018). This finding follows the research results by Wu & Adams (2006), who found that problem understanding is crucial to solving problems. Jha (2012); Prakitipong & Nakamura (2006) suggested that errors in the comprehension stage occur when students can read the questions but cannot fully understand the problem. This condition becomes a matter of concern because comprehension is crucial to word problem-solving (Garderen, 2004).

As well as at the comprehension stage students also experience errors at the problem transformation stage. This stage is related to how the information presented in the word problem is related to the choice of strategy or the formula for the solution. At the comprehension stage, errors occur when students are determining the strategy to solve the problem as the result of students being failed to determine the relationship between the information in the problem. This condition is in line with the results of research by Radatz (2013), which showed that mathematical language is one of the dominant factors of students' errors in word-problem solving. Moreover, Jha (2012), Prakitipong & Nakamura (2006), Singh et al. (2010) explained that errors at the transformation stage occur when students fail to determine the mathematical operations and procedures. Besides, Singh (2004) also argued that sharpness in mathematics is about reading problems and determining the course of a solution to a mathematical problem. Therefore, selecting strategies for solving mathematical problems related to mathematical knowledge content is a form of interpretation and comprehension of mathematics (Parvanehnezhad & Clarkson, 2008).

Another stage of the Newman Error Analysis, where students experience errors, is the process skill stage. This stage is related to skills in performing selected mathematical formulas. Based on students' answers, the errors in this stage occur in algebraic operations, and they misrepresent and operate mathematical calculations appropriately. Jha (2012) and Singh et al. (2010) found that errors in the process skill stage occur when students can determine the mathematical operations or formulas but cannot understand the order of operations completely.

Students also experience errors at the encoding stage. At this stage, students fail to conclude the goal of the word problem. It means that students cannot interpret the solution from the mathematical process. This finding is in line with the results of Wijaya et al. (2014) that the encoding phase in the Newman error stage is the final stage in the process of word-problem solving, which interprets the solution in the form of real-world problem situations. That the modeling process of the problem-solving process begins with a real-world problem and ends up with a real-world problem as well. In other words, interpreting and returning the solutions found from mathematical operations into the word problem context is considered the final stage of the whole process. Sepeng & Madzorera (2014), in their research, also found the fact that mathematical language is crucial to students' understanding of the problems in solving a mathematical word problem. Therefore, mathematical language is used from the beginning to the conclusions related to the context of the problem.

Students' errors that begin in the comprehension stage to the encoding stage are interrelated and lead to incorrect answers or solutions. However, following the results of research by Radatz (2013), every error in solving a mathematical problem is interrelated and can cause new errors that impact the wrong answer or solution. Nuryadin et al (2014) also found
that the causes of students' errors in solving mathematical problems were students' weak understanding of mathematical problems, and students were not used to solving mathematical word problems, which are considered non-routine problems. The implication of this research is the discovery of the most dominant stage of errors in solving mathematic word problems based on the Newman Error Analysis stages. So with this fact, it is hoped that the teachers can improve their ability to help students minimize these errors, and teachers must also help students used to solve mathematical word problems.

CONCLUSIONS
Students still experience errors in solving mathematical word problems, particularly in algebraic and the Pythagorean Theorem operation. Students' errors while working on word problems certainly affect error solutions. Following the Newman Error Analysis stage, students' errors in solving mathematical word problems occur at the comprehension, transformation, process skill, and encoding stages. In other words, it is only at the reading stage that students do not experience errors. However, the Newman Error Analysis stage is a cycle of unity that makes errors in one stage can trigger the occurrence of errors in the next stages and lead to incorrect problem-solving.

Furthermore, error at the comprehension stage is the most crucial error in mathematical problem-solving. The limitations of this study are that it only used one theory of error analysis. The next research expected is to develop an analysis of student errors in solving mathematical word problems based on other error theories and a larger number of subjects.

AUTHOR CONTRIBUTIONS STATEMENT
MZ developed ideas, methods, and data analysis. AM was responsible for designing the instruments and collecting the data.

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