Exploring mathematical technological knowledge of teachers with Geogebra application

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Abstract
Geogebra is a mathematical software that can be used in mathematics, especially geometry, calculus, and dynamic algebra, with various features, especially visualization. Geogebra helps students identify and improve their visual skills in understanding mathematical concepts. This study aims to explore the mathematical technology knowledge of teachers based on Geo-Gebra applications in mathematics learning by integrating content and technology. This research is qualitative research that uses a single-case design to explore the teacher's mathematical technology knowledge and involved teachers (N = 8) from the field of mathematics at universities in the Central Region of Indonesia. Three types of data were collected during one semester, weekly observations, progress report description; and the developed learning design. This research shows that there are two categories of teachers' mathematical technology knowledge characteristics. First, moderate teachers produce simple designs and are still influenced by numerical and algebraic approaches. Second, expert teachers can use analytical skills to design step-by-step learning, involving animation to explain concepts, and the resulting designs are quite complex and structured. These two findings are useful for building teacher competencies regarding using GeoGebra application-based technology in learning mathematics.

INTRODUCTION
In Indonesia, the contemporary era of digitization has resulted in changes to the mathematics curriculum (Lubis et al., 2022). The demands of mathematics teachers on the use of technology in learning are getting bigger, mathematics teachers must meet the needs of society in an era of constantly changing technology and be experts in content and teaching skills (Foulger et al., 2017). In particular, the use of technology impacts the learning and teaching of mathematics (Misfeldt & Zacho, 2016). In addition, national policies require that technology in learning requires teachers to increase their knowledge (Granberg, 2016). Teacher demands on the use of technology encourage researchers in the field of education to re-examine the theory of knowledge teachers need in learning (Hernández et al., 2020). In this case, teachers not only need to develop Pedagogical Content Knowledge (PCK) and Mathematical Knowledge for Teaching (MKT) but need Technological Pedagogical Content Knowledge (TPACK) knowledge (Zambak & Tyminski, 2020). Therefore, TPACK needs to be developed by teachers to answer the demands of curriculum changes. So, this paper try to elaborate mathematical technology knowledge in teachings by using geogebra application. First, the teacher's obligation to use technology in mathematics at primary and secondary levels. Second, teacher demands to increase innovation, creativity, and collaboration in learning.

The use of technology in learning is an important part of improving students' understanding, reasoning, and creativity in the classroom, this is a new perspective on the importance of integrating digital technology and visualization into mathematics learning so that
teachers have the opportunity to align learning objectives and expected outcomes for students (Bowers & Stephens, 2011). Several digital tools have been introduced and greatly impacted content, learning, and teaching mathematics, such as dynamic geometry, computer algebra, spreadsheets, and GeoGebra (Misfledt & Zacho, 2016). However, most teachers have difficulty using digital technology in learning because of their inability to incorporate technology into mathematics learning (Psycharis & Kalogeria, 2018). This also happens in Indonesia, where most teachers perceive the use of digital technology as difficult to realize. Therefore, digital technology in mathematics learning requires capacity building among mathematics teachers. This raises the question of how to align mathematics learning with effective technology.

The effective use of information technology in the curriculum is important for integrating technology and content in learning. The mathematics curriculum in various countries refers to the need and importance of integrating technology into mathematics learning (Yildiz & Gokcek, 2018). In increasing students' interest in presenting learning materials, teacher creativity is required to use technology and information-based learning media (Leow & Neo, 2014). One application that is very helpful in learning mathematics is GeoGebra. GeoGebra is a mathematical software that can be used in mathematics, especially geometry, calculus, and dynamic algebra, with various features, especially visualization (Reis & Ozdemir, 2010). The use of GeoGebra in learning mathematics is very helpful for teachers to explain mathematical material in the form of interesting visualizations so that it can affect students' understanding (Granberg, 2016; Misfeldt & Zacho, 2016; Yildiz & Gokcek, 2018). GeoGebra is a cognitive tool to increase teacher knowledge and guide students to learn geometric concepts.

GeoGebra helps students identify and improve their visual skills in understanding mathematical concepts (Granberg & Olsson, 2015). Furthermore, GeoGebra allows users to create graphs by writing algebraic formulas and displays algebraic and graphical representations. If anything is added or changed in the algebraic representation, the graphical representation will automatically change. GeoGebra can also build knowledge about linear functions, where students will understand when a function has one solution, no solution, and many solutions. The observations made in April 2021 on 20 mathematics teachers in junior high schools (Marufi et al., 2022) showed that only 40% of teachers could use the GeoGebra application. This provides information that exploring the teacher's mathematical and technological knowledge needs attention to be studied in depth.

Several previous studies in mathematics, which focus on technology-based mathematics learning, have investigated how students acquire mathematical knowledge (Pierce & Stacey, 2013). However, this focus has shifted to involving teacher knowledge in learning in recent years (Mata-Pereira & da Ponte, 2017). The use of various technologies is complex for teachers in learning mathematics (Maher et al., 2014). Meanwhile, according to Okumus (2016), this can allow teachers to explore and reflect on the various capabilities of technology to build connections and examine various representations of mathematical concepts (Okumuş et al., 2016).

The teacher's knowledge framework, referred to as Mathematical Knowledge for Teaching, has received considerable attention in mathematics education (Jankvist et al., 2020) (Ball et al., 2008). The impact of technology on mathematical content knowledge has been discovered by previous research (Zambak & Tyminski, 2020). In addition, several previous studies have linked knowledge of mathematical content to the TPACK framework. Therefore, the demands of integrating technology challenge teachers to explore mathematical concepts that are stronger and more in-depth understanding (Govender, 2018). In addition, technological knowledge encourages teachers to participate in modeling the representation of the material being taught (Psycharis & Kalogeria, 2018). Furthermore, technology in learning encourages teachers to integrate technology with pedagogy (Wachira & Keengwe, 2011). However, the literature discussing technology integration with learning is still limited. This is what underlies
the research that aims to explore the mathematical technology knowledge of teachers using the GeoGebra application.

**METHODS**

*Design*

Geogebra application in learning mathematics through a qualitative research approach with a single-case design so that it can provide an overview of the extent to which mathematics teachers can utilize technology in the learning process. So in this study the author uses a single-case design approach developed by Yin (2011), the single case design is a method of conducting research studies that focuses on conducting research only on a single case unit at a time. making it simpler for the author to devote more on the research. This study aims to explore the mathematical technology knowledge of teachers based on Geo-Gebra applications in mathematics learning by integrating content and technology. This research is a qualitative research that uses a single-case design to explore the teacher's mathematical technology knowledge (Yin, 2011).

*Participants*

This research was conducted in the odd semester of the 2021/2022 academic year, starting from September to December 2021. The research subjects involved teachers (N = 8) from the field of mathematics at universities in the Central Region of Indonesia in South Sulawesi. Research subjects are registered as postgraduate students in mathematics education aged 27 – 31 and have attended a series of lectures ranging from developing models and evaluating learning to learning design. In addition, the subjects had diverse educational backgrounds and teaching experiences, but one unifying factor was their interest in technology development in education (Table 1 shows the participants' backgrounds).

<table>
<thead>
<tr>
<th>Participants</th>
<th>Gender</th>
<th>Age</th>
<th>Educational Background</th>
<th>Teaching Experience (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>M</td>
<td>28</td>
<td>Mathematics Sciences</td>
<td>2</td>
</tr>
<tr>
<td>S2</td>
<td>F</td>
<td>30</td>
<td>Mathematics Education</td>
<td>3</td>
</tr>
<tr>
<td>S3</td>
<td>F</td>
<td>29</td>
<td>Mathematics Education</td>
<td>2</td>
</tr>
<tr>
<td>S4</td>
<td>M</td>
<td>28</td>
<td>Primary Teacher Education</td>
<td>1</td>
</tr>
<tr>
<td>S5</td>
<td>M</td>
<td>30</td>
<td>Mathematics Education</td>
<td>3</td>
</tr>
<tr>
<td>S6</td>
<td>F</td>
<td>27</td>
<td>Mathematics Sciences</td>
<td>1</td>
</tr>
<tr>
<td>S7</td>
<td>F</td>
<td>31</td>
<td>Primary Teacher Education</td>
<td>2</td>
</tr>
<tr>
<td>S8</td>
<td>F</td>
<td>29</td>
<td>Mathematics Sciences</td>
<td>2</td>
</tr>
</tbody>
</table>

*Data Collected and Analysis*

Subjects enrolled in the geometry class had no previous experience with Geogebra. Matters are supervised by two supervisors who are experts in using Geogebra so that the issue shows progress every week. Several subjects (3 out of 7) were called to present their designs. These two participants were chosen because the three subjects significantly improved designing geometry learning designs. The general description of the activities for each 1-8 week meeting is (1) discussing learning design using Geogebra, (2) sharing briefly the results of the design, (3) participants sharing their experiences regarding the material described, and sharing knowledge between one participant and another, (4) discuss the shortcomings of the developed...
design, (5) identify learning outcomes for the developed design, (6) discuss the challenges faced by students related to the developed design, (7) re-design the material described.

Three types of data were collected during one semester, namely: (1) weekly observations; (2) progress report description; and (3) the developed learning design. The collection of various data types allows researchers to triangulate from multiple sources. The researcher observed all participants in the class during the meeting and recorded it on video. In addition, field notes were used by the observation protocol designed to capture the results of discussions and input from participants during class presentations. Researchers also collect descriptions or progress reports of participants each week to measure how their knowledge has improved. In addition, subjects were asked to design geometric situations using GeoGebra and represent findings in class. The project-based task given by the subject is open, as shown in Table 2. The study focuses on using Geogebra on the material chosen by the subject. The design is based on mathematical content developed by the subject, including the material of flat shapes and spatial shapes.

Table 2. Task Design

<table>
<thead>
<tr>
<th>Tasks</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task Based Project</td>
<td>This task was first given at the initial meeting as the final learning achievement. This task is done individually by selecting mathematical material by applying GeoGebra in the learning design. Participants were given eight weeks to design lessons by selecting materials. Furthermore, participants were asked to present the design results to see the increase in their mathematical technology knowledge.</td>
</tr>
</tbody>
</table>

Subjects were given the opportunity for 10 minutes to explain the design of the material using the developed GeoGebra. Researchers made observations related to participant explanations and asked other subjects to provide input associated with the resulting design. The aim is to obtain more detailed information about the resulting design. Observers already have an observation sheet that is used as report material. Observers were asked to report: (1) steps taken by the subject; (2) the resulting visual image; (3) the usefulness of the learning design using GeoGebra; (4) novelty of the design; and (5) the legibility of the design results.

Triangulation is done by comparing observations on (1) weekly observations, (2) progress report description, and (3) the developed learning design. The results of observations and field notes from each meeting are used to identify specific characteristics and essential points in the research results. Researchers carried out the analysis by looking back at video recordings, observation sheets, and field notes. The aim is to describe the resulting findings. The researcher made a data collection for each meeting of 8 subjects. The data is coded to identify the increase in their mathematical technology knowledge. Researchers identify the resulting designs. Researchers coded the results of their designs when using GeoGebra as a tool for designing learning. In addition, the researcher coded his knowledge of the selected material.

RESULTS AND DISCUSSION

The results showed two tendencies of the teacher's character in increasing knowledge of mathematical technology, namely Moderate Teachers and Expert Teachers. First, 6 out of 8 subjects were categorized as moderate teachers because they were still influenced by classroom teaching. In this case, moderate teachers still do not understand the use of GeoGebra, so the resulting design is still simple. Moderate teachers are also still influenced by algebraic and numerical approaches to designing problems. Moderate teachers tend to choose material at the intermediate level in developing learning, for example, determining the area and perimeter of flat shapes, calculating the maximum/minimum value of curves, and looking for algebraic
forms. This can be seen in Figure 1, a design developed by one of the subjects (moderate teacher). Second, 2 out of 8 subjects were categorized as expert teachers because they used a real context and dynamic approach in designing learning. All learning activities, starting from the initial training, this activity, to evaluation using Geogebra. This is because expert teachers already have prior knowledge of Geogebra. This distinguishes between moderate teachers and expert teachers, where expert teachers develop teaching by choosing material at the top level, for example, the cone problem (shown in Figure 2). The general description of the design created by each participant is shown in Table 3 below.

### Tabel 3. Overview of Research Results

<table>
<thead>
<tr>
<th>Participants</th>
<th>Geogebra Knowledge</th>
<th>Developed material</th>
<th>Completion time</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Moderate</td>
<td>Multiplication with algebra</td>
<td>5 Weeks</td>
</tr>
<tr>
<td>S2</td>
<td>Moderate</td>
<td>Find area and parameters of kites</td>
<td>7 Weeks</td>
</tr>
<tr>
<td>S3</td>
<td>Expert</td>
<td>Spatial applications: the cone problem</td>
<td>8 Weeks</td>
</tr>
<tr>
<td>S4</td>
<td>Moderate</td>
<td>Space applications: cubes and blocks</td>
<td>7 Weeks</td>
</tr>
<tr>
<td>S5</td>
<td>Moderate</td>
<td>Find the area and perimeter of the trapezoid</td>
<td>5 Weeks</td>
</tr>
<tr>
<td>S6</td>
<td>Expert</td>
<td>Find the area and perimeter of a triangle</td>
<td>4 Weeks</td>
</tr>
<tr>
<td>S7</td>
<td>Moderate</td>
<td>Find the area and perimeter of the rectangle</td>
<td>8 Weeks</td>
</tr>
<tr>
<td>S8</td>
<td>Moderate</td>
<td>Searching for algebraic forms</td>
<td>6 Weeks</td>
</tr>
</tbody>
</table>

**Figure 1.** Design developed Subject 7
The character of teacher knowledge in the use of technology in mathematics learning integrating technology and content in mathematics learning found two characteristics, namely a numerical approach and a dynamic approach, shown in Table 3. This finding is in line with the research of Michos et al., (2018) on the conceptualization of the type of software suitable for the integration of technology in mathematics teaching. To describe and distinguish between characteristics, the researcher focuses on using Geogebra in a complex way to make it easier for students to learn. The integration of knowledge about technology and content in learning mathematics supports the development of teacher knowledge is shown in Table 4 below.

<table>
<thead>
<tr>
<th>Character</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical approach</td>
<td>Teachers can use Geogebra. They design mathematical material by utilizing geometric transformations. However, the resulting design is still simple. They sometimes choose materials that are at the intermediate level.</td>
</tr>
<tr>
<td>Dynamic approach</td>
<td>Teachers can use analytical skills to design step-by-step animation learning to understand concepts. The resulting design is quite complex and structured. They already have prior knowledge of Geogebra, so the material chosen is at the top level.</td>
</tr>
</tbody>
</table>

Source: Results of research data

1. Moderate Teacher's Mathematical Technology Knowledge

Analysis of the data from the assignments indicated that most subjects were often limited to using a dynamic approach when planning the design of the presented teaching. Teachers with moderate character are limited to the ability to apply GeoGebra in learning and tend to try still to design education involving conventional aspects. In this case, moderate teachers do not realize the importance of visualizing teaching materials to make it easier for students to understand a concept. This can be seen from the teacher's design that seeks to provide students with an understanding of flat shapes, for example, looking for area and perimeter. The teacher in this character only uses Geogebra to solve problems. For instance, students enter length or width values in the application. This results in the teacher not realizing the importance of students constructing the studied concepts. Another indication is that they do not understand the features of GeoGebra, relying solely on numerical and algebraic approaches in designing teaching. This shows that the teacher's mathematical technology knowledge is still limited in that it has not been able to be developed.

The design designed is the result of the transfer of conventional teaching. The moderate teacher first analyzes teaching materials to determine the part of the material that uses GeoGebra. Furthermore, moderate teachers need three weeks to integrate the design that will be
designed using Geogebra. The moderate teacher seems to use Geogebra for certain cases, for example, determining the area and perimeter of a flat shape. Thus, the visualization of the material being taught is not involved in the design. However, some of them try to apply a dynamic approach, but the limited knowledge of Geogebra results in not being able to produce the desired format. The resulting structure is shown in the following image is shown in Figure 3 below.

![Image](image.png)

**Figure 3.** Moderate teacher design that involves numerical and algebraic approaches

2. **Knowledge of Mathematical Technology Expert Teacher**

In this characteristic, the teacher is already aware of using Geogebra, which is integrated with teaching, for example, determining coordinate plans, plotting points, and changing/transforming shapes. In a sense, teachers are aware of the procedural nature of the chosen material, but they are also mindful of the importance of visualizing and applying mathematical concepts using Geogebra. Expert teachers already have insufficient knowledge about GeoGebra obtained from previous lectures. Some subjects also require time to study GeoGebra independently. This shows that teachers understand the pedagogical understanding of the material being taught, but they also develop technological knowledge. In addition, teachers are also able to integrate pedagogy with technology so that they can produce animations that require a high level of expertise.

Teachers who utilize technology-based learning using the GeoGebra application can minimize students' difficulties in understanding a concept, make generalizations about geometric visual problems, and are also able to understand complex issues for advanced math problems. The knowledge of mathematical technology developed by the teacher is interrelated with the teacher's conceptual understanding, so there needs to be an effort to visualize the problem through GeoGebra. It is more effective to explore the improvement of teachers' technological knowledge.

Most teachers develop their knowledge by integrating teaching with technology. In this case, the use of Geogebra in teaching design can be categorized as effective for increasing teachers' technological knowledge. Most teachers who choose teaching materials try to compare the situation in the classroom with the technology used, in the sense that the teacher seeks to involve GeoGebra in learning. Some teachers experienced significant changes in teacher design, where they used to use paper and pencil tests, then switched to using technology. However,
Geogebra's unfamiliarity causes teachers to still hesitate in designing teaching (Agyei & Benning, 2015). So very few teachers have previous experience using Geogebra in teaching. This affects the design, where teachers tend to integrate Geogebra with numerical and algebraic approaches. The choice of this approach is influenced by the teacher's thinking preferences when creating learning (Haciomeroglu et al., 2010) and the influence of their teaching experience (Zambak & Tyminski, 2020). Furthermore, the integration between technology and teaching is a new situation that increases teachers' mathematical technology knowledge to be aware of the importance of visualization in learning.

Teachers who have minimal experience using Geogebra learn independently with the help of the Learning Management System. This shows the teacher's efforts to improve their mathematical technology knowledge. In addition, the teacher conducts discussions to help each other learn Geogebra in detail. This shows that collaboration in designing learning is an essential aspect for teachers (Ratnayake et al., 2020). In addition, the use of Geogebra affects teacher involvement in learning the technology (Misfeldt & Zacho, 2016).

Geogebra is a tool for working mathematics, understanding concepts, and developing representations, teachers are motivated to involve technology in learning (Ma’Rufi et al., 2019). This shows that they regard Geogebra as an essential tool to integrate with teaching (Hernández et al., 2020). This indicates that Geogebra should be used to improve students' visualization and teachers' technological knowledge. Furthermore, teachers are invited to develop visual designs and stimulate students to produce their preferences in various ways (Anggraini et al., 2022).

**Implications**

This study found two teacher characteristics in increasing knowledge of mathematical technology, namely moderate teachers and expert teachers (Seidel et al., 2021). The difference between the two can be seen in the resulting design. Teachers reported improvements in their design results as a project assignment over eight weeks. From the analysis, moderate teachers tend to be less motivated to use Geogebra, but they can develop designs. This is a technical problem because they are not used to using Geogebra in teaching and only rely on algebraic and numerical approaches in their design (Sari et al., 2018).

In addition, moderate teachers spend quite a long-time designing education. In contrast to expert teachers, they take the time to re-learn Geogebra Applications, discuss with supervisors, integrate classroom teaching with technology, and minimize the involvement of algebra and numeric in the design. The fundamental difference between these two teachers is based on their perspectives on implementing learning (Haciomeroglu et al., 2013). The findings show that moderate teachers and expert teachers can design and implement teaching scenarios using Geogebra through project-based tasks. These results indicate that the use of technology in learning is a current educational need resulting from the increased use of GeoGebra. However, the challenge of integrating technology with teaching needs to consider teacher-generated designs (Michos et al., 2018).

Moderate teachers try to respond to learning progress by integrating technology into education. Meanwhile, expert teachers need adjustments and encouragement to further develop their teaching by looking at situations from various directions. In a sense, teachers should be allowed to explore the problems of a collaboratively designed technology-based curriculum (Hollebrands & Lee, 2020). Thus, teachers can feel the goals and need to master technology in
learning. Furthermore, the designed problem must require technology as a differentiator from the previous teaching. Researchers realize that the achievement of expert teachers for technology will be a challenge for education. Although based on experience in using technology, teachers must design teaching by integrating content, technology, and classroom practice (Ogrodzka-Mazur et al., 2017). Furthermore, this finding is in line with the teacher classification found by Zambak & Tyminski (2020) regarding the Mathematical Technology Knowledge (MTK) level, where the isolated MTK level is similar to the characteristics of moderate teachers, and the MTK expert level is identical to the parts of expert teachers.

It is essential to discuss how teaching design and technology impact teachers. When teachers develop designs, they tend to actively take a stand on the learning objectives. Furthermore, collaboration and discussion have a positive impact and position teachers as designers who support them in developing their materials. However, we recognize that the designs created by the teachers push them out of their comfort zone. Moderate teachers tend to perceive this situation as a complexity for big problems, in contrast to expert teachers who already have experience with GeoGebra. Therefore, teaching design integrated with technology can be a challenge for some teachers. Aware of the challenges faced by teachers related to the development of technology-based teaching materials. However, many teachers do not need adjustments to understand the tools to be used. Through GeoGebra, they independently need further guidance to design learning scenarios.

The findings of this study can be compared with the results of previous studies in various countries. In Europe (Belgium, Germany, and France), their teaching requires integrating content and technology (Törner et al., 2014). Meanwhile, in Indonesia, the use of technology is minimal. In the sense that technology is not used to form students' mathematical concepts. This is where the role of expert teachers in designing teaching designs that aim to develop students' mathematical abilities is here. However, in educational practice, this effort is not in line with the primary education goals in Indonesia. In a sense, technology in learning is only focused on certain materials, for example, geometry. In addition, teachers are not aware of the importance of technology, especially in constructing mathematical concepts.

**Limitations and Suggestions**

The researcher suggests that integrating teaching and technology is essential for teachers. The learning design, the ultimate goal to be achieved, the sequence of activities, and the use of technology are the core of teacher competency development. This supports the teaching of relevant and innovative mathematics. However, this creates difficulties for teachers who are less motivated to use technology. This knowledge needs to be developed by the teacher outside of independent teaching practice.

**CONCLUSIONS**

Two characteristics of mathematical technology knowledge were found: moderate teachers and expert teachers. Moderate teachers produce simple designs and are still influenced by numerical and algebraic approaches. In contrast, expert teachers can use analytical skills to design step-by-step learning involving animation to understand concepts. The resulting design is quite complex and structured. Finally, the study results indicate that the two characteristics can be concluded to categorize the competencies possessed by teachers in developing teaching. The
data the research results are based on weekly observations, descriptions of progress reports, and expanded learning designs, so the preliminary information is based on teacher responses based on rubrics developed by researchers. Therefore, it is impossible to discuss student responses when the subject applies the set design. However, we identified several trends that are gaps for further research concerning student responses to the plan. This refers to the usefulness of the strategy designed by the teacher.

**AUTHOR CONTRIBUTIONS STATEMENT**

MI as research coordinator who contributed to developing ideas and methods. MM and SS are responsible for developing theory, designing instruments, and collecting and analyzing data.

**REFERENCES**


