Student’s obstacles in learning surface area and volume of a rectangular prism related to mathematical representation ability

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Abstract
The surface area and volume of cubes and cuboids are mathematical concepts related to everyday life. However, many students need help learning this concept. This study aims to analyze the learning obstacles junior high school students encounter when solving mathematical representation problems on the concept of surface area and volume of cubes and cuboids. This study uses an interpretive paradigm which is part of Didactical Design Research (DDR). Students at one of the public junior high schools in Jambi City were given six questions related to mathematical representation after the researchers conducted classroom observations, which were then followed by interviews with students with high, moderate, and low representation abilities. Data analysis techniques include data reduction, display, and conclusion drawing and verification. The results showed that students experienced three learning obstacles: ontogenic, epistemological, and didactical. Furthermore, the results of this study can be a consideration for teachers to create learning designs for surface area and volume of cubes and cuboids that can anticipate student learning obstacles and develop students' mathematical representation abilities.

INTRODUCTION
Learning must prioritize the role of students actively in constructing their knowledge, making connections between ideas and with the new information they get (Candra & Retnawati, 2020). But in fact, learning mathematics is done not with knowledge construction but with instruction patterns. Thus, learning activities like this do not allow students to construct their knowledge independently, so learning mathematics at school is only rote without training students' mindsets (Gazali Yuliana, 2016). This rote mathematics learning can create student learning obstacles (Rachma & Rosjanuardi, 2021).

Brousseau (2002) defines obstacles as a part of knowledge that arises from implementing didactic situations where the knowledge produced is incomplete. There are three types of obstacles that students may experience, namely ontological obstacles, epistemological obstacles, and didactical obstacles. Ontological obstacles occur because of students' limitations in cognitive development. Epistemological obstacles occur due to limited context in the concepts students learn. Didactical obstacles are related to inappropriate learning processes, such as the methods used by the teacher, inappropriate teaching materials, etc. (Brousseau, 2002; Suryadi, 2019b).

The concepts of surface area and volume are important concepts to teach because these concepts are related to everyday life. In addition, the concepts of area and volume are learned in elementary schools and at the junior high school level (Hatziminadakis, 2018; NCTM, 2000; Tan Sisman & Aksu, 2016), so students need to understand these concepts well. Based on the
 mathematics curriculum in Indonesia, cubes and cuboids are the first topics to be taught before other prism shapes in studying surface area and volume. According to Sung et al. (2015), cubes and cuboids form the basis for studying the prism concept. Therefore, students need to understand the concept of surface area and volume of cubes and cuboids because they relate to other prism concepts. However, in the learning process, students make mistakes: students assume that the volume of a geometric shape is the total area of the sides of that geometric shape (Tekin-Sitrava, 2018). Students also wrote the wrong formula for the surface area of a cuboid $L = p \times l \times t$ (Pedai et al., 2021; Ramli & Prabawanto, 2020). The existence of this error is caused by the teacher not facilitating students to be able to construct their knowledge using previous student learning experiences. Learning obstacles that arise can result from errors in the learning methods used by the teacher (Hariyomurti et al., 2020).

Obstacles to learning related to student errors in representing the surface area formula of the cuboid indicate a relationship between mathematical representation abilities and students' understanding of concepts. The learning obstacles experienced by students in the form of misrepresenting formulas have an impact on revealing the meaning of a concept to students (Ramli & Prabawanto, 2020). In this regard, mathematical representations support understanding mathematical concepts (Afriyani et al., 2019; Bolden et al., 2015; NCTM, 2000). According to Gülkılık et al. (2015) and Purwadi et al. (2019), when students understand a concept, students can represent mathematical concepts accurately. Not only playing a role in understanding concepts but mathematical representation abilities are also needed in facilitating the process of solving mathematical problems (Cai & Lester, 2005; Goldin, 2014). The ability to represent mathematically is the ability to restate a problem through things such as: selecting, interpreting, translating, and using graphs, tables, images, diagrams, formulas, equations, and concrete objects to express problems so that they become clearer (OECD, 2018). In this study, there are three aspects used, namely visual, symbolic, and verbal. Each aspect has two indicators used to measure students' mathematical representations, adapted from Sianturi (2021). Firstly, visual indicators represent data or information from form of representation to representations of images, diagrams, graphs, or tables, and using picture representations, diagrams, tables, graphs, or tables to solve problems. Secondly, the indicators of symbolism are created equations or mathematical models from other forms of representation and problem-solving involving symbolic representation. Thirdly, the verbal indicators are writing down the process and solution to the problem through words or sentences and the interpretation of a form of representation.

Several previous studies have carried out obstacles to student learning on the surface area and volume of cubes and cuboids. Research conducted by Indasari & Ratna (2019) found ontogenic and epistemological obstacles in elementary school students when learning the concepts of volume and cubes. Furthermore, research conducted by Ramli & Prabawanto (2020), learning obstacles experienced by students when learning the concept of surface area and volume of cubes and blocks are in the form of epistemological obstacles (limited understanding of concepts) and ontogenic obstacles (students' mental readiness). These facts show that students experience errors in understanding the concept of surface area and volume cubes and cuboids, which indicates that students experience learning obstacles. The involvement of mathematical representations in understanding the concept of surface and
volume cubes and cuboids indicates that it is necessary to reveal the learning obstacles experienced by students that relate to the ability of students' mathematical representations.

Therefore, research on learning obstacles related to the mathematical representation abilities of junior high school students on the concept of surface area and volume of cubes and cuboids has never been done. Thus, this study aims to analyze the learning obstacles experienced by junior high school students when solving mathematical representation problems on the concept of surface area and volume of cubes and cuboids.

**METHOD**

The research approach used in this study is qualitative research with a phenomenological design (Creswell, 2014). The research method used is Didactical Design Research (DDR). The Didactical Design Research consisted of 3 stages: the prospective analysis stage, the metapedidactic analysis stage, and the retrospective analysis stage (Suryadi, 2010). This research is the first step in DDR which can be used to create a new didactical design. This study uses an interpretive paradigm to identify students’ learning obstacles related to cubes and cuboids' surface area and volume. The following is the flow of this research method, which is shown in Figure 1.

![Figure 1. Flow-Chart of Research Method](image)

The research subjects were selected using a purposive sampling technique. The subject is junior high school students of grade VIII at a junior high school in Jambi City. The data obtained in this study used classroom observation, mathematical representation tests, and interviews. The test items consist of 6 questions related to indicators of mathematical representation abilities, validated by experts, and related to the surface area and volume of cubes and cuboids. Twenty-eight students did the test after learning the surface area and volume of cubes and cuboids, with a time of 70 minutes. Based on students' answers, students with visual, symbolic, and verbal representation abilities were selected; high mathematical representation (HMR), moderate mathematical representation (MMR), and low mathematical representation (LMR), which indicates that students experience learning obstacles. They were followed by conducting interviews with the selected students. Semi-structured interviews were used to identify the factors that caused students to experience learning obstacles and were reinforced by the results of learning observations that had been made. Data derived from observations, test answers, and interviews were analyzed to conclude the types of learning obstacles experienced by students. Data analysis techniques were carried out through the following steps: (1) collecting data by recording main things according to the research focus, namely students' obstacles in learning...
surface area and volume of cubes and cuboids related to the ability of students' mathematical representation, (2) organizing data systematically based on categories and classifications, (3) analyzing and presenting data in a descriptive form, (4) interpreting the data, and (5) drawing conclusions and recommendations for further research (Miles et al., 2018).

RESULT AND DISCUSSION

Based on the results of students' mathematical representation ability tests, learning observations, and interviews, students experienced three types of learning obstacles: didactical, ontogenic, and epistemological. The six questions on the concept of surface area and volume of cubes and cuboids show difficulties or errors from each question, indicating learning obstacles.

a) Visual Representation Capability

Problem number 1 relates to indicators of visual representation ability in representing information from word problems to image representation. Given the information in the form of illustrations about the two cubes in word problems, students are expected to be able to make an image representation of the illustration. Figure 2 shows a question regarding the first visual representation indicator. Figures 3(a.1) and 3(a.2) show students' answers with high representation (HMR), Figure 3(b) shows students' answers with moderate representation (MMR), and Figure 3(c) shows students' answers with representation low (LMR).

Translation:

Tony has two cube-shaped toys that are the same size, and both toys are magnetic. Each of these toys has a side length, which measures 12 cm. One side of each new toy is a combination of two cubes.

a. Make a picture of the new toy by combining the two formed cubes!

b. What is the new shape of the combination of the two cubes?

c. If Tony is going to wrap his new toy in the paper, how much area of the paper is needed?

Figure 2. Test Question Number 1
Based on the four answers above, students with HMR and MMR could present information on word problems as an image representation in cuboids. Student RT and RS could correctly represent two cubes attached to form a cuboid. Although students with MMR could represent the information in the problem to an image representation, they could not use the information in answer 1a to answer question 1c. It can be seen from the students' answers using the formula for the surface area of a cube instead of the formula for the surface area of a cuboid. Meanwhile, students with LMR could not present information on word problems in image representation because he needed help understanding the word problem in question number 1 (Figure 2). When interviewed, students with LMR said that in answer 1b, the two combined cubes would become a rectangle. Then, in answer 1c, he used the formula for a cube's surface area instead of a cuboid's surface area because the question contains the word ord "cube."

Based on the interview shows that students' learning obstacles in representing a concept can show students' understanding of the concept. In addition, it was also seen that students needed more understanding regarding cubes and cuboids, seen from students who thought that two cubes would be longer when combined, so it was called a rectangle. The lack of reasoning carried out by students was seen from RR students who used the cube surface area formula because there needed to be a description of the cube in the problem without any reasoning process. Based on learning observations, students are given questions with procedural solutions without any reasoning process. In addition, the teacher gave students excessive scaffolding when solving problems. Thus, students need help to apply the reasoning process in solving mathematical problems. The RR student errors indicate that students experience didactical obstacles due to teacher errors in using strategies or methods in learning (Fauzi & Suryadi, 2020).

Another didactical obstacle relates to the second indicator of visual representation ability, using image representation to solve problems, where the achievement of this indicator can be seen in the students' answers in 1b and 1c. To be able to answer questions in 1b and 1c, good skills are needed in presenting existing information back into an image representation. Based on Figure 3(b), RS students can restate the information in question number 1 into an image representation. However, students need help using the image representation to solve the
problem in 1c. When interviewed, the RS student admitted that he used the formula for the surface area of a cube because the RS student remembered that formula. This shows that learning that emphasizes students memorizing formulas rather than discovering formulas by students. The mistakes made by the students show that students experience didactical obstacles. In addition, learning like this also causes students to write the wrong formula incorrectly. In the results of the RT’s answers (Figure 3(a)), he needed to write the formula for a cuboid’s surface area correctly. The results of the interviews found that the students knew that “2” in the formula of cuboids’ surface area also applied to \(lt\) and \(pt\), not just to \(pl\).

In the students’ RT work (Figure 3(a)), it was also found that some students needed to write the unit for surface area correctly. This mistake was caused by the teacher’s explanation of using the concept of exponential numbers to be learned in grade 9 instead of using the student learning experience. Thus, it shows that students experience conceptual ontogenic obstacles due to a discrepancy between the conceptual level of the material and student learning experiences (Suryadi, 2019a).

Problem number 2 relates to the use of image representations to solve problems. Given a picture of a cube where one side area is known, students are expected to be able to use the information on the given cube to solve problems related to surface area and volume. Figure 4 shows questions related to the second visual representation indicator. Figures 5(a.1) and 5(a.2) show students’ answers with high representation (HMR), Figure 5(b) shows students’ answers with moderate representation (MMR), and Figure 5(c) shows students’ answers with low representation (LMR).

Translation:

<table>
<thead>
<tr>
<th>2. Perhatikan daerah yang diwarnai pada kubus berikut!</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="https://via.placeholder.com/150" alt="Image with colored areas" /></td>
</tr>
<tr>
<td>a. Informasi apa yang dapat kamu peroleh dari gambar kubus di atas?</td>
</tr>
<tr>
<td>b. Berdasarkan informasi yang kamu peroleh pada bagian a, tentukan luas permukaan dan volume dari kubus di atas!</td>
</tr>
</tbody>
</table>

Looks for the colored areas on the following cubes!

a. What information can you get from the cube image above?

b. Based on the information you got in part a, determine the surface area and volume of the cube above!

**Figure 4. Test Question Number 2**
Based on the four student answers above, students with HMR, MMR, and LMR could not use image representation for problem-solving. They could not answer question 2 correctly: they misinterpreted the number "16 cm$^2$". However, students with HMR had better knowledge regarding cubes than students with MMR. It can be seen because he answered from 2a first, followed by 2b. Besides that, in the answer of students with HMR on number 2a, there is additional information related to the cube he wrote, namely, "every side has the same side size." In the answers of the students with MMR, they answered question number 2b first, followed by 2a. This process shows that student with MMR does not see any connection between answers 2a and 2b, unlike student with HMR. Different from students with HMR and MMR, students with LMR did not answer question number 2a. Student with LMR only works on problem number 2b by directly entering the number 16 into the formula for a cube's surface area and volume.

Based on the interview, all students considered "16 cm$^2$" as the side length of a cube instead of the side area of one side of the cube. In addition, students also assume that the symbol "s" in the formula for a cube's surface area and volume is a side, not a side length. When interviewed, students said that the teacher usually gives questions where the length of the side is known, so students plug them into the formula for the surface area and volume of a cube. This shows that students with HMR, MMR, and LMR experienced epistemological obstacles due to limited contexts when learning the surface area and volume of cubes and cuboids was first taught (Rachma & Rosjanuardi, 2021).

\textit{b) Symbolic Representational Ability}

Problem number 3 relates to indicators of symbolic representation ability by making equations or mathematical models from verbal representations. The area of one side of the cube is shown. Students are expected to be able to make a mathematical equation from the area of the top side, front side, and sides side of the cube. Figure 6 shows questions related to the first symbolic representation indicator. Figure 7(a) shows students' answers with high representation (HMR),

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{The Answers of Students with HMR, MMR, and LMR for Number 2}
\end{figure}
Figures 7(b.1) and 7(b.2) show students' answers with moderate representation (MMR), and Figure 7(c) shows students' answers with low representation (LMR).

Translation:
A rectangular box has an upper side area of 120 cm\(^2\), one of the front side areas of 96 cm\(^2\), and one of the side areas of 80 cm\(^2\). What is the surface area of the box?

Figure 6. Test Question Number 3

Figure 7. The Answers of Students with HMR, MMR, and LMR for Number 3

The student’s answers in Figure 7 shows that students with HMR and MMR had the same thoughts regarding solving problem number 3 (Figure 6). They assumed that the three values (120, 96, and 80) were length, width, and height. It happened because the learning that the teacher applies accustomed the students to working on questions where the length, width, and height of a cuboid were known. Thus, they had difficulty answering the question in a different context than usual. The students experienced epistemological obstacles due to limited context when the concepts of surface area and volume of cubes and cuboids were first taught (Rachma & Rosjanuardi, 2021).

The completion process is correct based on LMR's answer (Figure 7(c)). However, the results of the interviews that have been conducted found that the student LMR did it randomly. Thus, the student LMR is right, but only by chance. When interviewed, it was found that the students did not understand the problem, so because the information contained three numbers, namely 120, 96, and 80, the students plugged these numbers into the formula for the surface area of a cuboid. After being asked more deeply, it was found that the reason students answered incorrectly was that students did not know the visual representation of cuboids, so this caused students to be unable to carry out the reasoning process to be able to answer question number 3 (Figure 6). This shows that students experience instrumental ontogenic obstacles because
students need help understanding the key technical matters of the studied topics (Suryadi, 2019b).

Question number 4 relates to indicators of symbolic representation ability by solving problems involving symbolic representations. Given the cuboid's side area and the cuboid's width, students are expected to be able to find the height and volume of the cuboid from the known information. Figure 8 shows questions related to the second symbolic representation indicator. Figure 9(a) shows students' answers with high representation (HMR), and Figure 9(b) shows students' answers with low representation (LMR).

Translation:
A cuboid has a base area of 54 cm² and a side area of 30 cm². If the width of the cuboid is 6 cm, then find the height and volume of the cuboid!

**Figure 8. Problem Number 4**

Based on the answers of students with HMR and LMR, no one could correctly answer question number 4 (Figure 8). Students with MMR did not answer problem number 4. The following is a translated transcript of the researcher (R) interviewing student with HMR (HMR) to show their understanding:

R : "Does the question say that 30 is the length of the block?"
HMR : "No"
R : "So what is 30 cm²?"
HMR : "The problem is the same as above."
R : "It means that if the area of one of the sides, it means what measurements can be used to get the area?"
HMR : "Width and height."
R : "Are the widths known yet?"
HMR : "Yes"
R : "So how do you get the t? Try using this piece of paper to find the t."
HMR : "So the height is 5."
R : "How do you get 5?"
HMR : "This width is already known, so we look for a multiplication of 6 which produces 30, the result is 5."

Based on the students with HMR transcripts above, he could solve problems involving symbolic representations, as seen when he answered the method used to obtain the height of
the cuboid. RT said that the cause of his inability to answer question number 4 was because, during the lesson, the students were never given a context like that. They cannot solve problems related to symbolic representations due to context limitations, where there is no contextual variation on the problems given during learning. Thus, it also shows that students experience epistemological obstacles.

Unlike students with HMR, student with LMR obtained the formula used to find t from her friend, so she did not know the truth of the formula. Students with MMR did not answer problem number 4 because they did not know the formula. This problem happened because the teacher was focused on memorizing formulas. Learning that focuses on memorizing formulas will make it easy for students to forget what they have learned, causing obstacles to the student learning process (Rachma & Rosjanuardi, 2021).

The interviews between the researcher and student with LMR showed that student's motivation and interest were low in studying surface area and volume of cubes and cuboids. When interviewed, the student said that according to their interest in learning the surface area and volume of cubes and cuboids, only because these materials were included in the school examination material. This shows the low interest of students in learning the topic of surface area and volume of cubes and cuboids. Students think this concept is unimportant and can be applied in everyday life. Students' low motivation and interest in studying this topic indicate that students experience psychological ontogenic obstacles because children's unpreparedness to participate in the learning process is caused by psychological aspects such as low motivation and interest in the material being taught (Suryadi, 2019a).

c) Verbal Representation Ability

Question number 5 relates to indicators of verbal representation ability in writing down processes and problem solutions through words or sentences. Given an illustration related to the cube, it is expected that students can state the process and solution to the problem using their own words. Figure 10 shows questions related to the first verbal representation indicator. Figure 11(a) shows students' answers with high representation (HMR), Figure 11(b) shows students' answers with moderate representation (MMR), and Figure 11(c) shows students' answers with low representation (LMR).

Translation:

Mr. Dani has an aquarium shaped like a cube with 30 cm long sides. The aquarium is filled with as many as \( \frac{2}{3} \) parts, then 3.000 ml of water is added.

a. What steps will you take to determine the volume of the water in the aquarium after adding 3.000 ml of water? Explain using your own words!

b. What is the volume of water in the aquarium after adding 3.000 ml?
c. Check if there is any water spilled out of the aquarium. If so, how much water was spilled from the aquarium (in ml)? If not, why?

**Figure 10. Problem Number 5**

![Image: Problem Number 5](image1)

**Figure 11. The Answer of Students with HMR, MMR, and LMR for Number 5**

Based on Figure 11, only RT and RS can write down the process and results in their own words. However, the completion process written by students RT and RS was not correct. The RT students could not work on problem number 5 (Figure 10) because they did not understand the questions in the sentence "the water is filled with as much water as \( \frac{2}{3} \) it is". This shows that students experience epistemological obstacles due to limited connections between the concept of volume and the concept in the previous material, the concept of fractions (Istiqomah et al., 2016).

Unlike student RT, student RS could not solve problem number 5 because the student needed help understanding the story problem in number 5. This is because the learning applied by the teacher does not familiarize students with the reasoning process, so students are accustomed to procedural questions without involving the reasoning process in them. Learning like this can cause students to experience didactical obstacles (Wahyuningrum et al., 2019).

Question number 6 relates to indicators of verbal representation ability in writing down interpretations of a form of representation. Given problems related to cuboids, students are expected to be able to write down interpretations of the problems given. Figure 12 shows questions related to the second verbal representation indicator. Figure 13(a) shows students' answers with high representation (HMR), Figure 13(b) shows students' answers with moderate representation (MMR), and Figure 13(c) shows students' answers with low representation (LMR).

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Translation:
A bakery sells black forest cakes (as pictured on the side) with two different size options. The first cake with size $31 \text{ cm} \times 20 \text{ cm} \times 10 \text{ cm}$ and costs IDR 310,000. The second cake with size $38 \text{ cm} \times 25 \text{ cm} \times 10 \text{ cm}$ and costs IDR 380,000. Which cake is more economical? Give your reason!

Note: "Economical" is a situation where we can get goods at the lowest possible price but as many as possible.

**Figure 12. Problem Number 6**

Based on Figure 13, Student with HMR, MMR, and LMR could not write down the interpretation of the word problems. Based on the interview, it was found that student HMR and MMR used the concept of the volume of a cuboid to determine a more economical cake. In contrast, student LMR multiplied the size in problem number 6 without knowing that multiplication refers to the formula for the volume of a cuboid. This happens because students with HMR and MMR, in giving interpretations, were guided by the volume of the cuboid without being related to the value of each cake. These students' inability is due to the questions given by the teacher when learning is only limited to finding the volume of a cube or cuboid.

When interviewed, students said that students understood the questions given, but students needed to learn the method used to determine which cake was more economical. This is because when learning, the teacher only questions the volume and area of the base of the beam. Thus, the inability of students to determine the price of more economical cakes is caused by the need for more variety of questions given by the teacher. During learning, the teacher is limited to asking students to find the volume of cubes and cuboids without any deeper exploration process regarding the volume of cubes and cuboids found by students. This shows that students experience epistemological obstacles due to the limited context of questions the teacher gives during learning (Istiqomah et al., 2016).

Based on ontogenic obstacles, epistemological obstacles, and didactical obstacles related to students' representational abilities, it initially describes students' conditions when learning the concept of surface area and volume of cubes and cuboids. Thus, the results of this study can provide an overview to the teacher regarding the conditions experienced by students when studying this topic, as well as an overview of students' mathematical representation abilities. With these results, it can be a consideration for teachers to create learning designs of surface
area and volume of cubes and cuboids that can anticipate student errors identified as learning obstacles and designs that can develop students’ mathematical representation abilities.

CONCLUSIONS
Based on the results of the analysis of the results of student answers and interviews with students, students' mistakes in representing problems can reflect the learning obstacles experienced by students. In solving problems related to these mathematical representations, students experience three types of learning obstacles: ontogenic, epistemological, and didactical. Students experience ontogenic obstacles because students cannot understand the core concept of the surface area and volume of cubes and cuboids when students' motivation and interest are low in learning the concept of surface area and volume of cubes and cuboids and when there is a discrepancy between the conceptual level of material and student learning experience. Students experience epistemological obstacles due to limited context, so students are not accustomed to applying one mathematical concept to another. The didactical obstacles experienced by students were caused by the learning process applied by the teacher.

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AUTHOR CONTRIBUTIONS STATEMENT
GS designed the main conceptual research and ideas. D and DD are the coordinators of this research activity. All authors contributed to the interpretation of the results.

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