Students’ mathematical communication through math-talk learning community: Describing levels and components

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Abstract
This research aims to analyze and describe the role of the Math-Talk learning community in improving students' mathematical communication. A qualitative approach was adopted in this study, which involved observing the learning process related to patterns, sequences, and number sequences through the Math-Talk learning community method. The written test involved 34 grade VIII junior high school (SMP) students at a public school in Bandung, Indonesia. The results show that the learning components of the Math-Talk learning community, namely: (a) questioning techniques; (b) explanation of mathematical thinking; (c) sources of mathematical ideas; and (d) responsibility for learning, have the potential to improve mathematical communication skills in students. The results of this study show the importance of implementing the Math-Talk method in the school mathematics curriculum to facilitate better and more effective mathematics communication between students and enrich their mathematics learning experience.

INTRODUCTION
Mathematics is very important; besides being learned at school, mathematics is also used in everyday life, for example, in trade, development, and much more (Rachma et al., 2020; Sulastri & Haq, 2013; Sutarsa & Puspitasari, 2021). This is in line with the opinion of Islami (2018) that mathematics is a science that has an important role in the progress of human civilization. In learning mathematics, several abilities must be possessed by students, one of which is mathematical communication which is students' ability to express mathematical ideas orally or in writing in mathematical in the form of diagrams, graphs, or tables (Femisha & Madio, 2021; Widiyanto & Yunianta, 2021; Wulandari et al., 2021). Barker (2011) wrote data on the importance of communication skills, "In 2003, the American Management Association asked its members what skills go to make an effective leader. Number one skill—way ahead of the others—was communication (84 percent)." To be a leader, you need communication skills. Communication skills enable a person to process and convey the information they receive in a better language. According to Umar (2012), an important reason why learning mathematics is focused on communication, namely: mathematics is a language. Then, Mayasari (2015) stated the importance of mathematical communication in learning mathematics is that it can help teachers understand students' abilities in interpreting and expressing their understanding of the mathematical concepts being studied. Dewi et al. (2020) also said that if students' mathematical communication is good, this can stimulate students' ability to develop ideas and knowledge in finding mathematical concepts.

Given the importance of students' mathematical communication skills, a learning model to optimize these abilities is needed. Various types of learning methods can be applied, but in this study, I chose a method that is not widely known, namely learning with the Math-Talk learning community. A Math-Talk Learning Community is where individuals assist one another's mathematics learning by engaging in meaningful mathematical discourse" (Hufferd-
Math-Talk Learning Community (MTLC) is a learning community that involves each individual actively helping each other or interacting in learning mathematics with meaningful mathematical learning groups. This interaction occurs not only between teachers and students but also between students and other students. Students can explain, express opinions, defend opinions, and develop their thinking processes confidently. In addition, MTLC has a framework that can describe learning developments in two ways, including (1) seeing the stages of development through case study changes that occur in the classroom, namely the change from conventional learning (phase zero) to meaningful collaborative math-talk learning in stage three; (2) see the stages of development of each component (Hufferd-Ackles et al., 2004).

According to "Connecting Practice and Research in Mathematics Education", the key components of MTLC are questions, explanation of mathematical thinking, sources of mathematical ideas, and responsibility for learning. Within the MTLC framework, teachers are directed to guide the learning process and assist students in developing mathematical thinking (Mhuirí, 2012, 2017). The main goal of MTLC learning is to understand and develop one's ideas and those of others in the class, changing learning from teacher-centered to student-centered (Mhuirí, 2013). Additionally, MTLC is "defining the means of assistance, or teacher discourse behavior, as essential in shaping it to guide students to move to the next level" (Hufferd-Ackles et al., 2004). This follows the findings of Hadi (2011), which suggest that MTLC provides guidelines for teachers to guide students in developing mathematical communication from the lower to higher stages. Meanwhile, recent research by Irvine (2017) found that the use of MTLC can help students formulate and convey their mathematical ideas more effectively." By having students talk about informal strategies, teachers can help them identify and build implicit informal knowledge” (NCTM, 2000).

Several studies have attempted to understand and define the various MTLC components and how they work together to support mathematics learning (Hufferd-Ackles et al., 2004; Woods, 2022). In this context, the role of questions, explanation of mathematical thinking, sources of mathematical ideas, and responsibility for learning have been emphasized. Several studies have shown that MTLC can improve students' mathematical communication skills (Nurhayati et al., 2017). For example, research by Saputro et al. (2017) showed that students who studied in an MTLC environment had better mathematical communication skills than those who studied through traditional methods. Similar research by Setiawaty (2016) supports this finding, showing that using MTLC can help students formulate and convey mathematical ideas more effectively. This shows that the MTLC does not only function as a tool to develop students' math skills but also as a tool to support changes in class structure and dynamics.

Although research related to MTLC has progressed rapidly, some knowledge gaps still need to be filled. First, most of the previous research has focused on the context of higher education, and there are not many studies evaluating the effectiveness of this method at the secondary education level. Second, although research has shown that MTLC can improve students’ math communication skills, it is still unclear how this process occurs and how the various MTLC components interact to support this development. Therefore, deeper research into this mechanism would be invaluable. Based on this, this research aims to analyze and describe the role of the Math-Talk learning community in improving students' mathematical communication in junior high schools.

**METHODS**

This study applies a qualitative research design, which focuses on describing the value of a particular object or symptom (Abdussamad, 2022). This design aims to produce a deep and holistic understanding of the effect of implementing learning with the math-talk learning community on students' mathematical communication. Participants in this study involved 34
grade 8 junior high school students in Bandung. This sample selection is based on the consideration that students at this level of education are considered mature enough to follow and understand learning methods such as the math-talk learning community (Nurhayati et al., 2017).

The research instrument consists of three main parts: learning instruments, observation instruments, and questionnaires. Learning and observation instruments monitor and record learning activities and students' mathematical communication skills in the math-talk learning community. Meanwhile, a questionnaire measured students' interest in the math-talk learning community. The measurement scale of the questionnaire uses a Likert scale, designed to solve technical problems that arise in connection with the quantitative aspects of research on social attitudes (Moreno-Garcia et al., 2022).

The data collected through this research instrument were then analyzed for emerging patterns and themes. This approach includes identification, categorization, and interpretation of patterns in data, aiming to provide deeper insight into how the math-talk learning community influences students' mathematical communication (Creswell & Poth, 2016). This analysis ensures that research results are relevant and can be generalized to similar educational contexts.

RESULTS AND DISCUSSION
Mathematical Communication Skills with Math-Talk Learning Community
In this article section, lectures discuss the growth in each part of the math-talk learning community.

Component A: Questioning
The focus of the questioning on learning with the math-talk learning community is the questions that arise during the teaching-learning process. In this component, the teacher knows the students' mathematical communication and mindsets in answering questions. Component questioning makes students confident in giving answers and asking each other questions to understand their mathematical thinking. Therefore, questioning is an important learning component within the math-talk learning community.

Based on the questioning created from level 0 to level 3, it moves from the teacher as the questioner to the student as the questioner with the teacher. Another concurrent change in the questioning of the math-talk learning community is from a focus on questions that seek answers to questions that show the mathematical thinking behind the answers.

The first questioning component in learning with the math-talk learning community level 0 is in the form of conventional learning or teacher-centered learning students only respond by giving short answers. The teacher asks prompting questions to explore students' abilities related to the Number Pattern material. Then students respond to the teacher's questions with short answers.

Figure 1. Questioning Component on Patterns, Series, and Number Sequences
Level 1 Questioning: The teacher pursues student thinking.

Teacher: Write the numbers next to each other on the first line. What pattern can be formed from the sequence of numbers in the first row? (Then students are asked to observe the pictures/photos contained in the book or through video showings presented by the teacher as shown below)

Students: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

Teacher: What patterns can be formed from the sequence of numbers?

Student: Odd number

Teacher: How do you know?

Student: From the numbers in the first row, that number contains odd numbers.

In level 1 questioning, the teacher asks questions. Then students are asked to observe the pictures in the book or through the video presentation presented by the teacher. Students give short answers and mathematical thinking of the answers. There was interaction between students; one student answered the question, and the other students listened to the answer.

Level 2 questioning: students begin to give or make questions. In level 2, the teacher proceeds to the material "Patterns and Number Sequences" Starting with "Number Patterns," Students are asked to open LKS 1. Illustrate some pictures and facilitate students to discuss and students prepare conclusions. Students ask questions from the conclusions given, listen to explanations, and discuss.

Level 3 questioning: students take the initiative to ask. In stage 3, the teacher asks students to present the results of group discussions and ask each other questions. The teacher asks, "What exactly is the pattern and numbers sequence?" to liven up the discussion and communicate with each other. Students discuss their initiative, asking "why" and keep asking until they find an answer. Students interact in the classroom; students feel comfortable expressing their opinions in the discussion process.

This study shows an increase in the level of questions in the math-talk learning community. As the question level increases, student interaction and participation in the discussion also increases. These results align with research from Legesse et al. (2020), which states that effective questions from teachers and students in the math-talk learning community can encourage deeper mathematical discussions and foster students' critical thinking skills. At level 1, the teacher dominates the questioning process, and students only give short answers. This process is similar to what Manouchehri & Enderson (1999) stated: in the early phases of learning, the teacher usually plays a more dominant role. However, interaction between students has started to show, a sign that a collaborative learning environment is being formed.

When moving to level 2, students begin to take the initiative to ask questions, indicating that they are starting to feel comfortable with the learning environment and be more active in the learning process. This concept supports research conducted by Hufferd-Ackles et al. (2004), who found that, in a math-talk learning community, students who were more active in the learning process tended to benefit more in terms of increased communication and understanding of mathematics. At level 3, students fully take the initiative to ask questions and discuss. They feel comfortable expressing their opinion and trying to understand the material. This is in line with research from (Sfard, 2000), who found that ineffective mathematics learning, students must be able to actively participate in the process of mathematical communication and actively contribute to the formation of their mathematical understanding.

Thus, this study's results indicate that applying a math-talk learning community can effectively encourage students' mathematics communication. This also confirms previous
research findings showing that a collaborative and interactive learning environment can enrich students' mathematics learning experience and help them better understand mathematics (Cengiz et al., 2011; Sherin, 2002).

**Component B: Explaining Mathematical Thinking**

Now let's look at the second component of the math-talk learning community: explaining mathematical thinking. Although this component is closely related to the questioning, the researcher focuses on explaining through each level in the math-talk learning community. In this context, students go from level 0 to level 3, learning becomes easier, and student can explain their answers (the teacher helps students during this activity).

Class situations that support explaining mathematical thinking are needed to increase students' confidence in expressing their opinion. This is necessary because, for some students, especially those who are shy, getting them to stand in front of their friends to convey a math idea can be daunting. Many students prefer to return to their seats after writing on the blackboard rather than standing in front of the class explaining the answers they found.

However, as explaining mathematical thinking in the math-talk learning community became more comfortable, students' efforts in explaining were assisted by their groupmates and other students. This support helps explain mathematical thinking progress from level 0 to level 3. As students learn to explain their mathematical thinking more fully and fluently, they make significant contributions that other students can then question or build upon and assess by the teacher.

**Figure 2. Explaining Mathematical Thinking Component on Patterns, Series, and Number Sequences**

Level 0 explains mathematical thinking: the teacher assists in improving students' answers. Students give short answers to questions given by the teacher. Teachers sometimes answer questions on their own without waiting for student responses. The interaction at level 0 of explaining mathematical thinking showed that students' explanations about their work focused on giving the correct answer, and the teacher did not seek more explicit strategies or thinking from students. In other words, the teacher sees student responses and provides answers.

Level 1 explains mathematical thinking: the teacher helps students by giving short answers. At this stage, the teacher transitions to a complete student explanation of mathematical thinking in the classroom by examining students' answers more deeply. In level 1, explaining mathematical thinking, an explanation is given when students share information about their mathematical thinking in response to questions posed by the teacher. The first attempt at a complete explanation was tiring for the students as they were reluctant to stand at the front of the room and answer several questions in a row. Many students prefer to talk privately to the teacher (who is often nearby).
Teacher: There is a difference between the numbers 1 with 3, 3 with 5, 5 with 7, and so on. What's the difference?

Student: Two

Teachers: Yes, that's right. So, he has a subtraction, that is, two. Well, then, from the difference between the numbers, we can make a number pattern like this:

\[1 = 1 \text{ or } 1 = 2.1 - 1\]
\[3 = 4 - 1 \text{ or } 3 = 2...-1\]
\[5 = ... - ... \text{ or } ... = ... \ldots - ...\]
\[7 = ... - ... \text{ or } ... = ... \ldots - ...\]

We write 1 is equal to 1, or we can write 1 equal two times one minus one. Then three equals four minus one or two times; how much less one equals three?

Student: Times two

Teacher: Yes, that's right. So, what numbers are the dots filled with for the number five?

Student: Six and one

Teacher: Why did you fill in six and one?

Student: (Students are silent momentarily) Because six subtract one, the result is five.

Teacher: Is there any other number which, if you subtract it, the result is five?

Student: (Silent, no one answered)

Teacher: Five equals six minus one, or we can write that five equals two times three minus one. If each point is filled in, we will find the formula for the tenth for the odd number pattern.

In the conversation, the teacher asks students to solve problems by giving examples and explaining solving strategies (Kazemi & Hintz, 2014). This conversation shows that the teacher needs the patience to help students explain their ideas, in line with what was expressed by Serow et al. (2014). There are many long pauses while students think about what to say, in line with the research of (Cengiz et al., 2011), which emphasizes the importance of giving students time to reflect and organize their thoughts. As indicated by students, many are reluctant to explain their answers in the teaching and learning process (Allen & Tanner, 2005). Students are accustomed to conventional learning, where students only say one or two words and then sit down; they are not used to identifying and explaining thought processes.

At level 2, explaining mathematical thinking begins after students feel comfortable communicating their ideas (Hufferd-Ackles et al., 2004). At this level, students still need probing and help to explain their thoughts to teachers and classmates. Students begin to gain confidence in expressing mathematical ideas, supporting the argument of Goos et al. (2002) that students need to develop confidence in communicating mathematics. Students feel that short answers alone cannot provide important information in the conclusion; they need more information. Furthermore, the class situation began to generate questions and answers in line with what was revealed (Chapin & Anderson, 2013).

At level 3, explaining mathematical thinking, students explain their answers confidently without help from the teacher, supporting the argument of Hufferd-Ackles et al. (2004) that teachers should encourage students to become more independent in communicating mathematics. The teacher encourages students to give more competitive answers, explains the meaning of points, and stimulates students to think creatively, supporting Sfard's (2000) finding that mathematical communication is an important part of understanding mathematics.
**Component C: Source of Mathematical Ideas**

Level 0 Source of Mathematical Ideas: teacher explains the answers to the questions. In the arrangement of the balls, there are 1 ball, 2 balls, 3 balls, 4 balls, and 5 balls in a row, so an array of numbers 1, 2, 3, 4, and 5 is formed, a natural number pattern. Students pay attention, understand the teacher's explanation, and respond to questions from the teacher.

![Figure 3. Source of Mathematical Ideas Component on Patterns, Series, and Number Sequences](image)

Level 1 Source of Mathematical Ideas: the teacher begins to use students' thinking as part of the mathematical content. The teacher becomes the main resource in learning, asking several questions to generate students' ideas: "If each of the dots is filled with patterns, what can be concluded from the number sequence?" Students give ideas in discussion.

Level 2 Source of Mathematical Ideas: students' strategies in solving math problems. The teacher follows up on the explanation and builds students' ideas for comparison. If there is an error, it is used as a lesson. Students are confident in expressing mathematical ideas and strategies even though they differ.

Level 3 Source of Mathematical Ideas: student strategies are constructed as mini-lessons. The teacher provides opportunities for students to explain new, different strategies and allows students to interrupt while others are explaining. Students spontaneously construct their mathematical ideas, positioning themselves as peer tutors.

**Component D: Responsibility for Learning**

Level 0 Responsibility for Learning: Students listen to the teacher's explanation, follow/imitate the work steps explained by the teacher, and do not respond to the work of their friends.

Level 1 Responsibility for Learning: The teacher gives feedback, asking students who understand to help other students. Students help other students, working on the "prerequisite test" practice questions on the LKS consisting of two questions.

![Figure 4. Responsibility for Learning Components on Patterns, Series, and Number Sequences](image)
Level 2 Responsibility for Learning: figure 3 shows that the teacher asks students about mathematical ideas and the reasons for these ideas and asks students to write conclusions. Students use their own words to explain conclusions and clarify if there are errors.

Level 3 Responsibility for Learning: the whole class acts as a teacher; when students do not understand, students help other students to understand. The teacher directs students to take responsibility for evaluating each other, providing support, and helping when needed. Students take the initiative to clarify each other, help to understand, and correct mistakes.

The class moved quickly from level 0 to level 3 in all math-talk learning community framework components. This movement can be attributed in part to the use of the curriculum that supports a focus on student thinking and explaining ideas. At the end of 2020, the Ministry of Research, Technology, and Higher Education Indonesia said that the idea of learning in Indonesia is to create learning that favors students, learning that liberates thinking, and create a positive learning mindset that builds their thinking (Hadi et al., 2022).

Students no longer depend on the teacher to ask questions and argue. Students become more comfortable and can explain their answers. This is because class situations that support explaining mathematical thinking are needed to increase students' confidence in expressing their opinion. If teachers can manage the classroom, the teaching and learning process will run well, but if they cannot, the students will be difficult to control (Sevrika et al., 2022).

Sometimes students use the language of the domain of mathematics to carry on math talk both to explain their ideas or extend the work of the order. Hikmah & Maskar (2020) stated that the difficulties experienced by students in learning mathematics were that they tended not to pay attention to the material and did not read the questions well; besides that, students also did not know the names and shapes of mathematical symbols. This makes it difficult for students to learn mathematics, making them less than optimal in achieving learning outcomes. With a math-talk learning community, students gain self-efficacy to explain in front of the classroom. Students give more competitive answers, explain the meaning of the point, and stimulate students to think creatively. From the math-talk learning community, students make strategies for solving math problems.

Murray (2020) found that math-talk communities are an effective way for children to improve conceptual mathematical understanding. This aligns with this study, which found that the math talk learning community could improve students' mathematical communication.

**Student Responses to Math-Talk Learning Community**

The researchers analyzed student responses to determine how they fit into the emerging math-talk learning community. The student's answers were evaluated in a closed questionnaire using a Likert scale. The assessment criteria for students in this math-talk learning community are (1) interest in mathematical patterns, series, and number sequences material; (2) cooperative education; (3) willingness to participate in the study; (4) participation in education; (5) confidence in asking and answering questions; (6) communicate bold ideas; (7) prepared to resolve any issues; (8) like working in a group; (9) enjoy working individually; (10) make conclusions from the study. The following results were obtained from 30 sentences with 11 categories:
Students' behavior motivates them to achieve their academic goals. The feeling of support for the object influences behavior. There are many theories that there is a positive relationship between student behavior and academic achievement. In other words, students who have a positive attitude towards a particular subject will be more motivated to study, which leads to positive results. Conversely, students with a negative attitude towards the classroom are less motivated to study and earn less. Positive behavior is defined as behavior that can help students learn math, such as having fun in class, and negative behavior is defined as behavior that interferes with learning.

Figure 5 shows that the percentage of students responding to the math-talk learning community is good. Students have the courage, enthusiasm, and interest in learning using the math-talk learning community. This can be seen from the percentage showing good results for each category. The lowest percentage is 77%, and the highest is 85%, meaning that more than 50% of students respond well to the math-talk learning community, and activities in that learning can improve students' communication at each level. MTLC has benefited many aspects of student learning, including interpersonal skills, deeper learning of mathematical concepts, and improved motivation and engagement (Irvine, 2017b).

Mathematical communication is an ability that students should have. Students’ mathematical communication still needs to be developed so that mathematics teachers not only teach mathematics but also encourage students’ mathematics communication through activities that initiate creative and innovative learning (Rohid et al., 2019). Students' mathematical communication is stimulated at each stage of the math-talk learning community. Saylor et al., (2015) said, "After participating in math-talk learning community, all of the preservice teachers had a better understanding of math-talk learning community as a pedagogical approach, and many had experienced 'aha' moments in their own understanding of mathematical concepts."

From this research, it can be seen that the Math-Talk Learning Community (MTLC) method can have a significant impact on students' mathematical communication. This shows that this approach is feasible to be applied in the context of learning mathematics in other schools. It can also be a consideration for educators and researchers to further explore and expand the use of MTLC in different mathematics topics and grade levels. These results also show that MTLC can increase students' motivation and participation in learning mathematics. Therefore, applying this method can help overcome challenges such as students' disinterest or aversion to mathematics.

This study only involved a sample of students from one school in Bandung, which may not reflect the general student population in Indonesia. Therefore, the results should be interpreted cautiously when applied to other contexts. The research method used is qualitative, which focuses more on in-depth descriptions and understanding of phenomena rather than statistical generalizations. While this provides a good understanding of how MTLC can affect
students' mathematics communication, it does not provide a quantitative effect size that can be compared directly with other teaching methods.

For further research, it would be useful to involve a wider and more diverse sample, perhaps from schools in different parts of Indonesia, to evaluate how MTLC can be applied in different educational contexts. In addition, longitudinal studies can also be carried out to see how the MTLC effect on students' mathematical communication develops over time. This could provide further insight into the long-term effects of this approach. Investigating the impact of MTLC on other aspects of mathematics learning, such as problem-solving, conceptual understanding, or attitudes toward mathematics, could also be an interesting research subject.

CONCLUSIONS
Based on the results of research that has been done, it can be concluded several important things. First, the Math-Talk Learning Community (MTLC) learning method has proven effective in improving students' mathematical communication skills. Through this method, students can discuss, ask questions, and explain their thoughts in the context of mathematics, which ultimately contributes to a better understanding of mathematical concepts. Second, after learning with MTLC, students become more comfortable and able to explain their answers better. Students feel more confident speaking in front of the class and can develop various strategies for solving math problems. In addition, MTLC also encourages students' active participation in the learning process, which is an important aspect of education. Third, although developed some time ago, the Community Math-Talk framework remains relevant and provides important guidance for teachers in implementing this method. However, further research is recommended to update and extend this framework, especially to further explore how Community Math-Talk level-3 and level-4 can be applied.

ACKNOWLEDGMENT
The writers thank the Indonesian Endowment Fund of Education (LPDP) for financial support.

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