Mathematical-Analytical Thinking skills: The Impacts and Interactions of Open-ended Learning Method & Self-Awareness (Its Application on Bilingual Test Instruments)

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Abstract

Analytical thinking is a skill to unite the initial process, plan solutions, produce solutions, and conclude something to produce conclusions or correct answers. This research aims to 1) determine whether there are differences in students' mathematical, analytical thinking skills between classes that use the Open-ended learning method and classes that use the lecturing method, 2) to find out whether there are mathematical, analytical thinking skills differences between students with high, moderate, and low self-awareness criteria, and 3) to find out whether there is an interaction between Open-ended learning method and self-awareness toward students' mathematical-analytical thinking skills. This research employs a quasi-experimental design. Based on the data and data analysis, this research is mixed-method research, and the design used in this research is the posttest control group design. This research was conducted on students who have studied the Real Analysis Courses. Based on the results of hypothesis testing, it was found out that, first, there are differences in students' mathematical-analytical thinking skills between the class that uses the Open-ended learning method and the class that uses the lecturing method. Second, there are mathematical-analytical thinking skills differences between high, moderate, and low self-awareness criteria. Third, there is no interaction between the Open-ended learning method with self-awareness of students' mathematical-analytical thinking skills.

Keywords: Mathematical Analytical Thinking Ability; Open-ended; Self-Awareness

Introduction

Thorough mathematical learning emphasizes pattern discovery or general properties of variables, numbers, and other things. However, mathematics learning tends only to calculate, memorize, remember, and understand. The opportunity to seek mathematic experiences is not by memorizing but instead applying (Setiyani et al., 2020; Thahir et al., 2019). Mathematics learning in formal education is divided into several parts (levels), namely elementary school, middle school, and college. Low-level mathematical thinking skills can be found at elementary and junior high school levels. In contrast, high-level mathematical thinking skills can be found at high school and college levels.

One of the high-level mathematical thinking skills is analytical thinking skill. According to Bloom's theory, analytical thinking is a part of the cognitive domain (Amelia et al., 2016; Giani et al., 2015; Yuliandini et al., 2019). Thinking Skills in the cognitive domain are divided into six levels, namely: remembering (C1), understanding (C2), applying (C3), analyzing (C4),
evaluating (C5), and creating (C6) (Yusuf & Widyaningsih, 2018). Analytical thinking is higher than remembering, understanding, and applying. This skill is needed to respond to teaching materials and find concepts to solve problems (Dafrita, 2017). Before possessing the analytical skill, the students must possess three lower levels of skills: remembering, understanding, and applying (Ilma et al., 2017).

One of the Mathematics Education Study Program courses at UIN Raden Intan Lampung is the Real Analysis Course. In general, the Real Analysis Course is the most challenging subject for students to understand. Various learning strategies have been implemented, but the results are still unsatisfactory. Students’ scores were unsatisfactory, as shown in the Real Analysis course’s learning outcomes in the 2016-2017, 2017-2018, and 2018-2019 academic years, where the average score was not more than 60. These results might be caused by students’ minimum theorem analysis activities (Perbowo & Pradipta, 2017). Students need to possess analytical thinking skills to develop new knowledge and innovation (Astriani et al., 2017). The Real Analysis Course emphasizes proof that cannot be separated from how a student uses theorems and axioms. Mathematical proof plays a vital role for mathematicians because proof is an effective method to test knowledge (Kartika & Yazidah, 2019).

Several studies on analytical thinking have been carried out, including mapping prospective physics teachers’ analytical thinking skills conducted by Faiz Hasyim. He explains that analytical thinking skills are the weakest ability possessed by students (F. Hasyim, 2018). Astriani explain that the indicators for determining the point of view of an object in the general biology course were low (Astriani et al., 2017). Ilma also describes students' analytical thinking profiles in terms of visualizer and verbalizer cognitive styles in solving problems were good (Ilma et al., 2017).

Furthermore, one factor that can also affect the Real Analysis learning outcomes is the attitude factor (affective). Self-awareness covers the thinking awareness in choosing a method, strategy, and algorithm (Akbar et al., 2018; Chairunnisa et al., 2019; Fatmawati & Primanita, 2019; Ismayani et al., 2016; Kurniawati & Kusumurious, 2018; Kusumaningrum et al., 2016; Nuraida & Sunaryo, 2017; E. T. Putri et al., 2019; Y. Putri & Primanita, 2019; Sari et al., 2019). Silaholo argues that self-awareness is a physical and psychological process that is inextricably linked to mental life in terms of life goals, emotions, and cognitive processes (Sihaloho, 2019). The main aspects of self-awareness are attention, wakefulness, architecture, recall of knowledge, and emotion. Self-awareness is the foundation of almost all elements of emotional intelligence. It is an essential first step to understand themselves and to change. A person cannot control something he does not know (Hilapok, 2017). Patton states that self-awareness is a trait in Emotional Intelligence, and it is at this point that EQ development can begin (Ningtyas & Risina, 2018).

Research on self-awareness has been conducted. One research examines the relationship between Self-awareness and scientific literacy in Chemistry learning. This research finds no connection between scientific literacy and students’ self-awareness (Pujiati & Retariandalas, 2020). Another research was conducted by Nu’man who states that there is no Self-awareness difference between male and female students (Nu’man, 2019). Dewi states that there is no significant correlation between self-awareness and students’ learning motivation (Dewi et al., 2020). The difference in this research is that the researcher wants to see each category of self-awareness (high, moderate, low) on students’ analytical skills.

The Open-ended learning method emphasizes Open-ended problems to students so that their mathematical analytical thinking ability can be developed through problem-solving, various
techniques, method, or strategies to find solutions (Baba & Shimada, 2019; Bernard & Chotimah, 2018; H. T. Damayanti & Sumardi, 2018; Kurniawan et al., 2018; Lubis et al., 2019; Sapta et al., 2019; Wulansari, 2019), appreciates students’ answers, and pay attention to their cognitive domain (Magelo et al., 2019). Open-ended learning method can develop mathematical-analytical thinking skills in obtaining correct conclusions or answers. In an Open-ended learning method, several different ways or algorithms are often found but lead to the same conclusion. The Open-ended learning method allows students to investigate various ways based on their ability (Hidayat & Sariningsih, 2018). This learning requires students to improvise and develop various methods or approaches to obtain answers (M. Hasyim & Andreina, 2019).

Open-ended learning method can make mathematical proof free from thinking rigidity and develop flexible thinking (fluently). Sariningsih & Herdiman found that the Open-ended learning method is better than conventional learning in developing statistical reasoning and creative thinking skills (Sariningsih & Herdiman, 2017). Students’ increased critical thinking skills and self-confidence in mathematics learning are influenced by the Open-ended learning method (Novtiar & Aripin, 2017). The Open-ended learning method can also increase students' interest in mathematics learning (Aedi, 2018). Besides, the Open-ended learning method also affects mathematical reasoning (Lestari et al., 2016).

Based on the facts and several previous studies, Open-ended learning method learning is needed for Real Analysis Course to contain mathematical-analytical thinking skills. It is expected that Open-ended learning methods can produce positive impacts on Real Analysis Course. This research examines this learning on analytical thinking skills that have never been done before. Thus, this research is focused on the effect of the Open-ended learning method on mathematical-analytical thinking skills based on tertiary level students' self-awareness.

**The Research Methods**

This research employed the Quasi-experimental of pretest-postest control group design. This design has a control group but does not fully function to control the external variables that affect the implementation of the experiment. The population of this research was the fifth-semester students of the Mathematic Study Program of UIN Raden Intan Lampung in the academic year of 2019/2020. The students of classes A and B were chosen as the samples. The Open-ended learning method was used in the experimental class, while the lecturing method was used in the control class. The research data were collected through observation, interview, questionnaire, test, and documentation. The instruments used in this research were a questionnaire for self-awareness and a mathematical, analytical ability test. A good instrument must meet two requirements, namely valid and reliable. The tests distributed to the students were description tests to measure their mathematical, analytical thinking skills. The collected data were tested using a two-way ANOVA test.

**The Results of the Research and the Discussion**

The results of the research and discussion of this research came from several test stages. The tests consisted of instrument tests, prerequisite tests, and hypothesis testing, which are the characteristics of mixed-method research. The first test was the research instrument test. This research consisted of three variables: the learning method (Open-ended learning method and lecturing method), mathematical, analytical thinking ability, and self-awareness. The learning method variables were applied to the experimental class and the control class. Simultaneously, the tests and the self-awareness questionnaires were used to investigate students' mathematical
analytical thinking ability and self-awareness. The following research step was testing the hypothesis using ANOVA. The tested data are described in Table 1.

**Table 1. The Descriptive Data of Mathematical-Analytical Thinking skills Based on Learning Method and Self-Awareness Category**

<table>
<thead>
<tr>
<th>Learning Methods</th>
<th>Self-Awareness</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open-ended learning method</td>
<td>High Self-awareness</td>
<td>56.6667</td>
<td>17.81853</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Moderate Self-awareness</td>
<td>16.8667</td>
<td>6.03403</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Low Self-awareness</td>
<td>4.6667</td>
<td>2.17945</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>24.3939</td>
<td>22.92234</td>
<td>33</td>
</tr>
<tr>
<td>Lecturing Method</td>
<td>High Self-awareness</td>
<td>44.5455</td>
<td>9.56414</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Moderate Self-awareness</td>
<td>13.0588</td>
<td>6.38818</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Low Self-awareness</td>
<td>.7273</td>
<td>1.00905</td>
<td>11</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>18.4615</td>
<td>18.50867</td>
<td>39</td>
</tr>
<tr>
<td>Total</td>
<td>High Self-awareness</td>
<td>50.0000</td>
<td>14.83594</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Moderate Self-awareness</td>
<td>14.8438</td>
<td>6.42128</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Low Self-awareness</td>
<td>2.5000</td>
<td>2.56495</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>21.1806</td>
<td>20.71288</td>
<td>72</td>
</tr>
</tbody>
</table>

Table 1 displays the two learning methods applied, namely the Open-ended learning method and the lecturing method. Furthermore, the category of self-awareness consisted of high, moderate, and low. In the Open-ended learning method, the high self-awareness category obtained a mean score of 56.667. It was much higher than in the moderate and low self-awareness categories (16.867 and 4.667). Therefore, it can be assumed that in the Open-ended learning method, the higher the students’ self-awareness, the higher their mathematical-analytical thinking skills. In the lecturing method, the high self-awareness category obtained a mean score of 44.545. It was much higher than the moderate and low self-awareness categories (13.059 and 0.727). Based on the observed data, it can also be assumed that in the lecturing method, the higher the students’ self-awareness, the higher their mathematical-analytical thinking skills. The next step was testing these data to determine whether there were differences in each hypothesis or research question using the ANOVA test assisted by SPSS 17.

**Table 2. The Hypothesis Testing Results**

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>25795.069a</td>
<td>5159.014</td>
<td>5</td>
<td>72.980</td>
<td>.000</td>
</tr>
<tr>
<td>Intercept</td>
<td>35202.620</td>
<td>1</td>
<td>35202.620</td>
<td>745482</td>
<td>.002</td>
</tr>
<tr>
<td>Learning Methods</td>
<td>745482.10546</td>
<td>1</td>
<td>745482</td>
<td>25136124</td>
<td>.000</td>
</tr>
<tr>
<td>Self-Awareness</td>
<td>12568.062</td>
<td>2</td>
<td>177790</td>
<td>122199185</td>
<td></td>
</tr>
<tr>
<td>Learning Method Self-Awareness</td>
<td>122199.185</td>
<td>2</td>
<td>177790</td>
<td>4665584</td>
<td>.000</td>
</tr>
<tr>
<td>Error</td>
<td>122199.185</td>
<td>2</td>
<td>177790</td>
<td>4665584</td>
<td>.000</td>
</tr>
<tr>
<td>Total</td>
<td>62761.000</td>
<td>72</td>
<td>70691</td>
<td>30460653</td>
<td>.000</td>
</tr>
</tbody>
</table>

a. R Squared = .847 (Adjusted R Squared = .835)
The results of the hypothesis testing indicated that the first hypothesis or first research question, as can be seen in the "source" column (learning methods), obtained a significance value (Sig.) of 0.000 <0.05; therefore, H₀ was rejected. Thus, it can be concluded that there were differences in students' mathematical-analytical thinking skills between the experimental class and the control class. The second hypothesis or second research question, as can be seen in the "source" column (self-awareness), obtained a significance value (Sig.) of 0.000 <0.05; therefore, H₀ was rejected. It can be concluded that there were mathematical-analytical thinking skills differences between students who applied the Open-ended learning method and the students who applied the lecturing method in terms of high, moderate, and low self-awareness. The third hypothesis or third research question, as can be seen in the "source" column (learning methods *self-awareness), obtained a significance value (Sig.) of 0.185 > 0.05, which means that H₀ was accepted. Therefore, it can be concluded that there was no interaction between the Open-ended learning method, lecturing method, and self-awareness on students’ mathematical-analytical thinking skills.

The results of the hypothesis testing can be a reference for researchers to determine which learning is better to improve students' mathematical-analytical thinking skills and which self-awareness can affect students' mathematical analysis thinking skills. The ANOVA test showed a significant difference between the Open-ended learning method and the lecturing method. The difference was supported by the mean value of students' mathematical-analytical thinking skills in the Open-ended learning method (26.067) and the lecturing method (19.444). It can be concluded that the Open-ended learning method had a more significant influence to improve students’ mathematical-analytical thinking skills.

1. **The Effect of Open-ended Learning Method on Mathematical-Analytical Thinking Skills**

   Based on the results of hypothesis testing, there were differences in students' mathematical-analytical thinking skills between the class that applied the Open-ended learning method and the class that applied the lecturing method. These results can only point out the differences between the Open-ended learning method and the lecturing method. However, they cannot reveal which method had positive effects on students' mathematical-analytical thinking skills. To determine the positive influence, it was necessary to look at the average score of mathematical-analytical thinking skills between the two methods. The average score of mathematical thinking ability of students who applied the Open-ended learning method was better than the average score of students who applied the lecturing method. Therefore, the Open-ended learning method has a better effect on students' mathematical-analytical thinking skills than the lecturing method.

   The Open-ended learning method indirectly represents constructivism learning which emphasizes the uniqueness of each individual. It believes that students are not empty bottles; instead, they tend to have the ability to be developed (Hendikawati et al., 2019; Palapasari et al., 2017; Rizky & Zanthy, 2019). The Open-ended learning method allows students to discuss freely and participate actively in learning by systematically expressing their ideas. Those factors are related to mathematical-analytical thinking skills, which significantly support students’ thoughts. During the learning process, the discussion can help students mutually express their ideas and develop their mathematical reasoning and analytics. The lecturer can also identify students who have high, moderate, and low mathematical-analytical thinking skills during the learning process. This identification is necessary to equalize the transfer of knowledge to all students to achieve learning objectives.
The discussion process in the Open-ended learning method is beneficial to develop students' mathematical, analytical thinking processes so that three aspects or indicators of mathematical, analytical thinking skills (differentiating, organizing, and attributing) can be developed equally. The differentiating process in analytical thinking provides identification or early symptoms of students' thought patterns. In this process, the students must distinguish, sort, and select essential parts or components to compile a systematic structure in initiating a mathematical proof. The information is then discriminated against as relevant or irrelevant information to build the initial structure. The discrimination process of relevant and irrelevant and significant and insignificant information can be a strong foundation for the theorems. The Open-ended learning method develops a free discussion of issues that are relevant to the differentiating process. The lecturers can identify students' learning difficulties and understanding, which in turn, the correct understanding can be generated. The differentiating process is essential in developing students' mathematical-analytical thinking skills because this process is the initial process for students to receive and select important information. The following is an example of the differentiating process between the Open-ended learning method and the lecturing method.

**Figure 1. An Example of Mathematical-Analytical Thinking Skills in the Open-ended Method**

Figure 1 displays the differentiating process that becomes the basis for determining answers to the next question to become systematic, rational, and logical. There are two ways to prove the convergent theorem in the Real Analysis Course (if and only if) or known as bi-implication. The theorem which contains bi-implications must be proved in two directions. It can be seen that students who were given Open-ended learning methods can answer the initial questions listed in questions a, b, c, and d. Questions a, b, c, and d were differentiating questions that form the basis to collect essential or unimportant information to answer the following question. Therefore, if the differentiating stage is incorrect, the next question can usually be incorrect or unanswered. On the other side, the students could not correctly answer questions o, p, and q. Questions o, p, and q contained differentiating stages so that the student's answers to the next question were incorrect and not answered. In question o, students answered, "n ≥ K(ε)" which should be "k ≥ K_m(ε)." On question p, students answered "|x_n - x| < ε" which should be "|x_k - x| < ε." In question q, the student answered "X_m," which should be "X."
Figure 2. An Example of Mathematical-Analytical Thinking Skills in the Lecturing Method

Figure 2 shows that the students’ differentiating process was not well developed. Questions a, b and c were answered incorrectly so that the students were having difficulty in answering the next question. Consequently, all answers were incorrect or even not answered. Unanswered questions are the effect of flawed logical algorithms and basic algorithm understanding. The data clearly showed the answer differences between students given Open-ended learning methods and students who were given lecturing methods. In the Open-ended learning method, the students were grouped to discuss worksheets or open problems related to proving the theorem. The Open-ended learning method emphasizes fluency (flexibility in thinking). It will motivate students to openly express ideas, both right and wrong (Hidayat & Sariningsih, 2018; A. Muchlis et al., 2018; Rustyanti et al., 2019; Sholikhah et al., 2018).

The next stage or aspect of mathematical, analytical thinking skills is organizing. It is a process that involves the identification of elements of communication so that a coherent and mutually sustainable structure can be formed. At this stage, the students must develop communication to connect the elements or information obtained to be systematic, structured, and coherent. The Open-ended learning method properly facilitates this aspect because they could use their knowledge and mathematical skills. Knowledge and skills developed thoroughly and comprehensively will significantly assist the students in compiling pieces of information or elements. The details can be seen in students’ answers displayed in Figure 3.

Figure 3. An Example of Mathematical-Analytical Thinking Skills in the Open-ended Method

Based on the students’ answers in the second question, the students’ mathematical, analytical thinking skills, especially in the organizational aspect, have developed well. The steps or parts have been arranged logically to form a complete and meaningful unity to become a theorem. In the second question, the students were required to prove that there was only one limit
in the real numbers. The proof used by students was indirect proof, so that they must take an example. However, their argument contradicted the resulting theorem. The students’ thinking flow in the Open-ended learning method contained no confusion. Instead, they can logically connect the arguments. The good results in the organizing aspect in the Open-ended learning method were caused by open problems that invited the students to make extensive use of mathematical knowledge and skills without feeling doubtful and afraid of being wrong. Furthermore, these results can be compared to the students’ answers in the lecturing method.

Figure 4. An Example of Mathematical-Analytical Thinking Skills in the Lecturing Method

Figure 4 reveals that students made significant leaps in answering the theorem. Building a concept or knowledge structure can be better with significant leaps in thinking as long as they are strong. However, sometimes some people might experience a miss or loss (disappear suddenly due to memory loss). It is highly recommended to build knowledge or concept to build a continuous and logical small leap. This is a feature of constructivism in building knowledge (Argarini et al., 2019; E. E. Muchlis & Maizora, 2018; Mutmainnah et al., 2019; Supardi et al., 2019; Yati et al., 2018).

Attributing is an essential aspect of mathematical, analytical thinking skills. This stage involves deconstruction to determine an objective of the selected elements in building a structure. In another sense, this stage can also be said as the labeling or the conclusion stage of all supporting elements. This aspect can be seen at the end of an answer because it is related to the conclusion. In the Open-ended learning method, the students were encouraged to conclude their group works. The conclusions will be responded to by other students, and then differences will usually emerge. The differences are then analyzed to determine whether the differences are fundamental and tend to be wrong or just differences in the writing of mathematical symbols. At this stage, the lecturer as the facilitator provided confirmation and corrections to the differences to produce correct and logical conclusions. Figure 5 provides perspectives or views of the attributing aspect based on the students’ answers.

Figure 5. An Example of Mathematical-Analytical Thinking Skills in the Open-ended Method
Figure 5 displays that the students’ answer all the questions correctly. At the end of the answer, there is a confirmation question related to the theorem’s conclusion. The students can answer and conclude the direction and purpose of the theorem. The students in the Open-ended method made a decision based on the information and process they worked on so that the attributing process was easy to do. The aspects of mathematical, analytical ability are interrelated because they are connected and reciprocal. Simultaneously, each of these lines can be cut off or broken but can also be connected. It means that when students gather information and select relevant information, they must think about the impact and consequences of collecting the information so that conclusions can be obtained. The information should be organized to be logical and systematic. When the students carry out organizational processes, they must also think about the information obtained and labeling or concluding.

On the other hand, students’ answers in the lecturing method were different. In the aspect of drawing conclusions or attributing, many students made mistakes which indicated that they answered the questions carelessly without knowing the logical flow. The following is an overview of students’ answers during the lecturing method.

Figure 6. An Example of Mathematical-Analytical Thinking Skills in the Lecturing Method

Based on the answers, it can be seen that they can partially answer the question correctly. However, they cannot provide conclusions or labeling on the last answers. The students’ thought lines were not fully organized in their answers, as depicted in the students’ thought lines in the Open-ended learning method. It is clear that the Open-ended learning method positively influenced concept reconstruction or knowledge building.

One of the advantages of the Open-ended learning method is that the students with low mathematical skills can actively participate in responding to problems. In the Open-ended learning method, each individual is given the freedom to solve the problems openly. In the process, students with high or low skills can be seen and differentiated. There will be a noticeable difference between solving problems between students with high skills and students with soft skills during discussing and working on questions or problems. They will have a method or strategy that represents their thinking ability. Therefore, the Open-ended learning method provides various ways and strategies to solve problems (Dilla et al., 2018; Kadarisma, 2018; Ulya & Rahayu, 2018; Widiastuti & Putri, 2018; Witoko & Wardono, 2019). It will be helpful for lecturers to develop students’ mathematical, analytical thinking skills because students with low ability can solve problems using their way. The answer or strategy that leads to the correct answer or solution can be used as an alternative solution. However, if the strategy or method is wrong,
the lecturer can find out the errors in analyzing the problem. It can be used as a reference to correct students’ analytical mindset.

Students with low mathematical skills in the Open-ended learning method try to solve problems using the strategies or methods. In developing this strategy, the students are indirectly motivated to provide evidence, explanation, and other efforts. The same condition also occurs in students with high mathematical skills. All students try their best to motivate themselves to provide answers or evidence. The enthusiasm and motivation are the effect or impact of discussion and giving open problems in the Open-ended learning method. Discussions that involve all students affect their motivation and desire to solve the problems in their way. Curiosity and the desire to come up with the best ideas are also factors of students’ motivation.

Communication training is needed in delivering ideas in an Open-ended learning method. All the efforts made by students in the learning process are a collection of learning experiences that are very useful for them to build concepts or theorems. These learning experiences help them in choosing a strategy and answering problems.

2. The Influence of Self-Awareness (High, Moderate, and Low) on Students' Mathematical-Analytical Thinking Skills

The affective domain of attitude cannot be underestimated in the learning process. No matter how good the methods or techniques, if the affective domain is ignored, it will significantly affect the learning results. This research reveals various exciting facts and phenomena where the affective domain has a significant influence on the learning process. One of the affective domains that need to be paid attention to is self-awareness. In learning, someone cannot know whether he is right or wrong without awareness. Self-awareness and can also be referred to as sanity. It is a term to describe an awareness in realizing and understanding oneself, knowing his existence, knowing what strategy to choose, and knowing the condition and state he is in (Akbar et al., 2018; Chairunnisa et al., 2019; Fatmawati & Primanita, 2019; Ismayani et al., 2016; Kurniawati & Kusmuriyanto, 2018; Kusumaningrum et al., 2016; Nuraida & Sunaryo, 2017; E. T. Putri et al., 2019; Y. Putri & Primanita, 2019; Sari et al., 2019). In mathematics learning, self-awareness tends to be interpreted as a state or condition to question oneself whether the strategy, systems, logics, and rationale in solving mathematical problems are correct or not. Mathematical-analytical thinking skill is an ability that will be significantly helped by self-awareness because in building mathematical analysis skills or mathematical proof, students might be confused by where to start and what to do next. Self-awareness has become an affective domain that can help students develop their cognitive domain. Students with high self-awareness will become aware of the method or strategy in taking important information, synchronizing systematically, and concluding correctly.

The research results reveal differences in mathematical-analytical thinking skills between students with high, moderate, and low self-awareness. The research results also show that the average score of students with high self-awareness is better than the average score of mathematical thinking skills of students with moderate or low self-awareness, both in Open-ended learning method and in lecturing method. Self-awareness is crucial for every individual in the learning process. The high self-awareness possessed by students in learning will indirectly develop their self-confidence and avoid inferior feelings. A high self-awareness will make individuals aware of what will be done to solve problems. Therefore, self-awareness will affect students’ self-confidence. High self-awareness will make students find solutions to the problems.
After each problem has been appropriately resolved, self-confidence will appear, and inferior feelings will go.

Self-awareness in practice also helps students communicate effectively. Mathematical communication is an ability that students possess to express or convey ideas, opinions, and thought patterns, either orally or written (Aminah et al., 2018; D. Damayanti et al., 2018; Harahap & Harahap, 2018; Wahyuni et al., 2018; Wijaya & Afrilianto, 2018). Students with high self-awareness tend to master and can solve problems due to being confident in choosing strategy. This confidence will continue to develop students' mathematical communication ability. Confidence will make students communicate fluently with other students so that the communication can be effective to build knowledge, concepts, or theorems.

Many students complain that the Real Analysis Course is challenging. In Real Analysis Course, students are expected to solve problems and find solutions related to mathematical proofs in other mathematics courses such as calculus, geometry, trigonometry, and others. Many students do not know how to start a proof of a theorem which resulted in a sense of confusion that makes them give up easily. Honing students’ affective domain in self-awareness will make them genuinely aware that the problem at hand must have a solution and can be adequately resolved.

Self-confidence is an essential factor in solving problems. It will become a motivation for them to solve problems. Encouragement and motivation to solve problems ultimately lead to students' mathematical creative thinking skills. There is pressure and encouragement to make students try, choose, and develop strategies to solve problems (Dilla et al., 2018; Happy & Widjajanti, 2014). The creative thinking ability is expected to emerge in solving mathematical problems, especially mathematical proof. This research reveals how students with moderate and low self-awareness answer questions, both in Open-ended learning and lecturing methods. The following is an illustration of each level of self-awareness.

Figure 7. A Sample Answer by Student with High Self-Awareness in the Open-ended Method

Figure 7 shows that students with high self-awareness answered the question correctly. This problem is related to proving a convergent tail sequence. It is necessary to collect information and then organize and communicate it to obtain correct conclusions. Students' answers also indicate that they are confident because they can answer correctly with minimum scribbles. The students organized some of the previous theorems, namely the convergence theorem, the tail sequence theorem, the limit theorem, and even the sequence definition. There are at least four theorems that they must master and apply to prove the problem.
The students' answers indicated that they could partially provide correct answers. Based on these answers, it can be seen that the students were confident in answering, although they could not organize the critical information properly. The students did not fully understand the tails sequence theorem and the limit theorem. The students obtained some vital information. However, when they organize the information, they forgot or confused it with the previously obtained theorems. A high self-awareness allows students to acquire and structure concepts or theorems. Reinforcement is still needed by implementing better learning strategies. Furthermore, the student’s answers with moderate self-awareness in the Open-ended learning method can be seen in Figure 9.

The second question belongs to the moderate level. Students with moderate self-awareness could not correctly answer one question while the rests were correct and could be answered. The error was taking $K$ as a natural number which should correspond to epsilon ($\varepsilon$). However, the students answered with $K$ only. The error made by students were few because the problem factor only proved the convergence theorem. Furthermore, the answers by students with moderate self-awareness in the lecturing method can be seen in Figure 10.
The students with moderate self-awareness in the lecturing method only correctly answered three out of six questions. It can be seen that some students were indecisive in answering, especially in concluding the last answer. Many factors influence this phenomenon, including self-awareness, learning methods, and students’ mathematical skills. The following is a sample answer by students with low self-awareness in the Open-ended learning method:

![Image](image1.png)

**Figure 11. A Sample Answer by Student with Low Self-Awareness in the Open-ended Method**

The students' answers show that they can answer given definitions. The first question is a definition related to concepts understanding. Even though the questions are classified as easy, there are answer differences between students with low self-awareness in the Open-ended learning method and the lecturing method. The following is a sample answer by the student with low self-awareness.

![Image](image2.png)

Student's answers reveal that the students with low self-awareness cannot answer the questions correctly because they did not provide proof. The questions defined the sequence and understanding. The answers to these questions can be done by memorizing (the lowest level in mathematics). However, the students could not understand or remember the definition. The students have not mastered collecting necessary information, organizational processes, mathematical communication, and labeling or inference. The lecturing method did not support students' mathematical-analytical thinking skills development.

3. **The Interaction between Open-ended Learning Method and Self-Awareness**

Based on the results of hypothesis testing, it can be concluded that there is no interaction between Open-ended learning method and self-awareness on students' mathematical-analytical thinking skills. The test reveals that the Open-ended learning method, lecturing methods, and self-awareness do not affect mathematical-analytical thinking skills. Furthermore, students who applied the Open-ended learning method obtained a higher mean value than the students who applied the lecturing method. Furthermore, students with high self-awareness have better mathematical-analytical thinking skills than students with moderate self-awareness. Also, students with moderate self-awareness have better mathematical-analytical thinking skills than students with low self-awareness. Furthermore, the meaning of no interaction shows that
differences in students' mathematical-analytical thinking skills follow the characteristics of factor A (learning). Thus, it can be concluded that the Open-ended learning method influence students with high, moderate, and low self-awareness better than the lecturing method.

**Conclusion and Suggestion**

The conclusions obtained in this research are: 1) There are differences in students' mathematical-analytical thinking skills between classes using Open-ended learning method and classes using lecturing method; 2) there are differences in students' mathematical-analytical thinking skills who have self-awareness high, moderate, and low; 3) there is no interaction between the use of Open-ended learning method and lecturing method with self-awareness of students' mathematical-analytical thinking skills.

Based on the conclusions, the researchers suggest further researchers use the Open-ended learning method to see its impact on other skills. This learning can also be used as a solution to improve analytical thinking skills. Also, the researchers urge the teachers to pay adequate attention to the self-awareness so that the learning process can run smoothly.

**References**


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