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A predator-prey model of rice plant, sparrow, rat, and snake

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ABSTRACT

Rice is one of the most widely consumed food sources in Indonesia. The success of the rice harvest can be influenced by several factors, one of which is plant pests. This can threaten national food security. Predator-prey mathematical model can be constructed to explain the relationship between rice plants with plant pests. The predator prey model consist of ODE system describing 2 predators level one (rats and sparrows), 1 predator level two (snake) and prey (rice plants). In this model, nine equilibrium points are obtained with analysis of the behavior of the model at each of its equilibrium points. We successfully simulated the model using hypothetical parameters and the results are agrees with the analysis behavior. Several factors will make plant pest populations decrease even lost from the population is the natural death and interaction of rice with plant pests.

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INTRODUCTION

The Indonesia is a agrarian country where most of the people work as a farmer (Anitasari, 2019; Mulyadi et al, 2020). According to the Central Bureau of Statistics (BPS) consumption of rice every year always increasing in Indonesia. However, data from BPS also shows that production rice fell in 2019-2021. This matter will be a problem if inadequate rice availability to meet the demand for rice. The decline in the level of rice production can caused by crop failure as a result of plant pests. Manueke et al (2017) in his research proves that sparrow (Passer sp.) and mice (Rattus argentiventer) is a pest main rice crop. In its habitat at fields, rats and birds are prey for snakes. (Sabrina, et al. 2015).

The relationship between rice plants with plant pests as well as predators level two can be constructed into a mathematical model that is a predatorprey model (Ardestani, 2020; Padilah et al, 2018; Sun et al, 2016). The predator are organisms that eat prey. The assumptions used in this model is prey growth satisfying logistic growth model (Streipert et al, 2022; Taufiq and Agustito, 2020; Dawed

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et al, 2014). The predator-prey model in this study discusses the 1 prey model (rice plants), 2 predators of level one that is the pests of rice plants (rats and sparrow) and 1 predator level two (snake). In this research, a predator-prey model will be constructed in the form of a system of linear differential equations. We will determine the equilibrium point and carry out a stability analysis at the equilibrium point to determine the behavior of the system. Based on the behavior of the system, several influencing factors will be investigated so that the number of pests can decrease from the population. We perform numerical simulations using the MATLAB software to support our analysis.

Therefore, the purpose of this research is to analyze the behavior of the mathematical predator-prey model of rice plants, sparrows, rats and snake. The organization of this paper are as follows. Method are described in Sec 2. Here we provide the research step. In Sec. 3 we construct a system of linear ordinary differential equation of predator-prey model. In Sec. 4 we conduct a stability analysis whereas the simulation is conducted in Sec. 5 Conclusions and remarks will be given in the last section.

METHOD

The research steps in this paper are Determine follows: (1)model as assumptions. Assumptions are used to form the limits of modelling; (2) Determine the variables and parameters of the model; (3) Construct a model by presenting the relationship between prey and predator into a system of linear differential equations; (4) Determine the equilibrium point with the help of Maple software; (5) Analyze the behavior of the model by looking at the stability of the eigenvalues of the characteristic equation; (6) Simulation are conducted using MATLAB with hypothetical parameter to support and to see the suitability between

the analysis and simulation; (7) Interpretation and discussion are given.

Predator-Prey Model

Organisms in this predator-prey model were divided into: rice (P), sparrow (B), rat (T), and snake (U). The assumptions used in this model are as follows, (1) prey and predator populations are closed so there is no migration; (2) There is no competition between level 1 predators; (3) no disease in both prey and predators; (4) The parameters used are positive while the variables are nonnegative; (5) Prey growth follows logistic growth.

Changes in Population of Rice (Prey) with respect to time

Changes in the rice population P were influenced by interactions with 2 first-level predators, namely rats and birds. If there is no interaction, the rice will grow by following the logistic growth model (assuming a maximum rice growth of 100%). The interaction between rats and rice with an interaction level of γ and the interaction between sparrows and rice with an interaction level of α causes a reduction in the amount of rice. Therefore, the model of population change over time is given as follows:

$$\frac{dP}{dt} = pP(1-P) - \alpha BP - \gamma TP \qquad (1)$$

Changes in Population of Rats (Predator) with respect to time

The growth of the rat population is due to the large number of interactions between rats and rice for *y*. Meanwhile, the reduction in the number of rat populations was affected by the number of rats interacting with level 2 predators, i.e. snakes with the amount of δ and natural deaths experienced by rats for *t*. Based on this description, the model of rats population change over time is given as follows:

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$$\frac{dT}{dt} = yPT - \delta UT - tT \tag{2}$$

Changes in Population of Sparrows (Predator) with respect to time

The growth of the sparrow population is due to the large number of interactions between the sparrow and rice by α . Meanwhile, the reduction in the number of sparrow populations was affected by the large number of sparrows interacting with level 2 predators, i.e. snakes for ω and natural deaths experienced by sparrows for *b*. Based on this description, the model of sparrows population change over time is given as follows:

$$\frac{dB}{dt} = aPB - \omega UB - bB \tag{3}$$

Changes in Population of Snakes (Predator level 2) with respect to time

The growth of the snake population is due to the large number of interactions between snakes and sparrows of w and interactions between snakes and rats of d. Meanwhile, the reduction in the number of snake populations is influenced by the number of snakes that experience natural death of u. Based on this description, a model of changes in the rat population over time can be made as follows:

$$\frac{dU}{dt} = wUB - dUT - uU \tag{4}$$

From (1)-(4), we can construct mathematical model of predator-prey between rice plants, sparrows, rats and snakes as follows:

$$\frac{dP}{dt} = pP(1-P) - \alpha BP - \gamma TP$$

$$\frac{dT}{dt} = yPT - \delta UT - tT$$

$$\frac{dB}{dt} = aPB - \omega UB - bB$$

$$\frac{dU}{dt} = wUB - dUT - uU$$
(5)

The equilibrium point can be obtained if the system satisfying

$$\frac{dP}{dt} = \frac{dB}{dt} = \frac{dT}{dt} = \frac{dU}{dt} = 0.$$

Thus, we obtained 9 equilibrium points as given by the following:

$$\begin{split} E_{1} &= (P^{*}, B^{*}, T^{*}, U^{*}) = (0,0,0,0) \\ E_{2} &= (P^{*}, B^{*}, T^{*}, U^{*}) = \left(0, \frac{u}{w}, 0, -\frac{b}{\omega}\right) \\ E_{3} &= (P^{*}, B^{*}, T^{*}, U^{*}) = \left(0, 0, \frac{u}{d}, -\frac{t}{\delta}\right) \\ E_{4} &= (P^{*}, B^{*}, T^{*}, U^{*}) = (1,0,0,0) \\ E_{5} &= (P^{*}, B^{*}, T^{*}, U^{*}) = \left(\frac{t}{y}, 0, -\frac{p(t-y)}{\gamma y}, 0\right) \\ E_{6} &= (P^{*}, B^{*}, T^{*}, U^{*}) \\ &= \left(-\frac{dp + \gamma u}{pd}, 0, \frac{u}{d}, -\frac{dpt - dpy + \gamma uy}{d\delta p}\right) \\ E_{7} &= (P^{*}, B^{*}, T^{*}, U^{*}) = \left(\frac{b}{a}, \frac{p(a-b)}{a\alpha}, 0, 0\right) \\ E_{8} &= (P^{*}, B^{*}, T^{*}, U^{*}) \\ &= \left(-\frac{\alpha u - pw}{pw}, \frac{u}{w}, 0, -\frac{\alpha \alpha u - apw + bpw}{\omega pw}\right) \\ E_{9} &= (P^{*}, B^{*}, T^{*}, U^{*}) \\ &= \left(\frac{\frac{b\delta - \omega t}{a\delta - \omega y}}{(a\delta - \omega y)(-\alpha d + \gamma w)}, -\frac{(aa\delta u - a\delta pw - \alpha \omega uy + b\delta pw - \omega ptw + \omega pwy}{a\delta - \omega y}, -\frac{at - by}{a\delta - \omega y}\right) \end{split}$$

with the Jacobian matrix

$$\begin{array}{cccc} (p(1-P)-pP-\alpha B-T\gamma & -\alpha P & -\gamma P & 0 \\ Ba & Pa-U\omega-b & 0 & -B\omega \\ Ty & 0 & Py-U\delta-t & -T\delta \\ 0 & Uw & Ud & Bw+Td-u \end{array}$$

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Stability Analysis of Equilibrium Points

Stability analysis was carried out at nine equilibrium points by looking at the eigenvalues at each point to determine the behavioral of the model. Due to limitations and capabilities, there are several equilibrium points where it cannot be known whether the eigenvalues of that point are negative or positive. In addition, there are several equilibrium points that have negative values, so further analysis is not carried out, because population values in real life will not be negative.

a) Stability analysis at E_1

If the equilibrium point E_1 is substituted into the Jacobian matrix, we get

$$J_1 \coloneqq \begin{bmatrix} p & 0 & 0 & 0 \\ 0 & -b & 0 & 0 \\ 0 & 0 & -t & 0 \\ 0 & 0 & 0 & -u \end{bmatrix}$$

with eigen values: p, -b, -t, -u. Because the values of all the parameters used are positive, the first eigenvalue will always be positive, so that the stability at point E_1 becomes unstable.

b) Stability analysis at E_2

$$E_{2} = (P^{*}, B^{*}, T^{*}, U^{*}) = \left(0, \frac{u}{w}, 0, -\frac{b}{\omega}\right)$$

Because the equilibrium point value of U^* is negative due to the assumption that all parameters are positive, for stability at point E_2 no further analysis is carried out.

c) Stability analysis at E_3

$$E_3 = (P^*, B^*, T^*, U^*) = \left(0, 0, \frac{u}{d}, -\frac{t}{\delta}\right)$$

Because the equilibrium point value of U^* is negative due to the assumption that all parameters are positive, for stability at point E_3 no further analysis is carried out.

d) Stability analysis at E_4

If the equilibrium point E_4 is substituted into the Jacobian matrix, we get

$$J_4 \coloneqq \begin{bmatrix} -p & -\alpha & -\gamma & 0 \\ 0 & a-b & 0 & 0 \\ 0 & 0 & -t+y & 0 \\ 0 & 0 & 0 & -u \end{bmatrix}$$

with eigen values: -p, a - b, -t + y, -u. Taking into account that all parameter values are positive, then the second eigenvalue is negative if a < b and the third eigenvalue is negative if t > y. The stability of point E_4 depends on the four eigenvalues, so that the stability of point E_4 is local asymptotically stable if a > b and t > y.

e) Stability analysis at E_5

If the equilibrium point E_5 is substituted into the Jacobian matrix, we get

$$J_{5} := \begin{bmatrix} \left(1 - \frac{t}{y}\right) - \frac{pt}{y} + \frac{p(t-y)}{y} & -\frac{\alpha t}{y} & -\frac{\gamma t}{y} & 0\\ 0 & \frac{ta}{y} - b & 0 & 0\\ -\frac{p(t-y)}{\gamma} & 0 & 0 & \frac{p(t-y)\delta}{\gamma y}\\ 0 & 0 & 0 & -\frac{p(t-y)d}{\gamma y} - u \end{bmatrix}$$

with eigen values:

$$\frac{at - by}{y}, -\frac{dpt - dpy + \gamma uy}{\gamma y},$$
$$\frac{1}{2} \frac{-tp + \sqrt{p^2 r^2 + 4pt^{2y} - 4pty^2}}{y},$$
$$-\frac{1}{2} \frac{tp + \sqrt{p^2 r^2 + 4pt^{2y} - 4pty^2}}{y}$$

Taking into account that all parameter values are positive, so that the first eigenvalue is negative if $t < \frac{by}{a}$, the second eigenvalue will be negative if $t < -\frac{y(-dp+\omega u)}{pd}$, the third eigenvalue will be negative if t > 0, and the fourth eigenvalue will be negative if t > y The stability of point E_5 depends on the four eigenvalues, so that the stability of point E_5 is local asymptotically stable if t < by/a, $t < -y(-dp + \omega u)/pd$, t > 0, and t > y.

f) Stability analysis at
$$E_6$$

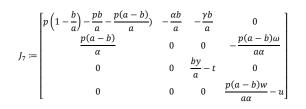
$$E_{6} = (P^{*}, B^{*}, T^{*}, U^{*})$$
$$= \left(-\frac{dp + \gamma u}{pd}, 0, \frac{u}{d}, -\frac{dpt - dpy + \gamma uy}{d\delta p}\right)$$

Because the equilibrium point value of P^* is negative due to the assumption that all parameters are positive, for stability at point E_6 no further analysis is carried out.

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g) Stability analysis at E_7

If the equilibrium point E_7 is substituted into the Jacobian matrix, we get



with eigen values:

 J_8

at - by	$a\alpha u - apw + bpw$	
_ <u></u> ,_	αα,	
$1-bp+\sqrt{-4a^2bp+4ab^2p+b^2p^2}$		
<u>2</u> a ,		
$1 bp + \sqrt{-4}$	$a^2bp + 4ab^2p + b^2p^2$	
2	а	
2	$\frac{a}{a^2bp+4ab^2p+b^2p^2}$	

Taking into account that all parameter values are positive, so that the first eigenvalue is negative if b < ta/y, the second eigenvalue will be negative if $b < -a(\alpha u - pw)/pw$, the third eigenvalue will be negative if b > 0, and the fourth eigenvalue will be negative if b > a The stability of point E_5 depends on the four eigenvalues, so that the stability of point E_5 is locally asymptotically stable if b < ta/y, $b < -a(\alpha u - pw)/pw$, b > 0, and b > a.

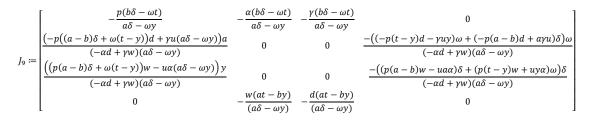
h) Stability analysis at E_8

If the equilibrium point E_8 is substituted into the Jacobian matrix, we get

$$\coloneqq \begin{bmatrix} \frac{\alpha u - pw}{w} & \frac{\alpha(\alpha u - pw)}{pw} & \frac{\gamma(\alpha u - pw)}{pw} & 0\\ \frac{\alpha u}{w} & 0 & 0 & -\frac{u\omega}{w}\\ 0 & 0 & \frac{\alpha \alpha \delta u - \alpha \delta pw - \alpha \omega uy + b \delta pw - \omega ptw + \omega pwy}{\omega pw} & 0\\ 0 & -\frac{\alpha \alpha u - apw + bpw}{\omega p} & -\frac{d(\alpha \alpha u - apw + bpw)}{\omega pw} & 0 \end{bmatrix}$$

The characteristic equations and eigenvalues are quite long and tedious so they will not be analyzed and presented i) Stability analysis at E_9

If the equilibrium point E_9 is substituted into the Jacobian matrix, we get here. For this reason, the stability analysis at E_8 will be investigated with numerical assistance.



The characteristic equations and eigenvalues are quite long and tedious so they will not be analyzed and presented here. For this reason, the stability analysis at E_9 will be investigated with numerical assistance.

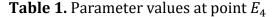
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RESULTS AND DISCUSSION

Here we conduct a numerical simulation using hypothetical parameter to support and to see the suitability of our previous analysis. The simulation for E_1 will not be carried out because at point E_1 stability is not achieved, while at points E_2 , E_3 , and E_6 the simulation will not be carried out because one of the values at the equilibrium point is negative.

Simulation in Equilibrium Point E₄

In this model simulation, the following parameters are used:



Parameter	Values
p	0,0344
α	0,6555
γ	0,1712
а	0,7060
У	0,0318
ω	0,2769
δ	0,0462
W	0,0971
d	0,8235
b	0,6948
t	0,3171
u	0,9502

Parameter values in Table 1 give simulation results as plotted in Figure 1

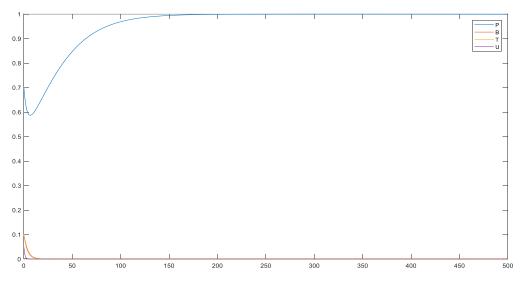


Figure 1. *P*, *B*, *T*, and *U* plots against time

P, *B*, *T*, and *U* will converge a value: 0.9999; 5.9672e-06; 9.5439e-47; and 9.7566e-08. This is in accordance with the search for the equilibrium point E_4 , where the rice population will live and the others will become extinct.

Simulation in Equilibrium Point E_5

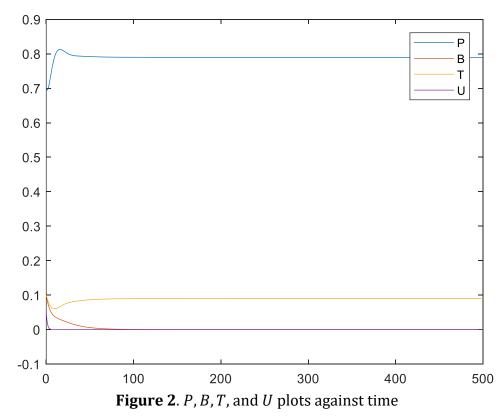
In this model simulation, the following parameters are used:

Table 2. Parameter values at point E_5

Parameter	Values
p	0.3922
α	0.4218
γ	0.9157
а	0.7922
У	0.9595
ω	0.6557
δ	0.0357
W	0.8491
d	0.9340
b	0.6787
t	0.7577
u	0.7431

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Parameter values in Table 2 give simulation results as plotted in Figure 2 below:



P, *B*, *T*, and *U* will *converge* a value: 0.7897; 7.6070e-25; 0.0901; and 1.4778e-07. This is in accordance with the search for the equilibrium point E_5 , where the rice population will live and the others will become extinct.

Simulation in Equilibrium Point E_7

In this model simulation, the following parameters are used:

Table 3. Parameter values at point E_7

Parameter	Values
p	0.0292
α	0.6028
γ	0.7112
а	0.2217
у	0.1174
ω	0.2967
δ	0.3188
W	0.4242
d	0.5079
b	0.0855
t	0.2625
u	0.8010

Parameter values in Table 3 give simulation results as plotted in Figure 3

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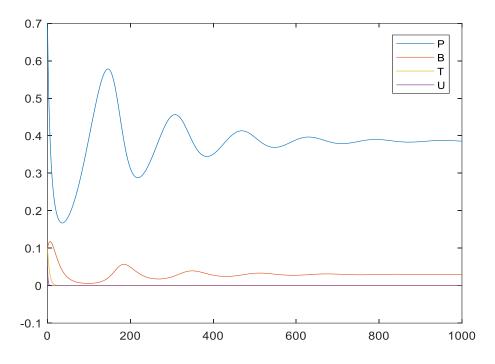


Figure 3. P, B, T, and U plots against time

P, *B*, *T*, and *U* will converge a value: 0.3855; 0.0301; 3.1888e-96; and 3.038e-08. This is in accordance with the search for the equilibrium point E_7 , where the rice population will live and the others will become extinct.

Simulation in Equilibrium Point E_8

In this model simulation, the following parameters are used:

Table 4. Parameter values at point E_8

Parameter	Values
p	0.6692
α	0.6663
γ	0.5391
а	0.6981
у	0.6665
ω	0.1781
δ	0.1280
W	0.9991
d	0.1711
b	0.0326
t	0.5612
u	0.7431

Parameter values in Table 4 give simulation results as plotted in Figure 4

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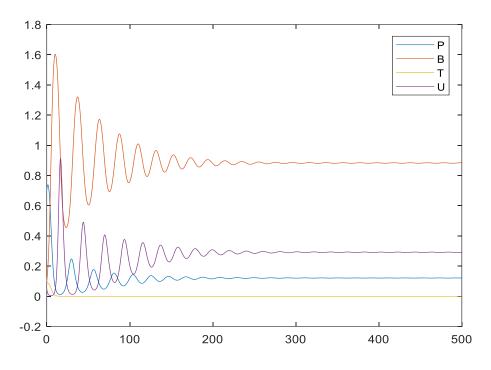


Figure 4. P, B, T, and U plots against time

P, *B*, *T*, and *U* will converge a value: 0.1211; 0.8826; 4.9041e-08; and 0.2913. This is in accordance with the search for the equilibrium point E_8 , where the rice, sparrows dan snakes populations will live and rats will become extinct. Although the stability analysis cannot be determined, numerical simulations can determine that the equilibrium point is towards local asymptotic stability.

Simulation in Equilibrium Point E₉

In this model simulation, the following parameters are used:

Table 5. Parameter values at point *E*₉

Parameter	Values
p	0.6538
α	0.0729
γ	0.0885
а	0.7984
у	0.9430
ω	0.6837
δ	0.1321
W	0.1321
d	0.7227
b	0.1104
t	0.1175
u	0.6407

Parameter values in Table 5 give simulation results as plotted in Figure 5

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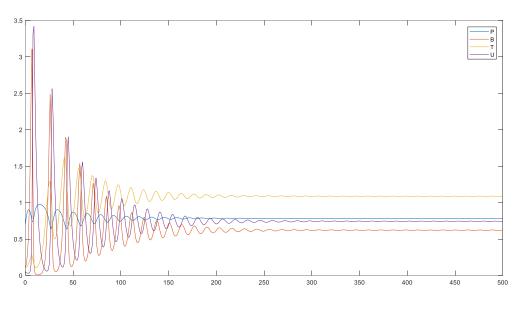


Figure 5. P, B, T, and U plots against time

P, *B*, *T*, and *U* will converge a value: 0.7835; 0.6192; 1.08614; and 0.7459. This is in accordance with the search for the equilibrium point E_9 , where all of the populations will live. Although the stability analysis cannot be determined, numerical simulations can determine that the equilibrium point is towards local asymptotic stability.

CONCLUSIONS AND SUGGESTIONS

Based on the description of the results and discussion, it can be concluded: (1) the predator-prey model that has been constructed is a system of linear differential equations with four variables; (2) 9 equilibrium points are obtained where the behavior of the model at point E_1 is unstable, E_2 , E_3 , E_6 is not analyzed further because one of the equilibrium point values is negative and E_4, E_5, E_7, E_8, E_9 is locally asymptotically stable, well proven through eigenvalues and numerical simulations; (3) The results of the model simulation using MATLAB are in accordance with the analysis.

We can conclude the influence factors are the natural death rate and the

interaction level of rice with both plant pests and the level of interaction between pests and second-level predators. Our results can be improved and elaborated in future research to simulate real world data in any province.

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