Tadris: Jurnal Keguruan dan Ilmu Tarbiyah 8 (1): 77-93 (2023)
DOI: 10.24042/tadris.v8i1.15176

TADRIS: JURNAL KEGURUAN DAN ILMU TARBIYAH
(Tadris: Journal of Education and Teacher Training)
P-ISSN: 2301-7562 | E-ISSN: 2579-7964
ejournal.radenintan.ac.id/index.php/tadris/index

Enhancing Students' Combinatorial Thinking for Graceful Coloring Problem: A STEM-Based, Research-Informed Approach in ATM Placement

Robiatul Adawiyah[1,2], Arika Indah Kristiana[1,2], Dafik[1,2]*, Muhammad Lutfi Asy'ari[2], I Made Tirta[3], Zainur Rasyid Ridlo[4], Elsa Yuli Kurniawati[1]
[1]Center of Excellence of the Combinatorics and Graph, Universitas Jember, Indonesia
[2]Department of Mathematics Education, Universitas Jember, Indonesia
[3]Department of Mathematics, Universitas Jember, Indonesia
[4]Department of Science Education, Universitas Jember, Indonesia

Article History:
Received: December 23rd, 2022
Revised: March 5th, 2023
Accepted: June 18th, 2023
Published: June 29th, 2023

Keywords:
Combinatorial thinking,
Research-based learning,
STEM

***Correspondence Address:**
d.dafik@unej.ac.id

Abstract: Combinatorial generalization thinking, a component of higher-order thinking skills, encompasses perception (pattern identification), expressions (pattern illustration), symbolic expressions (pattern formulation), and manipulation (combinatorial results application). Implementing a research-based learning (RBL) model with a Science, Technology, Engineering, and Mathematics (STEM) approach can effectively transform students' learning processes, promoting experiential learning through the integration of STEM elements. This study employs a mixed-method research design, combining quantitative and qualitative methodologies, to evaluate the impact of this RBL-STEM model on students' ability to solve graceful coloring problems, hence developing their combinatorial thinking skills. Two distinct classes, one experimental and one control, were analyzed for statistical homogeneity, normality, and independent t-test comparisons. Results indicated a significant post-test t-score difference between the two groups. Consequently, we conclude that the RBL model with a STEM approach significantly enhances students' combinatorial generalization thinking skills in solving graceful coloring problems. As this research provides empirical evidence of the effectiveness of a STEM-based RBL model, educators, and curriculum developers are encouraged to incorporate this approach into their instructional strategies for enhancing combinatorial thinking skills. Future research should consider various contexts and diverse student populations to further validate and generalize these findings.

INTRODUCTION

Education and technology advancements have always been parallel, with the sophistication of technology significantly elevating educational standards. Mathematics underlies this technological evolution and offers solutions to various problems (Boye & Agyei, 2023). Mathematical concepts are logically organized, systematically developed, and hierarchically structured from the simplest to the most complex, requiring strong mathematical thinking skills for successful learning (Mason et al., 1982; Hariadi et al., 2021). Among these skills, reasoning stands out as a crucial factor in drawing conclusions (Al-Maktoumi et al., 2016). In this context, the Combinatorial Generalization Thinking Skills (CGTS) emerge as a vital tool that facilitates the reasoning process by formulating general conclusions from specific observations (Upadhyay & Mohammed, 2022). These skills are instrumental in teaching students to reason and form assumptions based on repeating patterns observed in numbers or images (Anwarudin et al., 2020). Graph theory, a branch of mathematics, is particularly well-suited for students to demonstrate and cultivate their CGTS (Anggraeni et al., 2019).

Despite the existence of some research on graceful coloring, a component of graph theory (Khoirunnisa et al., 2021; Kristiana et al., 2022), there is limited research on the development of students' CGTS in relation to this topic. This gap in the literature demonstrates the need for innovative learning models that can enhance students' CGTS, such as the Research-Based Learning (RBL) model.

In this study, we have developed a learning tool using the 4D model, designed by Thiagarajan, Semmel & Semmel, to facilitate the RBL process (Creswell, 2012). This model incorporates four stages: define, design, develop, and disseminate. As this study employs a mixed-methods approach, data was collected qualitatively and quantitatively through interviews and observations (Hermawan, 2019; Jaafar et al., 2022).

The relevance of the RBL model in graph theory is evident, as it aligns with ongoing advancements in graph theory research. In response to the challenges of 21st-century skills, the RBL model has been combined with a STEM (Science, Technology, Engineering, Mathematics) approach (Fergus & Smith, 2022; Sabilah & Yolanda, 2022). Such innovative strategies are essential for higher education institutions to remain adaptable and relevant to current demands (Ervani et al., 2021).

STEM incorporates four interrelated components, when applied in real-life contexts, promote deeper understanding among students. This study aims to enable students to develop their understanding of graceful coloring in relation to ATM (Automatic Teller Machine) placement. Expected outcomes include students' ability to conduct generalizable tests using the concept of graceful coloring, mastery of graceful coloring, and determination of chromatic numbers in a graph. A STEM-infused RBL approach can foster students' knowledge and skills across scientific, technological, engineering, and mathematical fields (Yudha et al., 2018).

A novel aspect of this study involves the application of graceful coloring to ATM machine placement, whereby no two banks of the same type place ATMs within the same neighborhood radius. Despite maintaining the minimum number of ATM machines, this approach ensures optimal service (Anele et al., 2021; Ehiedu et al., 2021).

Despite relevant studies examining the implementation of RBL-STEM (Hidayati et al., 2020; Maylisa et al., 2020; Septory et al., 2019), research specifically exploring the impact of RBL-STEM on improving students' CGTS (particularly in solving graceful coloring problems) is scant. Therefore, this study intends to further develop and investigate students' CGTS in solving graceful coloring problems, focusing on the implementation of an RBL model combined with a STEM approach in the context of ATM machine placement. This represents a significant novelty and contribution to the existing literature.

THEORETICAL SUPPORT

The Research-Based Learning (RBL) model fosters a unique learning environment wherein students enhance their knowledge through direct research. Recognized for its effectiveness in transforming students' learning processes, the RBL model applies the 'learning by doing' principle (Komaria et al., 2022;

Kristiana et al., 2022). It employs diverse teaching methods, including contextual, authentic, problem-solving, cooperative, and direct learning, along with inquiry-discovery approaches. These methods align with the philosophy of constructivism, thereby promoting students' autonomy in learning, critical thinking, creativity, and communication skills (Hussen et al., 2019; Monalisa et al., 2019). This study anticipates that implementing RBL will cultivate students' proficiency in conducting scientific tasks and developing scientific understanding (Erdogan, 2015).

Table 1. Indicators of Combinatorial Generalization Thinking Skills (CGTS).

Indicator	Sub-Indicator
Perception of generality	Students can recognize graphs. Students can assign graceful coloring to graphs.
Expression of generality	Students can ascertain the cardinality of graphs. Students can formulate the generality of cardinality computation results in a graph.
Symbolic expression of generality	Students can identify graceful coloring on vertices and edges.
Manipulation of generality	Students can determine the graceful chromatic number. Students can discern graceful coloring patterns within a graph.

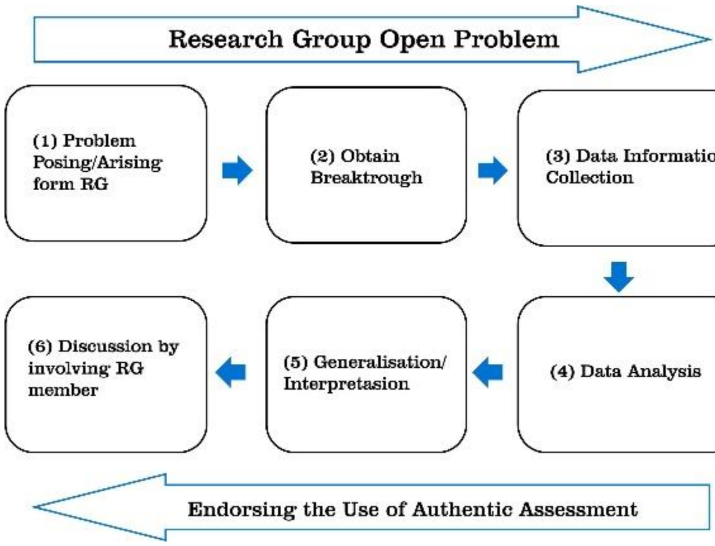

Figure 1. Framework of Research-Based Learning (RBL) Syntax.

The prevailing challenge in contemporary education is to equip students with the requisite thinking skills needed to navigate the complexities of the Fourth Industrial Revolution. It thus becomes imperative to devise multifaceted strategies aimed at nurturing students' ability to acquire and apply the skills pertinent to the 21st-century (Poonpan & Siriphan, 2001). STEM, an approach that encapsulates four key components: science, technology, engineering, and mathematics, proves instrumental in this pursuit. STEM literacy fosters the ability to identify, apply, and integrate concepts across these four components, thereby paving the way for innovation and problem-solving (Ardianti et al., 2020; Nadelson et al., 2012; Widana et al., 2021). STEM's inherent application-oriented, problem-based learning approach fosters a natural blend of these four disciplines, focusing on addressing practical issues in daily life. Integrating STEM content into real-world scenarios enhances students' learning experience and understanding (Geng et al., 2022).

In the present study, we employed a fusion of the Research-Based Learning (RBL) model and the STEM approach, discussing the urban design of Automated Teller Machine (ATM) placement via the graceful coloring concept. The proposed framework, derived from RBL-STEM and depicted in Figure 1, is structured as follows:

1. Problem Posing: This phase involves presenting a fundamental problem related to graceful coloring (Science).
2. Breakthrough Generation: Developing innovative solutions for Urban Design, specifically arranging ATM machines in urban areas using an application of graceful coloring within the syntax space (Engineering).
3. Data Collection: Accumulating data pertinent to Urban Design via web browsing and other media channels (Technology).
4. Data Analysis: Analyzing the collected data in relation to Urban Design, applying space syntax, and implementing graceful coloring (Engineering).
5. Combinatorial: This involves the arrangement of ATM machines using the graceful chromatic number (Mathematics).
6. Discussion: Presenting the outcomes of student research associated with Urban Design and the problems of graceful coloring (RBL Report).

METHOD

This study employs a mixed-methods approach, integrating both qualitative and quantitative research methodologies to develop research-based learning with a STEM approach and evaluate its impact on students' Combinatorial Generalization Thinking Skills (CGTS) (Suntusia et al., 2021). This dual-method design first involves gathering and analyzing qualitative data, followed by quantitative data analysis to generate a holistic conclusion from the research findings.

The qualitative data is obtained through interviews, and subsequently used to map out the phase portrait of the students. This research leads to the development of mathematics learning tools, which provide a practical, applicable component to the theoretical exploration. For the quantitative part, a sample was taken and used as the research subject for CGTS. Pre- and post-tests were administered to these subjects to measure the potential changes in their CGTS after the implementation of the research-based learning with the STEM approach.

Should the data obtained conform to normal distribution and exhibit homogeneity, a t-test analysis will be employed. This analysis aims to discern the differences in CGTS between the experimental and control groups. The t-test utilizes a null hypothesis (H_0) and an alternative hypothesis (H_1). If the significance value (sig) is greater than 0.05, the null hypothesis (H_0) is accepted; conversely, if the significance value is less than 0.05, the null hypothesis (H_0) is rejected. The hypotheses are stated as follows:

H_0: There is no significant difference in the effect of the Research-Based Learning (RBL) model with a STEM approach on students' CGTS between the control and experimental groups.

H_1: There is a significant difference in the effect of the Research-Based Learning (RBL) model with a STEM approach on students' CGTS between the control and experimental groups.

The research population consists of students enrolled in the odd semester of the academic year 2021/2022 at the Mathematics Education Study Program, Faculty of Teacher Training and Education, University of Jember. Following the experimental design, two groups were established: an experimental group and a control group (Jamaludin et al., 2022). The experimental group was exposed to a specialized treatment involving learning materials developed by the authors, utilizing the Research-Based

Learning (RBL) model combined with a Science, Technology, Engineering, and Mathematics (STEM) approach and based on Combinatorial Generalization Thinking Skills (CGTS) indicators.

In this study, purposive sampling was used to select two classes from the Mathematics Education program at FKIP UNEJ in the odd semester of 2021/2022. These classes were chosen based on the students' pre-existing knowledge of graph theory. Each class comprised of 25 students.

The goal was to compare the CGTS between the experimental and control groups, with the former receiving a specialized treatment consisting of the aforementioned RBL-STEM learning materials, while the control group followed the standard curriculum. The research design implemented is detailed in Table 2.

Table 2. Experiment Research Design.

Experiment class	R_1	X_1	R_3
Control class	R_2	X_0	R_4

In the proposed experimental design, R_1 and R_2 represent the pre-test administered to the students in the experimental and control groups, respectively, while R_3 and R_4 stand for the post-test provided to the students in both groups. X_1 corresponds to the treatment given to the experimental group, which includes the implementation of the Research-Based Learning (RBL) model coupled with a Science, Technology, Engineering, and Mathematics (STEM) approach, and the provision of learning materials developed by the authors. The control group, represented by X_0, was also exposed to the RBL-STEM model but without the specialized learning materials developed by the authors.

Table 3. Classification of Combinatorial Generalization Thinking Skills (CGTS).

Level	Indicator of Combinatorial Thinking
High	Students are able to fulfill the four indicator of critical thinking: 1. Perception of generality 2. Expression of generality 3. Symbolic expression of generality 4. Manipulation of generality
Medium	Students are able to fulfill the three indicator of critical thinking: 1. Perception of generality 2. Expression of generality 3. Symbolic expression of generality
Low	Students are able to fulfill the two indicator of critical thinking: 1. Perception of generality 2. Expression of generality

This study assessed students' combinatorial thinking abilities across three proficiency levels (high, medium, low), based on four critical thinking indicators: Perception, Expression, Symbolic Expression, and Manipulation of generality. High-level students met all four indicators, demonstrating comprehensive critical thinking. Medium-level students fulfilled the first three, indicating a lack of proficiency in manipulation. Low-level students showed competence only in perception and expression. This classification provides insights into students' critical thinking abilities, informing potential pedagogical improvements.

STEM problems leverage graph coloring and space syntax, notably through graceful coloring, a concept used in urban studies developed by Bill Hillier (2012) and his team. An application of this is in strategic ATM placement within cities, a crucial aspect of urban spatial planning. The placement, considering the diversity of banking services, requires a thoughtful strategy. The space syntax theory, using Google Maps Street View, facilitates city layout creation, representing streets and junctions as graph elements. The graceful coloring concept ensures varied bank ATM machines aren't placed within the same neighborhood radius. The strategy aims to minimize the number of ATMs while maintaining optimal service, using graceful coloring to find the graceful chromatic number. The results from this process provide strategic insights.

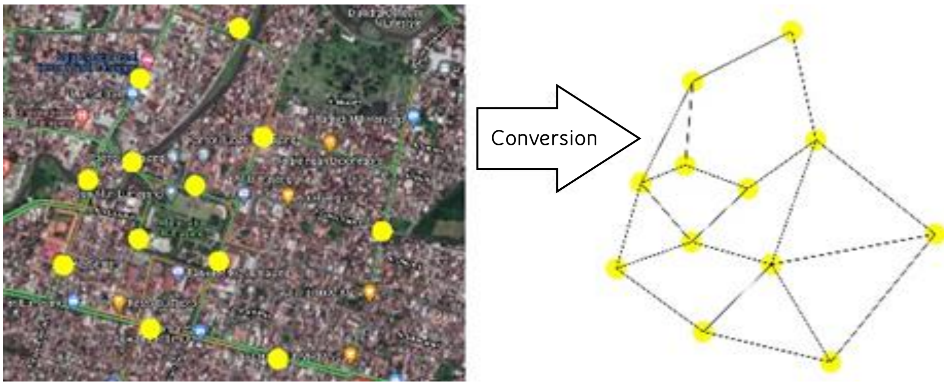

Figure 2. The Graph Representation of City Map Obtained from Google Map.

Data pertaining to the learning process is procured through observational techniques during instructional sessions. These observations employ a learning implementation sheet, which not only acts as a guideline for the process, but also provides a space for observers to record events beyond the scope of the structured plan. This method allows a nuanced view of the learning process and captures dynamics often missed in more rigidly structured approaches.

1. Test Method: The test method is employed to ascertain student learning outcomes following the execution of learning activities. This method typically involves the administration of post-tests, serving as a concrete measure of the acquired knowledge and skills resulting from these educational exercises.
2. Questionnaire Method: The questionnaire method is utilized to garner student feedback regarding various aspects of the learning experience. Specifically, questionnaires are distributed among students to gain insight into their responses to both the learning activities conducted and the instructional tools utilized. These questionnaires are instrumental in assessing learning tool effectiveness, offering valuable data for potential enhancements.

RESULT AND DISCUSSION

This study employed rigorous data analysis to assess students' Combinatorial Generalization Thinking Skills (CGTS), with the evaluation premised on the results of both pre-tests and post-tests from the experimental and control classes involved in solving the graceful coloring problem. As evidenced by the data in Table 3, the authors classified students' combinatorial thinking skills using both pretest and posttest results. The pre-test, administered to 50 students in the experimental class, revealed that 16% of students demonstrated high skills, 52% medium skills, and 32% low skills in combinatorial thinking. Contrastingly, the control class pre-test results showed slightly different percentages, with 24% of students displaying high skills, 48% medium skills, and 28% low skills. A detailed breakdown of these percentages can be viewed in Figure 3.

Table 4. Normality Test Results of Pre-test.

Class	Kolmogorov-Smirnov		Shapiro-Wilk	
	Statistic (df)	Sig.	Statistic (df)	Sig.
Eksperimental Class	0.150	0.152	0.920	0.050
Control Class	0.155	0.152	0.915	0.050

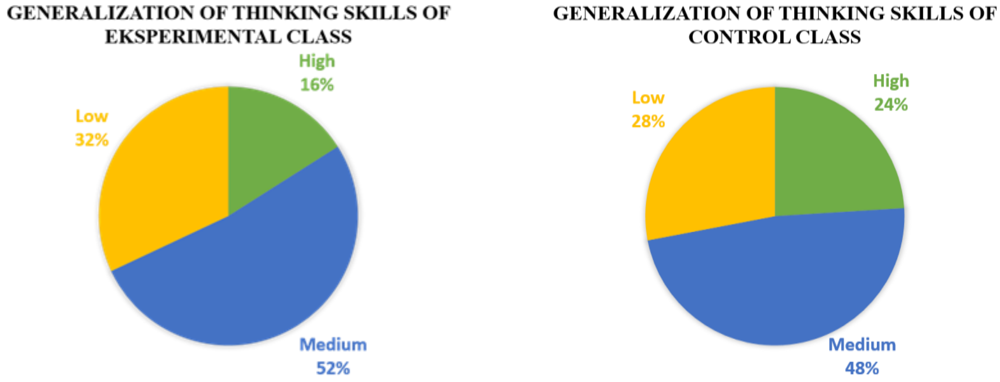

Figure 3. The Pre-test Result of Combinatorial Generalization Thinking Skills (CGTS).

Table 4 provides evidence that the pre-test data from both the experimental and control classes adhere to a normal distribution. This is a critical assumption that validates the use of certain statistical methods, including the independent sample t-test. This analysis, in the context of this study, is conducted to discern the differences in the effects of the Research Based Learning (RBL) model with the STEM (Science, Technology, Engineering, Mathematics) approach, as implemented in the experimental class, compared to the control class.

The independent sample t-test is a statistical method used to compare the means of two independent groups in order to determine whether there is statistical evidence that the associated population means are significantly different. In this case, the two groups are the experimental and control classes, and the t-test will provide insights into whether the RBL model with the STEM approach results in a significant difference in students' combinatorial thinking skills. This comparison provides a robust method for evaluating the efficacy of this pedagogical approach.

Table 5. T-test Result of Pre-test.

	Levene's Test for Equality of Variances		t-test for Equality of Means		Sig. (2-tailed)	Mean Difference	Std. Error Difference
	F	Sig.	t	df			
Equal Variances Assumed	0.004	0.948	0.246	48	0.807	0.520	2.114
Equal Variances Not Assumed	-	-	0.246	47.998	0.807	0.520	2.114

Table 5 reports the results of Levene's test for variance equality and the t-test for mean equality. Levene's test is used to check whether the variance among two or more groups is the same (homogeneous). If the Sig. value is greater than 0.05, the variance is considered the same or homogeneous. In this case, the Sig. value for Levene's test is 0.948, meaning the variance between the two groups is considered homogeneous for the equal variances assumption. The t-test is used to compare the mean differences between two groups. The t-value is 0.246 with degrees of freedom (df) of 48 and 47.998, for the equal variances and unequal variances assumption, respectively. The Sig. (2-tailed) value for both assumptions is 0.807, meaning there is no significant difference between the mean of the two groups as this value is greater than 0.05. The mean difference between the two groups is 0.520 with a standard error of 2.114, both for the equal and unequal variances assumptions. This mean difference, when compared to the standard error, helps to determine the significance of the difference. However, in this case, because the Sig. (2-tailed) value is greater than 0.05, we can conclude that there is no significant difference between the means of the two groups.

Referring to Table 5, we find that the value of sig (2-tailed) equals 0.807. When sig (2-tailed) > 0.05, we accept the null hypothesis (H_0), signifying that no difference exists between the impact of research-based learning (RBL) with a STEM approach on students' CGTS within the control and experimental classes. Following this, the experimental class was exposed to treatment via the RBL model integrated with the STEM approach, while a different treatment was administered to the control class. To measure the differential impact of the RBL model with the STEM approach on students' CGTS, both classes were administered the same post-test.

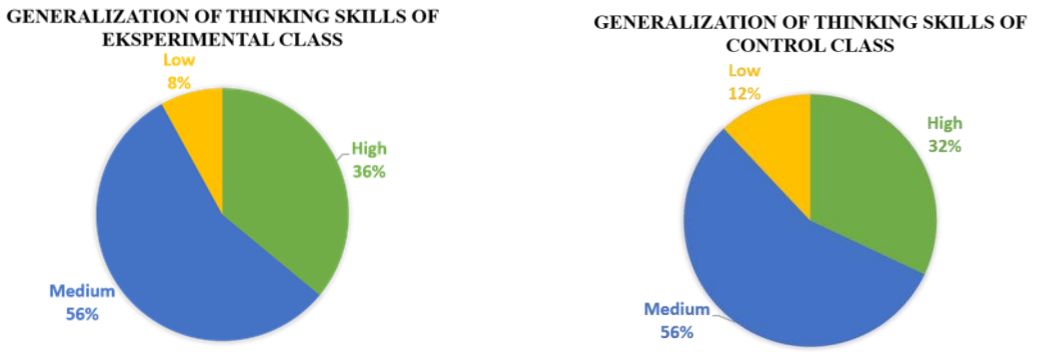

Figure 4. The Post-test Result of Combinatorial Generalization Thinking Skills (CGTS).

The post-test results from a total of 50 students revealed that out of 25 students in the experimental class, 36% demonstrated high skills, 56% exhibited medium skills, and 8% showed low skills. Conversely, the control class post-test results indicated that 32% of students had high skills, 56% possessed medium skills, and 12% had low skills. Figure 6 presents the proportionate representation of the post-test results.

As indicated by the normality test results in Table 6, a post-test was administered to both the experimental and control classes, with the subsequent normality test suggesting a normal distribution of data as the significance value for both classes was greater than 0.05. Based on the post-test results, we infer that data from the pre-test results of both the experimental and control classes exhibit normal distribution.

Furthermore, a t-test was conducted with the objective of identifying any difference in the influence of the RBL model with the STEM approach on students' combinatorial thinking skills in the experimental and control classes. Post-test t-test results presented in Table 6 indicate that the value of sig (2−tailed) is 0.022. Given that sig < 0.05, the null hypothesis (H0) is rejected, implying a significant difference in the impact of research-based learning with the STEM approach on the combinatorial skills of students in the control and experimental classes. It is therefore concluded that the average value of the experimental class exceeds that of the control class. Thus, we can deduce that research-based learning with the STEM approach significantly influences students' CGTS.

Table 6. Normality Test Results of Post-test.

Class	Kolmogorov-Smirnov		Shapiro-Wilk	
	Statistic (df)	**Sig.**	**Statistic (df)**	**Sig.**
Eksperimental Class	0.128	0.200	0.951	0.269
Control Class	0.137	0.200	0.947	0.215

Table 7. T-test Result of Post-test.

	Levene's Test for Equality of Variances		t-test for Equality of Means		Sig. (2-tailed)	Mean Difference	Std. Error Difference
	F	Sig.	t	df			
Equal Variances Assumed	0.090	0.765	2.376	48	0.022	7.000	2.946
Equal Variances Not Assumed	-	-	2.376	47.980	0.022	7.000	2.946

This study delineates the phase portraits of students based on the progression of their CGTS while resolving graceful coloring problems. Students' combinatorial skills were categorized into three groups: high, medium, and low, with one student selected from each category for further analysis.

As depicted in Figure 5, Student 1 (S1) demonstrates proficient capability in identifying the wheel graph (Wn) and assigning appropriate labels to the graph. S1 successfully determines the cardinality of a wheel graph with order 6 (W_6), however, their ability to generalize the cardinality remains undeveloped. Consequently, it can be inferred that S1 fulfills both the first and second indicators, albeit with a degree of deficiency in the latter.

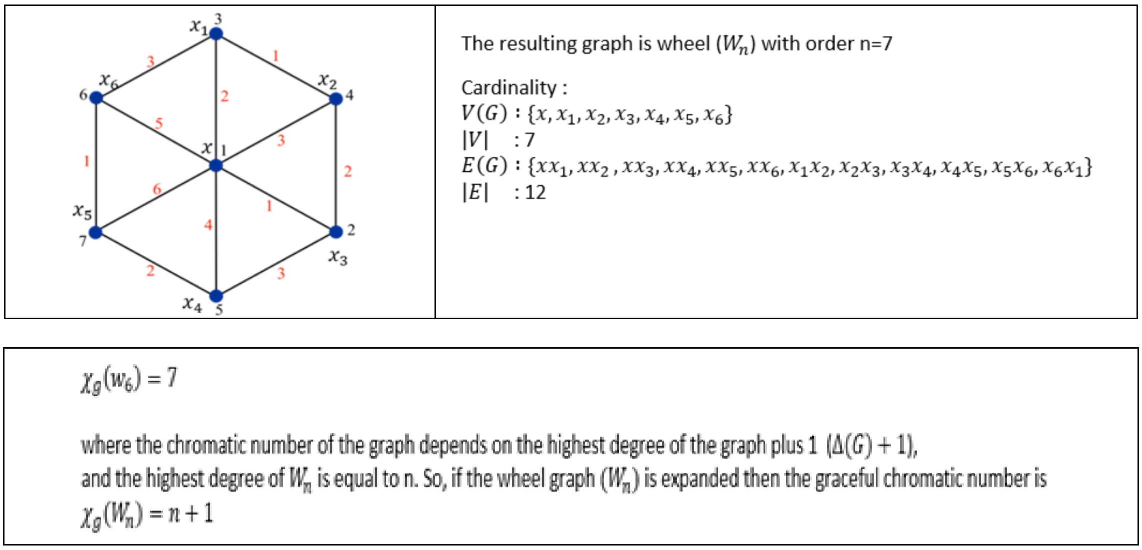

Figure 5. The Work of Student 1 (S1).

In addition, S1 successfully colors the vertices and edges of the wheel graph (W_6) correctly. S1 is adept at determining the chromatic number, expressing original thinking while discerning graceful coloring patterns on the graph. Based on the results of this work, S1 exhibits mastery of both the third and fourth indicators. Consequently, it can be inferred that while S1 demonstrates comprehensive proficiency in the application and manipulation of generality, their expression of generality in the context of cardinality generalization requires further development. This in-depth exploration of an individual student's performance can provide a nuanced understanding of the varying levels of students' CGTS, highlighting areas of strength and pinpointing areas in need of improvement.

The test outcomes demonstrate that S1 has effectively mastered the four indicators. However, there is room for improvement in mastering the second indicator, particularly in determining the general cardinality of a graph. Consequently, S1 falls within the high proficiency category. Subsequently, the resulting phase portrait for S1 is presented as follows.

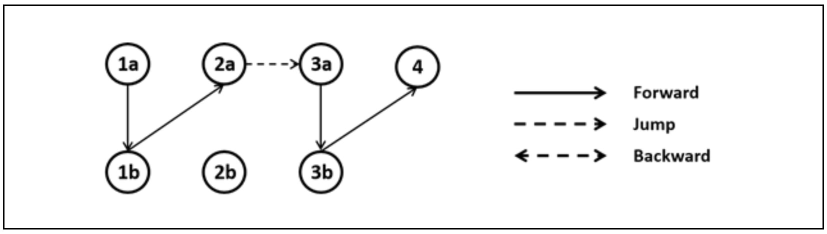

Figure 6. Phase Portrait of S1.

As depicted in Figure 6, Student 1's (S1's) Combinatorial Generalization Thinking Skills (CGTS) are nearly impeccable. In Indicator 1a, S1 successfully identifies and labels a graph correctly, and in Indicator 1b, S1 applies graceful coloring to the graph effectively. Progressing to Indicator 2a, S1 capably determines the graph's cardinality. However, S1 fails to apprehend the generality of the cardinality in Indicator 2b. S1 exhibits competence in determining graceful coloring on the vertices and edges and correctly ascertaining the graceful chromatic numbers in the graph. In Indicator 4, S1 articulates thoughtful insights concerning graceful coloring patterns on wheel graphs. Consequently, it can be concluded that S1 demonstrates near-flawless mastery of all four CGTS indicators, although there is a minor deficiency in understanding the generality of the cardinality.

Next, we turn our attention to the performance of Student 2 (S2). As Figure 7 illustrates, S2 demonstrates a solid understanding in graph identification, correctly labeling the ladder graph (Ln). Moreover, S2 shows the ability to create cardinality in a ladder graph where the order equals 5 (Ln), and can also effectively generalize this cardinality. These findings imply that S2 has successfully met the benchmarks set by the first and second indicators, showing a strong foundational understanding of these concepts.

Continuing with the analysis, Figure 7 also shows that S2 can correctly apply graceful coloring to the vertices and edges of the ladder graph (Ln), staying true to the formal definition of graceful coloring. Despite these positive points, S2 has shown certain shortcomings as well. Figure 7, for instance, reveals that S2 incorrectly determines the chromatic number, when the correct graceful chromatic number in the graph is actually 5. S2 also struggles with identifying a discernable pattern within the ladder graph and does not articulate a personal perspective on the subject.

Collectively, the test results suggest that S2 has a good understanding of three out of the four indicators, but struggles with the third indicator, specifically in determining the chromatic number of the graph accurately. This shortfall places S2 in the category of students with medium-level skills. These insights are helpful in tailoring future instruction to better meet the needs of S2. Following this, we present the phase portrait results for S2, which provides a more visual representation of S2's understanding and application of these concepts.

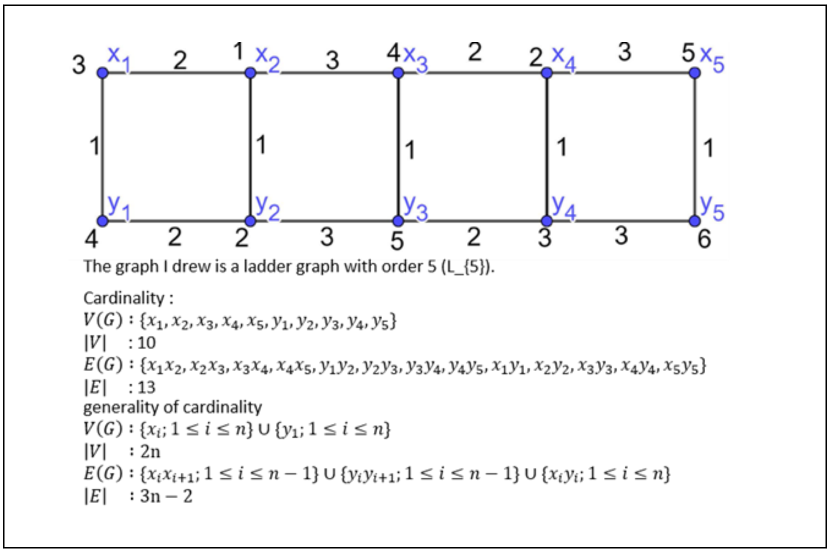

Figure 7. The Work of Student 2 (S2).

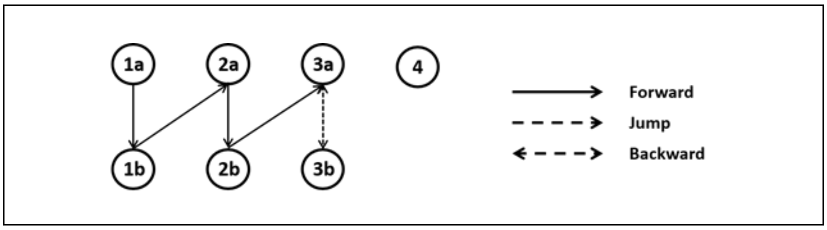

Figure 8. Phase Portrait of S2.

The Combinatorial Generalization Thinking Skills (CGTS) of Student 2 (S2), as depicted in Figure 8, reveal a degree of mastery that is less than perfect. Under the first indicator (1a), S2 demonstrates an ability to correctly identify the graph and accurately label it. Similarly, S2 applies graceful coloring to the graph effectively under indicator 1b. Moving to the second set of indicators, S2 correctly determines the cardinality of the graph (indicator 2a) and is able to successfully generalize this cardinality (indicator 2b).

However, despite these successes, S2's understanding falls short in other key areas. While S2 correctly applies graceful coloring to the vertices and edges of the graph, S2 fails to accurately determine the graceful chromatic number on the graph, suggesting a gap in comprehension. Additionally, S2 struggles to articulate a personal perspective regarding the graceful coloring pattern on the ladder graph (indicator 4).

Given these observations, it can be concluded that S2 has almost perfectly mastered three of the four indicators. However, owing to the shortcomings in understanding the graceful chromatic number and articulating personal insights on the graceful coloring pattern, S2 falls within the medium competency category in terms of combinatorial thinking skills.

We now turn our attention to Student 3 (S3), who exemplifies a low competency level in Combinatorial Generalization Thinking Skills (CGTS). As shown in Figure 9, S3 demonstrates difficulty in correctly identifying the graph's name. However, S3 is capable of properly labeling the graph. S3 manages to compute the cardinality of the graph but fails to generalize this cardinality. Consequently, S3 falls short of complete mastery of both the first and second indicators. Furthermore, S3 struggles to accurately apply graceful coloring to the graph's vertices and edges, falling short of the criteria established in the definition of graceful coloring. S3 is also unable to correctly compute the graph's chromatic number, and struggles to discern any discernable patterns within the graph. Additionally, S3 lacks the ability to articulate personal opinions regarding the graph pattern.

Given these observations, it is clear that S3 has only mastered two of the four indicators, having difficulty in identifying the graph's name and generalizing the cardinality, as covered by the first and second indicators respectively. As such, S3 is classified within the low competency category of students.

The combinatorial thinking skills of S3, as depicted in Figure 10, are underdeveloped. Under the first indicator (1a), S3 struggles to correctly identify the graph, despite being able to accurately label it. S3 also manages to apply graceful coloring under indicator 1b. Under indicator 2a, S3 correctly determines the graph's cardinality but fails to generalize this cardinality under indicator 2b. S3 encounters additional difficulties when progressing to indicators 3a to 3b, struggling to apply graceful coloring to vertices and edges in line with the defined criteria. Furthermore, S3 is unable to determine graceful chromatic numbers on graphs, and under the fourth indicator, S3 fails to articulate personal insights regarding graceful coloring patterns on graphs. Thus, it can be concluded that, while S3 exhibits near complete mastery of two indicators, S3 only truly demonstrates a mastery of two indicators of combinatorial thinking skills.

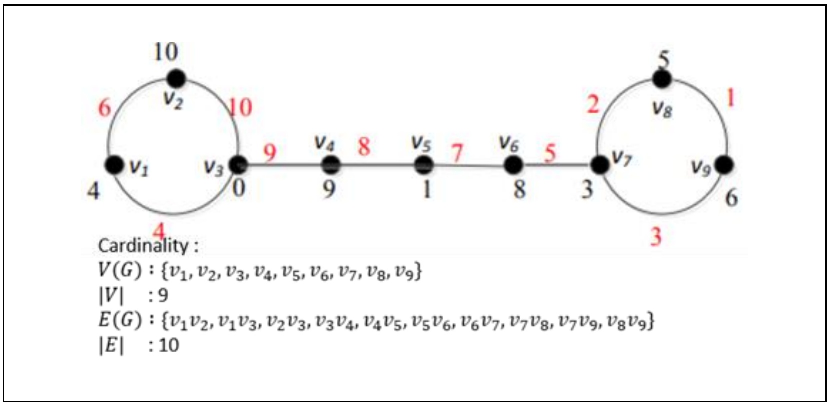

Figure 9. The Work of Student 3 (S3).

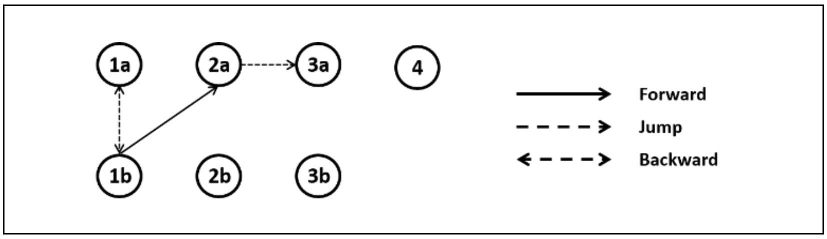

Figure 10. Phase Portrait of S3.

This study delves into the analysis of combinatorial generalization thinking skills through the application of research-based learning with a STEM approach. The subjects of this research are students in control and experimental classes, with combinatorial thinking skills being assessed via their post-test answer sheets. Among the participants, the highly skilled student (S1) met all indicators of combinatorial thinking skills, albeit not perfectly. The student with medium skills (S2) satisfied three of the indicators, while the low-performing student (S3) only fulfilled two indicators related to combinatorial generalization thinking skills. To further understand their cognitive process, we depicted the flow of their thoughts through phase portraits.

Comparative analysis of the pre-test results from both classes via a t-test yielded a significance value (sig, 2-tailed) of 0.807. Given that this value exceeds 0.05, no significant difference was discerned between the experimental and control classes. However, the independent sample t-test of the post-test results unveiled a significance value of 0.022, implying a substantial difference between the two groups since this value is less than 0.05. The control class averaged a score of 63.32, while the experimental class achieved a higher average of 71.32. Therefore, we concluded that students in the experimental class demonstrated superior combinatorial generalization thinking skills compared to their counterparts in the control class. This substantiates the impact of research-based learning with the STEM approach on this particular group of students.

The pedagogical strategy adopted in this study entailed the use of a STEM-based research learning approach to enhance students' combinatorial thinking skills. Prior to this approach, students were encouraged to foster their intellectual curiosity and classroom engagement, actively discovering patterns of graceful coloring. The primary objective was to evaluate the implementation of the research-based learning (RBL) model in conjunction with a STEM approach to foster active learning and improve combinatorial thinking skills. Given its successful deployment in the experimental class, we recommend wider application of this learning model.

A study by Izza et al. (2023) previously examined the use of an RBL-STEM Learning approach to bolster combinatorial thinking skills, focusing specifically on student capabilities in solving local (a,d)-edge anti-magic coloring problems. Their findings indicated that 82.75% of students achieved complete or near complete understanding, satisfying the set criteria for efficacy. However, the scope of their study was limited to combinatorial thinking skills alone, whereas our research further explores the specific aspect of generalization thinking skill. We observed an increase in students with high and medium generalization thinking skills from 16% to 36% and 52% to 56%, respectively, alongside a decrease in students with low combinatorial generalization thinking skills from 32% to 8%. Thus, the data suggests that the integration of RBL-STEM with graceful coloring learning media effectively enhances students' combinatorial generalization thinking skills.

While our study provides valuable insights into the effects of research-based learning with a STEM approach on the improvement of students' combinatorial generalization thinking skills, several limitations need to be acknowledged. Firstly, the sample size of the students was relatively small, which may limit the generalizability of the findings. Additionally, this study focused on specific types of combinatorial problems - graceful coloring problems and chromatic numbers - and hence may not encompass the broad scope of combinatorial thinking skills. Finally, the study was conducted within a single educational setting, thus its findings may not be applicable to other contexts with differing academic environments and student demographics.

These limitations suggest directions for future research. Future studies could expand the sample size to include a wider range of students from different educational settings and backgrounds, thereby enhancing the generalizability of the findings. Moreover, to provide a more comprehensive understanding of students' combinatorial thinking skills, future research could consider exploring other types of combinatorial problems beyond graceful coloring problems and chromatic numbers. Investigations could also be carried out to assess the effectiveness of the research-based learning with a STEM approach across different cultural and educational contexts. Furthermore, the inclusion of a longitudinal study design could provide insights into the long-term effects of the STEM-based research learning approach on the development of combinatorial generalization thinking skills among students.

CONCLUSION

This research investigates the effectiveness of a research-based learning approach incorporating a STEM framework in enhancing students' combinatorial generalization thinking skills. While initial assessments showed no significant discrepancy between the control and experimental groups, post-test evaluations highlighted a marked improvement in the experimental group's combinatorial skills. The study affirms the potential of this approach in cultivating active learning and enhancing combinatorial thinking abilities, with evidence of a notable increase in high (from 16% to 36%) and medium (from 52% to 56%) generalization thinking skills, and a concurrent decrease in low-level skills (from 32% to 8%). An independent sample t-test further demonstrated a higher post-test average in the experimental class, indicating a significant differentiation from the control class. However, the study, constrained by a small sample size, a focused problem scope, and a single educational setting, underscores the necessity for future research to expand its scope and improve generalizability. This will enable a more comprehensive assessment of the long-term effects of the STEM-based research learning approach on the development of students' combinatorial generalization thinking skills.

REFERENCES

Al-Maktoumi, A., Al-Ismaily, S., & Kacimov, A. (2016). Research-based learning for undergraduate students in soil and water sciences: a case study of hydropedology in an arid-zone environment. *Journal of Geography in Higher Education*, *40*(3), 321–339. https://doi.org/10.1080/03098265.2016.1140130

Anele, B. C., Ikeh, I. M., & Stanley, H. O. (2021). Isolation of microbes associated with automated teller machine (ATM) keypads studied at Rumuokoro Port Harcourt, Rivers State. *South Asian Journal of Research in Microbiology*, *11*(1), 10–17. https://doi.org/10.9734/sajrm/2021/v11i130241

Anggraeni, Dafik, & Tirta, I. M. (2019). The analysis of the application of discovery learning in improving student's combinatorial thinking skill to solve local super antimagic face coloring problem. *Journal of Physics: Conference Series*, *1211*, 012086. https://doi.org/10.1088/1742-6596/1211/1/012086

Anwarudin, M., Dafik, & Farisi, M. I. (2020). Students' combinatorial generalization thinking skills in solving tessellation coloring pattern problems and its enhancement through problem-based learning. *Journal of Physics: Conference*

Series, *1465*(1), 012066. https://doi.org/10.1088/1742-6596/1465/1/012066

Ardianti, S., Sulisworo, D., Pramudya, Y., & Raharjo, W. (2020). The impact of the use of STEM education approach on the blended learning to improve student's critical thinking skills. *Universal Journal of Educational Research*, *8*(3 B), 24–32. https://doi.org/10.13189/ujer.2020.081503

Boye, E. S., & Agyei, D. D. (2023). Effectiveness of problem-based learning strategy in improving teaching and learning of mathematics for pre-service teachers in Ghana. *Social Sciences & Humanities Open*, *7*(1), 100453. https://doi.org/10.1016/j.ssaho.2023.100453

Creswell, J. W. (2012). Educational research: Planning, conducting, and evaluating quantitative and qualitative research. In *Educational Research* (Vol. 4). https://doi.org/10.1017/CBO9781107415324.004

Ehiedu, V. C., Onuorah, A. C., & Ezekiel, O. (2021). Automated Teller Machine (ATM) penetration and financial inclusiveness in Nigeria: A tripod banking system approach. *Indian Journal of Economics and Business*, *20*(3), 1093–1104. http://www.ashwinanokha.com/IJEB.php

Erdogan, A. (2015). Turkish primary school students' strategies in solving a non- routine mathematical problem and some implications for the curriculum design and implementation. *International Journal for Mathematics Teaching and Learning*, 1–27.

Ervani, R. S. R., Dafik, & Tirta, I. M. (2021). The analysis of the implementation of problem-based learning students' generalization skills in obtaining the resolving total domination number. *Journal of Physics: Conference Series*, *1836*(1), 012060. https://doi.org/10.1088/1742-6596/1836/1/012060

Fergus, S., & Smith, C. P. (2022). Characteristics of proficiency-based learning and their impacts on math anxiety in the middle grades. *RMLE Online*, *45*(4), 1–19. https://doi.org/10.1080/19404476.2022.2045810

Geng, F., Srinivasan, S., Gao, Z., Bogoslowski, S., & Rajabzadeh, A. R. (2022). *An online approach to project-based learning in engineering and technology for post-secondary students* (pp. 627–635). https://doi.org/10.1007/978-3-030-96296-8_56

Hariadi, B., Jatmiko, B., Sunarto, M. J. D., Prahani, B. K., Sagirani, T., Amelia, T., & Lemantara, J. (2021). Higher order thinking skills based learning outcomes improvement with blended web mobile learning model. *International Journal of Instructionion*, *15*(2), 565–578.

Hermawan, I. (2019). *Metodologi penelitian endidikan (kualitatif, kuantitatif dan mixed method)*. Hidayatul Quran.

Hidayati, Y. M., Ngalim, A., Sutama, S., Arifin, Z., Abidin, Z., & Rahmawati, E. (2020). Level of combinatorial thinking in solving mathematical problems. *Journal for the Education of Gifted Young Scientists*, *8*(3), 1231–1243. https://doi.org/10.17478/jegys.751038

Hillier, B. (2012). Studying cities to learn about minds: Some possible implications of space syntax for spatial cognition. *Environment and Planning B: Planning and Design*, *39*(1), 12–32. https://doi.org/10.1068/b34047t

Hussen, S., Dafik, Monalisa, L. A., Murtikusuma, R. P., & Oktavianingtyas, E. (2019). Combinatorial thinking and creativity skills in solving a colored-square paving decoration problem. *Journal of Physics: Conference Series*, *1211*, 012062. https://doi.org/10.1088/1742-6596/1211/1/012062

Izza, R., Dafik, Kristiana, A. I., & Mursyidah, I. L. (2023). The development of RBL-STEM learning materials to improve students' combinatorial thinking skills in solving local \left(a,d\right)-edge antimagic coloring problems for line motif batik design. *European Journal of Education and Pedagogy*, *4*(1), 145–153. https://doi.org/10.24018/ejedu.2023.4.1.571

Jaafar, N. A., Rohani, S., Nor, M., Norrulashikin, S. M., Bazila, N. A. K., & Mohamad, A. Q. (2022). Increase students ' understanding of mathematics learning using the technology-based learning. *International Journal of Advanced Research in Future Ready Learning and Education*, *1*(1), 24–29.

Jamaludin, Z. Z., Unnafsyah, S., Agustin, E. S., Nuryadin, A., & Wahid Muharram, M. R. (2022). Development of gamification-based flipped learning in mathematics learning as an effort to overcome learning obstacles. *Indonesian Journal of Primary Education*, *6*(1), 106–114. https://doi.org/10.17509/ijpe.v6i1.41910

Khoirunnisa, S., Dafik, Kristiana, A. I., Alfarisi, R., & Albirri, E. R. (2021). On graceful chromatic number of comb product of ladder graph. *Journal of Physics: Conference Series*, *1836*(1), 012027. https://doi.org/10.1088/1742-6596/1836/1/012027

Komaria, N., Suratno, Sudarti, & Dafik. (2022). On cascara STEM education research-based learning activities: Analysing the caffeine, trigonelline substance of cascara fermented by magnetic fields to produce healthy herbal tea. *AIP Conference Proceedings*, 060039. https://doi.org/10.1063/5.0103043

Kristiana, A. I., Dafik, Ridlo, Z. R., Prihandini, R. M., & Adawiyah, R. (2022). Research based learning and STEM learning activities: The use of r-dynamic coloring to improve the students meta-literacy in solving a tessellation decoration Problem. *European Journal of Education and Pedagogy*, *3*(4), 52–60. https://doi.org/10.24018/ejedu.2022.3.4.381

Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically*. Addison-Wesley.

Maylisa, I. N., Dafik, Hadi, A. F., Wangguway, Y., & Harjito, L. O. (2020). The influence of research-based learning implementation in improving students' combinatorial thinking skills in solving local irregularity vertex r-dynamic coloring. *Journal of Physics: Conference Series*, *1538*(1), 012090. https://doi.org/10.1088/1742-6596/1538/1/012090

Monalisa, L. A., Dafik, Hastuti, Y., Hussen, S., & Oktavianingtyas, E. (2019). The implementation of research based learning in developing the students mathematical generalization thinking skills in solving a paving blocks design problem. *IOP Conference Series: Earth and Environmental Science*, *243*, 012168. https://doi.org/10.1088/1755-1315/243/1/012168

Nadelson, L. S., Seifert, A., Moll, A. J., & Coats, B. (2012). An integrated approach to teacher professional development in STEM. *Journal of STEM Education*, *13*(2), 69–84.

Poonpan, S., & Siriphan, S. (2001). *Indicators of research-based learning instructional process: a case study of best practice in a primary school*. Chulalongkorn University.

Sabilah, M., & Yolanda, F. (2022). Development of mathematics learning devices with a problem based learning model oriented on the mathematics problem solving ability. *International Journal of Trends in Mathematics Education Research*, *5*(2), 180–190. https://doi.org/10.33122/ijtmer.v5i2.136

Septory, B. J., Dafik, & Tirta, I. M. (2019). The analysis of students' combinatorial thinking skills in solving r-dynamic vertex coloring under the implementation of problem based learning. *Journal of Physics: Conference Series*, *1211*, 012084. https://doi.org/10.1088/1742-6596/1211/1/012084

Suntusia, S., Dafik, D., & Hobri, H. (2021). The level of critical thinking skill on solving two-dimensional arithmetics problems through research based learning. *Alifmatika: Jurnal Pendidikan Dan Pembelajaran Matematika*, *3*(1), 55–69. https://doi.org/10.35316/alifmatika.2021.v3i1.55-69

Upadhyay, P., & Mohammed, L. A. (2022). Challenges of online modules for task-based learning of mathematics for effective online teaching-learning process. *International Journal of Emerging Issues in Social Science, Arts, and Humanities (IJEISSAH)*, *1*(1), 74–80. https://doi.org/doi.org/10.31674/ijeissah.2022.v1i01.007

Widana, I. W., Sopandi, A. T., & Suwardika, G. (2021). Development of an authentic assessment model in mathematics learning: A science, technology, engineering, and mathematics (STEM) approach. *Indonesian Research Journal in Education |IRJE|*, *5*(1), 192–209. https://doi.org/10.22437/irje.v5i1.12992

Yudha, F., Dafik, D., & Yuliati, N. (2018). The analysis of creative and innovative thinking skills of the 21st century students in solving the problems of "locating dominating set" in research based learning. *International Journal of Advanced Engineering Research and Science*, *5*(3), 163–176. https://doi.org/10.22161/ijaers.5.3.21