Al-Jabar: Jurnal Pendidikan Matematika
Volume 13, Number 1, 2022, Pages 1 – 11
http://ejournal.radenintan.ac.id/index.php/al-jabar/index

Prospective mathematics teachers' understanding of proof in mathematics for high school

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Article Information
Submitted October 21, 2021
Revised February 21, 2022
Accepted February 26, 2022

Keywords
Belief;
Comprehension;
Conception;
Proof;
Secondary Mathematics.

Abstract
Changes in the mathematics education curriculum affect the preparation of prospective teachers to learn as well as possible and provide extensive opportunities and experiences regarding mathematical proof in secondary schools, especially those reflecting the nature and role of proof in the field of mathematics. Teachers' ability to respond to this depends on their understanding of proof in a mathematical context. This research examined the understanding of 36 prospective mathematics teachers at FKIP Untan regarding proof and their ability to prove problems in the context of high school mathematics. The data were collected through a series of interviews focused on their responses to the role of proof and from tests to prove math problems. This research is descriptive-analytical research, which describes the conception of prospective mathematics teachers as knowledgeable in mathematics about proof. However, some view proof as a tool for studying mathematics. This research also found that many of them had a limited view of the nature of proof. They lacked a clear understanding of the subject of demonstrating or explaining the process of proving a mathematical statement.

INTRODUCTION

There are many assumptions that proof a fundamental thing in mathematics and has become the habit of mathematicians (Knuth, 2002). On the other hand, (Bloch, 2011; Chazan, 1993; Doruk, 2019; Hanna, 1995; Mukuka et al., 2021; Stylianides & Stylianides, 2009) contradicts this point and states that "the lie of mathematics lies in the proof ."So far, the role of proof in learning mathematics in schools is usually not seen as the main thing. Proving process is limited to Euclid's geometry. In fact, according to Wu (1996), the lack of providing proof for fields outside of geometry is a mistake about the nature of proof in mathematics. He argues that this deficiency is a handicap in teaching mathematics in secondary schools today. Existence of fundamental facts outside of geometry given without proof is an anomaly for further education and is, even more an anomaly if the entire mathematical representation is falsified. Related to this, (Bardini et al., 2014; Joseph, 2020; Schoenfeld, 2020) states that proof is not something separate from mathematics. This is reflected in the mathematics curriculum. The proof is an essential component of performing, communicating, and recording mathematics. This content can be included in the mathematics curriculum at all levels.

Many mathematicians and math teachers agree with Wu and Schoenfeld. This is proven by the many quotes taken from their mouths. Over the past 20 years, they have become accustomed to defining the role of proof in mathematics education. These provisions have influenced the work of mathematicians, mathematics learning theory and mathematics curriculum (Durand-Guerrier et al., 2012; Hanna, 1990, 1995; Stylianides & Stylianides, 2009; Todd et al., 2010). In addition, proofs also played an important role in creating new mathematics. Todd et al., (2010) says that there are many examples in the history of mathematics where recent results had been

How to cite Mirza, A., Fitriawan, D., Zubaidah, R., & Nursangaji, A. (2022). Prospective mathematics teachers' understanding of proof in mathematics for high school. *Al-Jabar : Jurnal Pendidikan Matematika*, *13*(1), 1-11.
E-ISSN 2540-7562
Published by Mathematics Education Department, UIN Raden Intan Lampung

discovered or made purely deductively, such as non-Euclidean geometries. This follows the opinion of Robson (1946) that proof acts as a manifestation in the relationship of proof to problem-solving and conjecture. So, the role of proof can be said to be very mathematically unique, namely organizing or compiling results in the form of a deductive system (axioms, definitions, and theorems).

The current target of implementing mathematics education emphasizes the importance of developing understanding/understanding skills and critical, creative, and productive thinking skills, especially for prospective secondary school teachers. In line with that, several national reports in various countries have shown that instructional design to stimulate thought processes and problem-solving and generate complex solutions is urgently needed (Halpern, 2013; King et al., 2003; Moseley et al., 2005; Rif'at & Fitriawan, 2020; Wallbank, n.d.; Zohar et al., 2001) so that students can adapt to any changes (Halpern, 2013), acquire new knowledge more quickly and be able to face the complexities of real problems in life (Conklin, 2012; King et al., 2003; Sumar & Sumar, 2020). This will be achieved if students at the primary and secondary levels begin to be fostered or trained to develop such a mindset. The competence of teachers as teachers should not be ignored. About these expectations, Educational Personnel Education Institute (LPTK) needs to provide solid basic mathematical competencies for prospective teachers, especially in mathematics education.

However, the reality on the ground shows the opposite condition. Based on learning outcomes data in the last three years, it shows that more than 60% of students who program basic courses (calculus) and high school mathematics fail to achieve a minimum passing score (score 60). The results of the search for students who fail in this course show that the ability of students to reason (logic) and understand (understand) the concepts in question is still very weak, especially in proof. At the same time, the ability to prove is a competency that must be possessed in analytical courses such as calculus and real analysis. Mathematical proof is used to explain why a statement is true (Bloch, 2011; Branden, 1992; Sa'd, 2014). The proof is at the core of mathematics, and the foundation of proof is deductive reasoning (Doruk, 2019; Farida, 2011; Azkalia, 2018; Valleman, 2006). The weak ability of these students can be seen from the way they work when asked to prove properties or theorems. They mostly do it by giving examples using the properties they want to prove. In interpreting concepts/definitions/theorems, they only read symbols and cannot explain their meaning in a specific way. They have not been able to provide an explanation that is appropriate to the context using their language.

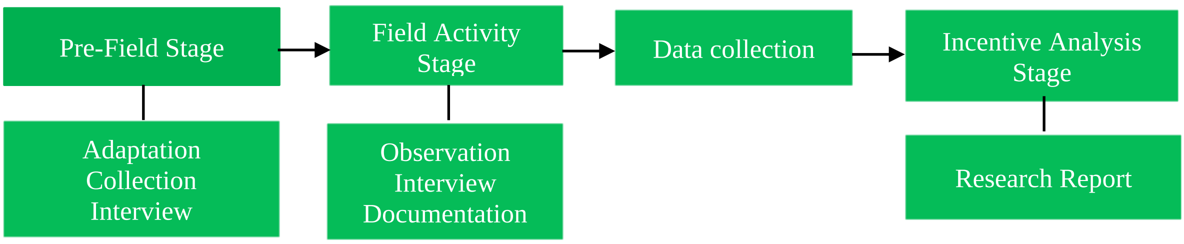
In a lecture activities, students tend to be passive, namely only receiving information from the lecturer and very rarely asking questions, giving opinions or ideas. The work of independent/group assignments tends to seem random and is not studied well by each individual. This can be seen when asked to present the results of their work in class. They still cannot convey it properly, especially without seeing the results of their work. This calculus is a basic subject that is very important to strengthen the understanding of mathematics in high school. As a producer of educational graduates, the Education Personnel Education Institute (LPTK) certainly has a big role in producing quality teachers. Therefore, LPTK must be selective in carrying out their role as filters to produce qualified prospective teachers, including encouraging a lecture system that can stimulate students' academic and educational competencies to develop optimally. In response to this, it is necessary to continue to strive for a learning system that can help students understand the material well, especially regarding proof in mathematics. The ability to prove is a competency that must be possessed in mathematics content courses. According to Knuth (2002), proof plays an important role in discovering or generating new mathematics.

Several previous proof studies support this research (Chazan, 1993; Knuth, 2002; Noto et al., 2019). The results show that many teachers have a limited view of the nature of proof in mathematics and demonstrate an inadequate understanding of what constitutes proof. The learning barriers are related to difficulties in applying concepts related to visualizing geometric objects, determining principles, and understanding problems. There are also related obstacles in mathematical proofs, such as understanding and expressing definitions, using definitions to build proofs, understanding the use of language and mathematical notation, and knowing how to start a proof. Mathematical proof states that one way to use geometric conjecture as a pedagogical tool is to ask students to explore geometric constructions and develop conjectures. This assumption is about the characteristics that students believe to be true for all diagrams resulting from certain constructions on certain types of shapes. After developing their conjecture by measuring geometric assumptions, they were asked to make arguments and write mathematical proofs. This research reports in-depth interviews and analyzes students' reasons for viewing empirical proof as proof and mathematical proof as only proof. The specific results that can be achieved from this research are determining the description of facts about the conception of prospective mathematics teachers in the Mathematics Education S1 study program, FKIP Tanjungpura University, regarding the role of proof in mathematics and determining the description of the competence or ability of prospective mathematics teachers in the Mathematics Education S1 study program, FKIP Tanjungpura University, in carrying out proofs in the context of mathematics.

In implementing mathematics learning in schools, teachers should provide opportunities and broad experience for students to enrich themselves with proof. The success of the teachers in responding to the needs of students in this context depends on the ability and breadth of the teachers' conception of proof. To see the competency profile of prospective teachers in the mathematics education study program, FKIP Untan, this research will examine their conception and understanding of proof in the context of high school mathematics. The purpose of this research was to describe the conception and understanding of prospective mathematics teachers at FKIP Untan regarding proof and its role in mathematics and to find out the knowledge and abilities of prospective teachers in carrying out the proof process and identify their beliefs about the role of proof in the context of high school mathematics, which includes the case of numbers, geometry, angular geometry and algebra.

METHODS

This research is descriptive-analytical research, which describes the conception of prospective mathematics teachers as knowledgeable in mathematics about proof in the discipline of mathematics. The research flow is described in the following flowchart:



Flowchart 1. The Descriptive Analytical Method

Data collection techniques in this research consisted of measurement techniques to obtain data on students' abilities regarding proof and interview techniques to explore their conceptions or understanding of the role of proof in mathematics. The research participants were 36 students of the Mathematics Education Study Program, FKIP Untan, who were in the final semester (Semester VII and IX). Data collection was carried out in two stages. At the first stage is to conduct semi-structured interviews to explore students' conceptions of the role of proof. The second stage describes the characteristics of students' abilities in carrying out the process of proving math problems for high school material.

Data collection instrument in this research was used to interview and test guidelines (questions of proof). Collected data is then described in tabulated form and is elaborated narratively. The description of the data includes identification of the role of proof according to prospective teachers based on the results of interviews and an analysis of the characteristics of their ability to provide arguments against the evidentiary process they do. To test whether prospective teachers have confidence in the findings or results of their work, they are asked to provide several arguments according to the context of the questions given.

RESULTS AND DISCUSSION

One of the characteristics of mathematics is its abstract nature. In addition, mathematics is deductive knowledge and emphasizes logical and axiomatic reasoning. Without good reasoning skills (logical), someone will experience obstacles in learning mathematics. Reasoning itself is a thought process in concluding the form of knowledge (Suriasumantri, 2010). The reasoning will produce knowledge related to thinking activities and not with feelings. Therefore, reasoning is a thinking activity with certain characteristics to get to the truth. The first characteristic is the existence of a certain pattern called logic. The second characteristic, the thought process, is analytic.

General conclusions are considered valid if the process is carried out according to a certain way called logic. Drawing conclusions related to this logic can be done inductively (inductive reasoning), namely drawing conclusions based on real special cases into general conclusions, and deductively (deductive reasoning), drawing specific conclusions based on specific statements or premises.

Based on the results of data analysis, it can be seen how the conceptual profile of prospective high school mathematics teachers in the mathematics education study program at the Faculty of Teachers Training and Education, Tanjungpura University, Pontianak. Specifically, the description of their conception of the role of proof in mathematics is known. The conception in question includes their understanding as knowledgeable people in mathematics, which is focused on their opinion about the usefulness and benefits of proof in mathematics, a description of the ability to do proof in the context of high school mathematics.

Table 1. Three arguments representing the sum of the first n positive integers is n(n+1)/2. (Adapted from (Hanna, 1990))

Problem 1: Prove that: the sum of the first n positive integers is $n(n+1)/2$.
Proof: a) For $n = 1$, the statement is true, because $1 = 1(1+1)/2$ Assume that the statement is true for any k, that is, $S(k) = k(k+1)/2$ Then we have to prove that the statement is true $n = k + 1$. Proof: $S(k+1)=S(k)+(k+1)=[k(k+1)/2]+(k+1)=[k^2+k+2(k+1)]/2=(k^2+3k+2)/2=(k+1)(k+2)/2$.

Therefore, the statement is true for $n = k + 1$.
So, based on the mathematical induction theorem, the statement is true for all n.

b) We can represent the sum of the first n positive integers as the number of vertices in the following triangular shape.

			O
		O	O O
	O	O O	O O O
O	O O	O O O	O O O O
1	1+2	1+2+3	1+2+3+4

The points of the isosceles triangle for the nth triangle contain:
S(n) = 1+2+3+4+ … + n
Overlay the isosceles triangle with a second isosceles triangle of the same size so that its corresponding diagonals coincide and produce a square with n^2 points plus n coincident diagonal points. As an illustration, the image below presents 4 points of an isosceles triangle shape (the one on the left) and another isosceles triangle of the same size. The two triangles are overwritten so that their corresponding diagonals coincide as follows.

O O O O
O **O** O O
O O **O** O
O O O **O**

In this case, the square contains 4^2 points plus 4 additional points that coincide on the diagonal. Therefore, in general (using n triangles), the number of points formed by the two overlapping triangles is $2S(n) = (n^2 + n)$.
So, $S(n) = (n^2 + n)/2$ atau $S(n) = n(n + 1)/2$.

c) $S(n) = 1 + 2 + 3 + \dots + n$
$S(n) = n + (n - 1) + (n - 2) + \cdots + 1$
$2S(n) = (1 + n) + (1 + n) + \cdots + (1 + n) = n(1 + n)$
So, $S(n) = n(n + 1)/2$

The context of the material used in this research includes arithmetic (numbers), geometry, trigonometry, algebra, and general. Two instruments are used to obtain data: a questionnaire containing questions about the role of proof in mathematics and a five-question mathematical proof test. The questionnaire on the role of proof contains questions about the meaning of proof, the benefits of proof, the purpose of proof, when can an argument become proof? And why can a proof be invalid? Thirty-six prospective mathematics teachers participated as respondents.

From the answers given by respondents regarding the role of proof in mathematics, the variations of their answers can be identified as follows.

Meaning of proof: 1) Efforts to find out the truth of an event with the help of logic; 2) To prove/show the truth of the theorem; 3) A process carried out to justify or blame; 4) So that the answers put forward are valid and acceptable; 5) To explain a solution described; 6) As a valid tool to check the truth; 7) Systematic and correct steps to get a conclusion.

Proof benefits: 1) To find out or confirm the truth of a thing, theorem or formula; 2) To prove the theorem; 3) Show or confirm statement: true or false; 4) Can know the origin of the theorem and be able to apply it; 5) To be able to understand why the problem is like that and can understand from the questions that have been proven; 6) Means of convincing others.

Purpose of proof: 1) So that students believe and know the origin/occurrence of the theorem so that they are amazed; 2) Show the truth of the theorem, not the result of thinking without proof; 3) Show the truth value of a theorem; 4) Test understanding.

Arguments can be proof: 1) Arguments can be proof if any statement is substituted for the hypothesis, and if all hypotheses are true, then it can be proof; 2) If the argument cannot be refuted (denied by the other party); 3) If facts support the argument; 4) If it has a truth value; 5) If the argument is clear, real and logical; 6) If the argument is true then the argument can be proved.

Proof of being invalid: 1) Proof is "invalid" if the hypothesis is true, but there is a wrong conclusion; 2) If the thing that underlies the proof is wrong or not yet true; 3) If the steps in the proof are wrong/not true; 4) If the proof is wrong; 5) If the hypothesis is correct, but the conclusion is wrong; 6) If the proof can be broken with a more valid proof based on properties, definitions, or theorems.

Problem number 2 is given to find out a firm understanding of proof in general for relatively easy proof by answering questions and the possibility of finding examples of disclaimers. The proof presented there is empirical proof. From this proof, the question can be asked, can the conclusions are drawn generally apply to all triangles? Alternative answers to this question can be presented in Table 2.

Table 2. Empirical Proof; Does the conclusion apply to all triangles? (Adapted from (Chazan, 1993))

Problem 2: Given Δ ABC where points D and E are the midpoints of AC and BC, respectively. Prove that AB is parallel to DE. Does the conclusion apply to all triangles?	
Proof:	
D is the midpoint of AC and E is the midpoint of BC	- Known
DC= (1/2)AC and EC = (1/2)BC	- Definition of midpoint
$< C \equiv < C$	- Reflexive Properties
Δ ABC $\approx \Delta$ DEC	- Congruent
$<$ CDE $\approx <$ CAB	- Facing Angle
AB Parallel DE	- If two lines are cut by a line (AC) at an angle congruent to the secant, then the two lines are parallel.

From the results of the test (answers of prospective mathematics teachers) regarding the proof of the given high school mathematics context problem, it can be seen that the method used to prove question number 1 uses induction and substitution. Problem number 2 proved geometrically. Problem number 3, geometrically and algebraically. Problem number 4, by way of substitution, induction, logic and algebra. Next, they prove problem number 5 by using examples and algebra. The answers given do not reflect the truth and accuracy or suitability of the proof of the questions posed. In general, it can be said that their ability to prove the questions given is still very weak because all of them have not been able to present the proof process properly and correctly. Their competence to provide proof is still very low.

Question number 3 is intended to evaluate the evidentiary argument empirically as well. There are two ways to prove the statement in this problem. Although it is not a formal proof, especially for the proof of part (a), it is still proof. Two ways to prove this problem are presented, as shown in Table 3. This proof is adapted from Winicki-Landman (1998). The proof of problem number 4 concerns trigonometry related to angles. Two kinds of proof are presented here. The proof of part (a) provides an argument quite different from the statement put forward. Both types of proof can be presented in Table 3.

Table 3. Two Kinds of Proofs $If\ x > 0,\ then\ x + \frac{1}{x} \geq 2$ (Adapted from (Stylianides & Stylianides, 2009))

Problem 3:
Prove: $If\ x > 0,\ then\ x + \frac{1}{x} \geq 2$

Proof:

(a) $x + \frac{1}{x} \geq 2$ is assumed to be true

$\frac{x^2+2x+1}{x} \geq 0$ subtract 2 from both sides and equalize the denominators

$\frac{(x-1)^2}{x}$ is factored $x \geq 0$ Since the numerator is always positive, and the division itself must be greater than or equal to zero, the denominator must be positive

That is, $x + \frac{1}{x} \geq 2$ is ekuivalen to $x \geq 0$. So, it's proven: $If\ x > 0,\ then\ x + \frac{1}{x} \geq 2$

(b) We can construct a right triangle whose sides satisfy the Pythagorean theorem.

Note: $Jika\ 0 < x < 1\ maka\ panjang\ sisi\ tegak\ (vertikal)nya\ adalah\ x - 1/x$

Therefore, the following statements are true : $\left(x - \frac{1}{x}\right)^2 + 2^2 = \left(x + \frac{1}{x}\right)^2$

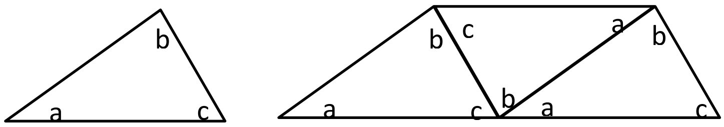
Based on the geometry of a right triangle, we know that the hypotenuse is longer than the other two sides, so it is clear that $x + \frac{1}{x} \geq 2$.

Table 4. Proof the Sum of Angles in Any Triangle is 180 Degrees (Adapted from (Schoenfeld, 2020))

Problem 4 :
Show that: The sum of the measures of the interior angles of any triangle is 180^o

Proof:

(a) We cut the corners of any obtuse triangle and place the corners together closely, as shown below.
The angles together form a straight line (c+b+a) = 180^o.



So, it is clear that the sum of the interior angles of any triangle is 180^o.

(b) Use a diagram like an image below.
Imagine that the sides BA and CA in triangle ABC change positions to BA' and CA, respectively, as shown in the first Table below. As a result, the side became perpendicular like the second picture. Next, we return it to be like the third image. Here, the displacement of the side BA' to BA forms the angle ABC, which is the right angle minus x, and the angle ACB is the right angle minus m. While the angle BAC is the sum of y and n. Make the line DA perpendicular to BC, so that DA//A'B and DA//A"C. If we let angle A'BA = x; and angle A"CA= m; angle BAD = y and angle DAC = n, then the magnitude of x = y, and the magnitude of m = n.

Thus the sum of the interior angles of ΔABC $=(90^o - x) + (90^o - m) + y + n$
$= 90^o + 90^o$; x=y dan m=n
$= 180^o$

So, it is proved that the sum of the interior angles of any triangle is 180^o.

Furthermore, question number 5 is a question given to understand proof for general cases related to numbers (arithmetic). In this case, only four respondents were able to provide proof.

The rest could not answer. In this case, the proof given is not generally proven but a special proof to be generalized. How to prove this problem is presented in Table 5.

Table 5. A Special Proof For General Proofs (Adapted from (Martin & Harel, 1989)).

Problem 5 : If the sum of the digits of an integer is divisible by 3, then the number is divisible by 3.
Proof: Without reducing generality, let's say the numbers are x,y and z. For example, if we take the number 756, a number whose sum of digits is divisible by 3, 756 is divisible by 3. View 756 = (7 x 100) + (5 x 10) + 6 = (7 x 99 + 7) + (5 x 9 + 5) + 6 ; commutative and associative properties = (7 x 99) + (5 x 99) + (7 + 5 + 6) Note that : (7 x 99) and (5 x 99) are divisible by 9. This means that it is also divisible by 3. Now, the next expression (7+5+6) =18 is divisible by 3 (as per the hypothesis). Thus, it is clear that based on the nature of integers, 756 is divisible by 3. Since all numbers can be expressed with the same expression, for any integer, if the sum of the digits is divisible by 3, then the number is divisible by 3. (Prove).

An assessment of the ability and belief of prospective mathematics teachers in providing arguments about why a statement is true is carried out by observing the various kinds of arguments given. Some of the characteristics of the arguments against the statement proof in the questions given, as presented in the picture, are a variety of different arguments used as guides or scoring rubrics.

Table 6. The Role of Proof in Mathematics According to Prospective Mathematics Teachers

The Role of Proof	Frequency
To compose/show the truth of a statement	
• Proof as a verification tool	36
Can proof be invalid? When?	
• If the proof contradicts the truth	13
• Depending on the axiom system	9
• When there are conflicting truths	14
Could an example of a disclaimer provide proof?	
• Through empirical testing	1
• Hesitating to give feedback	30
• "To try"	5
To explanation	
• Directing understanding	23
• Answering "why"	3
To communicate math	10
Creating knowledge/composing a mathematical system	1

According to Knuth (2002), the opinions of 36 prospective mathematics teachers who became respondents in this research can be identified as summarized based on the answers given from interviews regarding the role of proof in mathematics, when associated with the role of proof in mathematics in table 1. Meanwhile, during the process of proving the questions given, the characteristics of the arguments can be found according to the beliefs or beliefs of the respondents. Some of the characteristics of the arguments given when proving the statements in the questions given can be summarized as presented in Table 7.

Table 7. Characteristics of Arguments Given by Prospective Mathematics Teachers

Characteristics of Argument	Frequency of Question				
	1	2	3	4	5
Concretely (using an example/using a specific value or a visual representation)	18	23	27	8	4
As per custom	12			16	
In general		1	2	2	
Showing reason/cause				2	
Valid method (valid)	5	12	7	6	
Just convincing	1			2	4

Table 8 displays the abilities and beliefs of prospective mathematics teachers in providing arguments about why a statement is true, which is stated in each question.

CONCLUSIONS

This research aims to find out how students' understanding of prospective school mathematics teachers in the mathematics education study program FKIP Untan, regarding the role of proof in learning mathematics. They state that the proof plays an important role in mathematics, with various variations on the description of the conception as follows: (1) All respondents (36 people) stated that proof acts as a tool to compose or show the truth of a statement (as a tool to verify the truth of a statement); (2) A total of 26 respondents stated that proof is a tool to explain why a statement is true. For this context, 23 respondents believe that providing proof is an effort to facilitate understanding to lead to insight, and three respondents believe the proof is an attempt to answer the question why (why); (3) 10 respondents stated that proof is a means of communicating mathematics (mathematics communication); (4) There is only one respondent who provides information that proof is a tool to create knowledge (creation of knowledge) or to develop a mathematical system as an orderly knowledge (systematization of result); (5) There are six characteristics of arguments by the respondents in the process of proving the test questions, namely concretely (using examples or using certain values, or using a visual representation), follow the usual way, using a general method, display reasons or causes, use a valid method, and use a method that can provide confidence.

This research reveals facts about the conception of prospective mathematics teachers in the Mathematics Education S1 study program, FKIP Tanjungpura University, regarding the role of proof in mathematics. The conception includes the knowledge of those who have mathematical knowledge focused on their opinions about the usefulness and benefits of proof in mathematics. Besides, this research reveals the competence or ability of prospective mathematics teachers in the Mathematics Education S1 study program, FKIP Tanjungpura University, in carrying out proofs in the context of high school mathematics. The mathematical material includes arithmetic (numbers), geometry, trigonometry, algebra, and general mathematics.

AUTHOR CONTRIBUTIONS STATEMENT

AM is the chief executive and coordinator of this research activity. DF and AN as the author of the article until the revision. ZR as the maker of all instruments, look for the validators, and instrument data processors.

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