Al-Jabar: Jurnal Pendidikan Matematika
Volume 14, Number 02, 2023, Pages 253 – 266
http://dx.doi.org/10.24042/ajpm.v14i2.17064

Dissecting thought patterns: Analyzing how cognitive fragmentation affects conceptualization and problem-solving abilities in junior high school students

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Article Information
Submitted June 01, 2023
Revised June 30, 2023
Accepted July 15, 2023

Keywords
construction holes;
fragmentation;
mis analogical construction;
mis logical construction;
pseudo construction.

Abstract

Background: This research is rooted in the exploration of a nuanced understanding of the effect of cognitive fragmentation on the conceptual grasp and problem-solving competencies among junior high school students.
Aim: The principal aim of this investigation is to delve into the way cognitive fragmentation influences the conceptualization and problem-solving faculties of pupils aged 12-14, from varied academic milieus.
Method: Employing a qualitative research blueprint, specifically phenomenological inquiry, the study probes into the subjective experiences and cognitions of the participants. Purposefully chosen for this research, the participants consist of junior high school students. The multi-faceted data collection approach includes task-centered, in-depth individual interviews with students and Focus Group Discussions with educators. The amassed data are then meticulously examined through thematic analysis.
Result: Findings of the research reveal diverse manifestations of cognitive fragmentation among the learners. A phenomenon termed 'Pseudo construction' emerges when learners articulate correct responses without wholly comprehending the foundational concepts. 'Mis analogical construction' is recognized when incorrect analogies are deployed in problem-solving, culminating in fallacious solutions. 'Construction holes' are detected when learners exhibit inconsistent responses owing to an absence of alignment with scientific principles.
Conclusion: In summation, this inquiry furnishes invaluable insights and evidence-supported strategies to foster efficacious learning and surmount cognitive impediments within the sphere of junior high school education. The conclusions drawn herein contribute to a broader understanding of cognitive dynamics in mathematics education, offering a fresh perspective on enhancing educational practices.

INTRODUCTION

The learning of mathematics in junior high school is closely connected to elementary school learning, which often focuses on procedural use and memorization, leading to less meaningful learning (Clements & Sarama, 2020). Research findings indicate that students still tend to memorize mathematical concepts instead of truly understanding them (Hwa, 2018). For example, when discussing fractions, students may mention the symbolic representation of

How to cite Usodo, B., Sutopo, S., Nurhasanah, F., Chrisnawati, H, A., Kuswardi, Y., & Hendriyanto, A. (2023). Dissecting thought patterns: Analyzing how cognitive fragmentation affects conceptualization and problem-solving abilities in mathematics educations. *Al-Jabar: Jurnal Pendidikan Matematika*, *14*(2), 253-266.
e-ISSN 2540-7562.
Published by Mathematics Education Department, UIN Raden Intan Lampung.

concepts like $\frac{1}{2}$ or $\frac{3}{4}$ but struggle to express their meaning or significance (Permadi & Irawan, 2016).

When confronted with real-life problems involving fractions, many students face difficulties in applying symbolic concepts to solve them (Isnawan et al., 2022). For instance, when dealing with addition or subtraction of fractions, some students may know the symbols involved but fail to comprehend the problem's purpose (Sharp & Adams, 2002). This lack of understanding hinders their ability to solve fraction-related problems effectively. This suggests a persistent misconception of concepts that should have been established in elementary school. It is crucial to establish a correct conceptual foundation during the initial stages of mathematics learning (Hiebert, 2013). Therefore, it is essential to identify errors in concept construction and problem-solving approaches in order to facilitate effective learning.

Students' struggles in solving numerical problems in junior high school can be attributed to misconceptions about number concepts and operations acquired during elementary school (Smith III et al., 1994). Facts on the ground for example, when asked why -2×-3 equals 6, many students erroneously state that "negative times negative is positive." They believe they learned this concept in elementary school. Although the answer is correct, the underlying understanding of multiplying negative numbers is flawed. Mathematics learning involves the construction of knowledge, characterized by the development of mental schemas. The construction of mathematical knowledge relies on linking concepts within the subject. Experts have conducted studies on errors in mathematical concept construction and problem-solving (Brodie, 2009; Subanji & Nusantara, 2013). These errors include misconceptions in constructing number concepts and errors in performing numerical operations. (Subanji & Supratman, 2015) explains that concept construction errors can take two forms: true pseudo and false pseudo. True pseudo occurs when a student arrives at the correct answer, but their reasoning is flawed. For instance, a student may correctly determine that -3×-2 equals 6 but cannot explain why negative multiplied by negative yields a positive result. False pseudo occurs when a student's answer is incorrect, but their reasoning is correct. This often happens due to lack of thoroughness or carelessness in problem-solving.

Among various construction errors, students may exhibit mistakes in their thinking structures. Subanji (2016) refers to these mistakes as "fragmentation of thinking structures," borrowing the concept of data storage fragmentation in computers. Fragmentation of thinking structures occurs when information storage in the brain is inefficient, hindering the reconstruction and solution of mathematical problems. Fragmentation can be a result of less meaningful learning experiences. Subanji & Nusantara (2013) further explore concept construction errors and problem-solving, categorizing them into five forms: pseudo construction, construction holes, mis-analogical construction, mis-connection, and mis-logical construction.

Pseudo construction refers to a situation where a subject provides the correct answer to a problem, but upon further examination, it becomes evident that their reasoning does not align with scientific concepts (Subanji & Nusantara, 2016). According to Vinner (1997), pseudo thinking occurs when the process of problem solving does not stem from genuine mental activity. It is possible that the subject did not think accurately to arrive at the answer. In mathematical problem solving, there are two potential outcomes: a correct answer or an incorrect answer. Even though the subject's response may be correct, further investigation reveals a lack of proper understanding of the underlying concept. For instance, a subject may correctly solve $-7 - 4 = -11$ by associating negative numbers with "debt," but when faced with a problem like $-7 - (-4) = \ldots$, they struggle to apply the concept of negative numbers as "debt."

Construction holes occur when a subject incorrectly constructs a concept, rendering it meaningless. For example, a subject may use the analogy of "debt" to represent negative numbers. This understanding leads to difficulties when subtracting negative integers. Additionally, the subject may mistakenly believe that "negative numbers multiplied by negative numbers result in positive numbers." However, this statement is incorrect as it involves multiplying the numbers rather than performing operations. Mis analogical construction arises when a subject constructs concepts and solves mathematical problems using faulty analogies. For instance, a subject may attempt to apply the concept of fraction multiplication to fraction addition. They may incorrectly deduce that $\frac{1}{2}+\frac{1}{3}$ is equivalent to the product of $\frac{1}{2}+\frac{1}{3} = 1 \times \frac{1}{2} \times 3 = \frac{1}{6}$. Mis logical construction occurs when a subject is unable to provide a proper explanation for the correctness of the rules used. For example, a subject may assert that multiplying two negative numbers yields a positive result based on the understanding that "negative multiplied by negative is positive." However, they may struggle to explain why this statement holds true.

Previous research, such as that conducted by Subanji (2016), has primarily focused on describing the process of concept construction. However, this study aims to explore the impact of cognitive structure fragmentation on the construction of concepts and mathematical problem solving. By examining how fragmented thinking structures affect learning, this research provides a fresh perspective on the subject. Other studies, such as the one by Blanton et al. (2015) discuss the development of algebraic thinking but do not establish a connection between the thought development process and the fragmentation of existing thinking structures within the subject's cognitive framework. Jiménez-Fernández (2016) describe the challenges of learning mathematics and propose ways to overcome them but do not address the existence of fragmented thinking structures, which could potentially be a cause of learning difficulties. While other studies discuss the development of algebraic thinking or learning challenges in mathematics, they do not establish a direct connection between the thought development process and the fragmentation of existing thinking structures within the subject's cognitive framework. This research emphasizes the significance of fragmented thinking structures, introducing a novel link between thought development and cognitive fragmentation.

Junior high school mathematics learning is closely linked to elementary school learning, which often emphasizes procedural methods and memorization, lacking meaningful learning experiences. However, it is crucial to establish a solid foundation of correct concepts during the initial stages of formal mathematics education. Therefore, to ensure the proper process of concept construction and mathematical problem solving, it is essential to engage in appropriate learning practices by identifying instances of fragmented thinking structures and addressing mathematical problem solving. One way to trace the occurrence of fragmentation is by eliciting the thought process through in-depth interviews. In light of the aforementioned explanation, the purpose of this study is to describe the fragmentation of thinking structures and its impact on concept construction and mathematical problem solving among junior high school students in the context of numerical material.

The research on dissecting thought patterns and analyzing how cognitive fragmentation affects conceptualization and problem-solving abilities in junior high school students is urgently needed for several reasons. *Firstly,* understanding how cognitive fragmentation impacts students' ability to conceptualize and solve problems is crucial for their academic success. Junior high school is a critical period in a student's education where they begin to encounter more complex concepts and problems in various subjects, including mathematics. If students experience cognitive fragmentation, where their thinking structures are fragmented or disconnected, it can hinder their ability to grasp new concepts and effectively solve problems. By conducting this research, we can gain insights into the specific ways cognitive fragmentation affects students' learning processes and identify strategies to address and mitigate challenges.

Secondly, addressing cognitive fragmentation can have long-term implications for students' educational trajectories and future careers. If students struggle with conceptualization and problem-solving due to fragmented thinking patterns, they may develop negative attitudes towards subjects that require critical thinking, such as mathematics and science. This can lead to decreased motivation, lower academic performance, and limited opportunities for higher education and career advancement. By exploring the link between cognitive fragmentation and students' abilities, we can develop targeted interventions and instructional approaches to support students in overcoming these barriers and fostering their cognitive development.

Additionally, this research has the potential to contribute to educational theory and pedagogy. By examining how cognitive fragmentation manifests in junior high school students and its impact on their learning, we can expand our understanding of cognitive development during this crucial stage. The findings can inform the design of instructional methods, curriculum development, and teacher training programs to better accommodate the cognitive needs of students. By promoting more effective teaching and learning strategies that address cognitive fragmentation, we can enhance the overall quality of education and promote positive learning outcomes for junior high school students. By investigating this topic, we can gain valuable insights into the challenges students face and develop evidence-based strategies to promote effective learning and overcome cognitive barriers in junior high school education.

METHODS

Design:

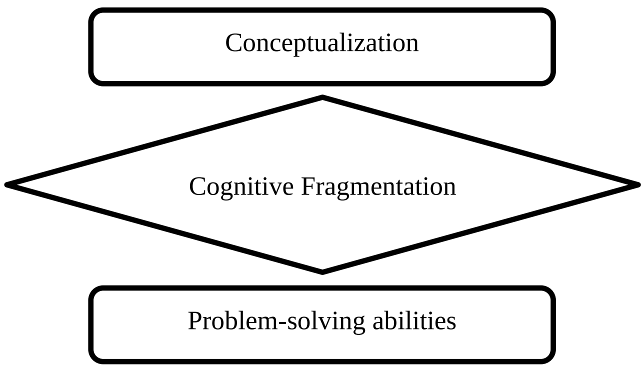

Figure 1. Research design visualization

The aim of this study is to gain a deeper understanding of how cognitive fragmentation affects the conceptualization and problem-solving abilities of junior high school students. A qualitative research design is chosen to explore the subjective experiences and perceptions of the participants in relation to cognitive fragmentation. The type of qualitative research appropriate for this study is phenomenological research. Phenomenological research aims to understand and describe the essence and meaning of human experiences as they are lived and perceived by individuals (Neubauer et al., 2019). In this study, the focus is on exploring the lived experiences and perceptions of junior high school students regarding cognitive fragmentation and its impact on their conceptualization and problem-solving abilities. Phenomenological research allows for an in-depth exploration of the subjective experiences and perspectives of the participants (Davidov & Russo-Netzer, 2022).

Participants:

The participants for this study will be junior high school students aged 12-14, purposefully selected from diverse academic backgrounds. This purposive sampling approach will ensure a range of perspectives and experiences are captured.

Instruments:

Several instruments utilized to collect data and gain insights into the phenomenon under investigation. The instruments in this study included problem solving tests, interview guidelines, and observation sheets. In this research, a combination of data collection techniques was employed to gather comprehensive insights into the experiences of junior high school students regarding cognitive fragmentation and its impact on conceptualization and problem-solving abilities. The primary data collection technique used was task-based in-depth interviews. Task-based in-depth interviews involved individual sessions with the participating students, during which they were presented with specific tasks or problem-solving scenarios (Mejía-Ramos & Weber, 2020). These tasks were designed to elicit their thought processes, strategies, and challenges related to conceptualization and problem solving. The interviews provided a platform for the students to express their experiences, perceptions, and reflections in a detailed and personalized manner.

Additionally, forum group discussion (FGD) were conducted with the teacher involved in the study. These forums provided an opportunity to gain insights from the teacher's perspective, including observations of the students' behaviors, instructional strategies used, and any specific challenges faced in addressing cognitive fragmentation in the classroom. The teacher's input added valuable context and enriched the understanding of the phenomenon under investigation.

Both the task-based in-depth interviews and FGD allowed for interactive and dynamic exchanges between the researchers and participants, enabling a deeper exploration of the students' experiences and the teacher's insights. These qualitative techniques facilitated open-ended discussions, encouraged participants to reflect on their experiences, and provided rich and detailed data that captured the complexities and nuances of cognitive fragmentation in the educational context (Gök, 2020). The data collected through the interviews and FGD were audio-recorded and transcribed for further analysis. The transcripts served as the primary data source for the qualitative analysis, allowing the researchers to immerse themselves in the participants' narratives, identify recurring themes, and gain a comprehensive understanding of the impact of cognitive fragmentation on conceptualization and problem-solving abilities.

***Data Analysis*:**
The collected data will be analyzed using thematic analysis, which involves identifying common themes and patterns across the interview transcripts, observational field notes, and student journals. A coding scheme will be developed to categorize and organize the data into meaningful units, allowing for a systematic and rigorous analysis. To ensure the reliability and validity of the qualitative data, triangulation will be employed. This involves comparing and integrating findings from multiple sources, such as interviews and FGD. The researchers will also engage in member checking, seeking participant feedback on the findings to validate the interpretations and ensure accuracy.

RESULTS AND DISCUSSION

The data in this study were obtained by giving a number problem, which is related to the operations of addition, subtraction, multiplication and division on integers, root lift and withdrawal operations and operations on the root shape. Based on the written answers and interview results obtained, it is described the fragmentation of thinking structures in the form of pseudo constructions, construction holes in analogy thinking, mis analogical construction and mis logical construction.

Pseudo Construction

Error due to the pseudo construction found in this research is the subject does not understand the concept of subtraction with negative numbers but the subject can work correctly.

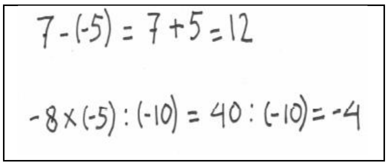

Figure 2. Students' written test answers about subtraction and multiplication operations on integers

From the answers to the subjects in Figure 2, it appears that the answers are correct. But researchers want to find out if the concepts they have are also true. For this reason, interviews were conducted with research subjects. The summary of the interview as follows. The researcher asks the research subject why $7 - (-5) = \cdots$ changes to $7 + 5$. The subject's answer is that it's the same as a sum, which is -5 to $+5$. Researchers ask again, where is $+5$ obtained. The subject's answer is that "negative meets negative becomes positive"? To ensure the subject's understanding of the concept of subtraction and the concept of negative integers, the researcher asks whether the meaning is the same as a reduction symbol with a negative number symbol and the subject's answers are the same.

To further explore the concept of multiplication and division in integers by research subjects, the researcher conducted interviews with research subjects whose summary of interview results is described as follows. In the interview, the researcher gives a problem, $-8 \times (-5) : (-10) = \cdots$. The subject can answer correctly, i.e., $-8 \times (-5) = 40$. The researcher wants to find out whether the subject understands the concept of multiplication on negative integers is true, by asking why negative numbers multiplied by negative numbers are positive. The subject's answer is because if "negative meets negative it becomes positive". The subject was asked to explain why the results were positive and the answer was time in elementary school the teacher taught, that negative multiplied by negative the results were positive

The summary of the interview showed that the research subjects did not understand the concept of subtracting with negative numbers. The subject in working on the problem is based on the statement "negative to meet negative is positive" Almost the same thing happens to different subjects. As seen in the results of the following subject work

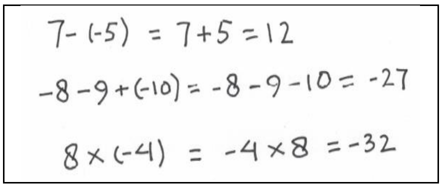

Figure 3. Other students' written test answers about subtraction and multiplication operations on integers

The subject's answer in Figure 3 shows that the subject's answer is correct but after further tracing, the subject's answer shows the mis construction. The following is a summary of the results of the interview. In the interview, the researcher asked the research subjects why how to calculate $8 \times (-4) = \cdots$, by changing to $(-4) \times 8 = \cdots$ The subject's answer is to make it easier to explain, because if changed $(-4) \times 8$, it means "-4 *ping* 8" (in Javanese language). Means $(-4) \times 8 = (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) = -32$. The reason the subject solved $8 \times (-4)$ problems done by flipping $(-4) \times 8$ was to make them easier to work on. Subjects were influenced by habits in Javanese, namely saying $(-4) \times 8$, with "-4 *ping* 8" meaning the number 8 is 4 times.

Even though it is different from the concept of multiplication as a repeated sum, that the numbers in front are multipliers and the numbers in the back are multiplied. This indicates that the subject's understanding of the concept of multiplication as a repetitive sum is not quite right. Turning it to $(-4) \times 8$ will certainly be more difficult, because it is not easy to solve with the concept of repeated addition. In the $(-4) \times 8$ summary of the interview, the reason to solve the question $8 \times (-4)$ is done by flipping $(-4) \times 8$ so that it is easier to indicate the subject does not understand the concept of multiplication as a repeated sum. By reversing to $(-4) \times 8$ it will certainly be difficult to be more difficult, because it is not easy to solve $(-4) \times 8$ by repeated addition.

The tracing of these two subjects shows that there is a concept construction error, in the form of pseudo construction. The subject tends to make a statement that does not know the basic truth. In addition, the subject also does not have the correct concept of the number symbol with a number operation. The incident shows that there is an inappropriate concept construction that causes pseudo construction. The possible impact of the fragmentation is that the subject will experience an error in the construction of subsequent concepts. These results are in line with Vinner (1997) findings that explain some facts related to the subject in solving problems, namely often the subject does not control when solving a problem, the subject only thinks to give the right answer, and the teacher only expects learning to get the right answer. This is what causes the subject to experience pseudo thinking or pseudo thinking (Vinner, 1997).

Mis analogical construction

Mis analogical construction or similes can be a way to solve a problem if it is constructed correctly, on the contrary it can lead to a wrong solution if an analogy error occurs. This analogy error occurs because the construction of the concept of division of fractions is wrong. Following is the exposure and analysis of data related to analogy errors.

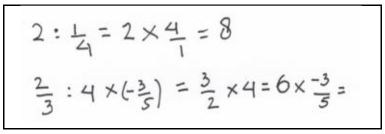

Figure 4. Subject's answer to the question of division and multiplication operations

In the subject's answer in Figure 4, The subject can correctly answer a division problem with fractions, but the subject incorrectly answers another question about the division operation with fractions. Researchers suspect, the subject only bases the understanding that the division operation is to be a multiplication operation with the fraction reversed, the numerator becomes the denominator and the denominator becomes the numerator.

For this reason, interviews were conducted with the following summary of interviews. The researcher asked the research subjects why $2 : \frac{1}{4}$ became $2 \times \frac{4}{1}$. The research subjects gave the answer that the division with fractions was carried out by means of the fractions that were reversed and turned into multiplication. The subject also explained that the understanding was obtained during elementary school, and the teacher taught that if the division is done by fractions, then by changing into multiplication and the fraction behind is reversed. But when given the problem $\frac{2}{3} : 4$, the subject answers $\frac{3}{2} : 4 = 6$. Next the researcher asks why doing it like that, the answer is because the fraction is reversed and it is no longer consistent with the original answer that the fraction that is reversed is the fraction that is behind (as a divisor)

From the summary of interviews, the subject uses a method that is considered correct to solve almost the same problem. But the subject is not appropriate in using this method. Supposedly what is reversed is the fraction as a divisor, but the subject considers what is reversed is a fraction, not necessarily the divisor. This provides information on the occurrence of analogy errors. Subjects cannot recognize the similarity of structural relations between known problems and new problems. This is in line with what is conveyed by Ruppert (2013), that the analogy reasoning indicator of subjects can identify each mathematical object at the source problem by looking at the similarity of properties and structures, looking for identical relationships from characteristics between the source problem and the target problem, can then solve the target problem using a solution or concept similar to the source problem, then can write down the answers to what the target problem wants. Thus, in analogy reasoning must recognize the similarity of structural relations between known problems with new problems. The impact of the analogy error includes understanding a mathematical concept less profoundly. Amir-Mofidi et al. (2012) also argues that analogy reasoning makes understanding deeper mathematical concepts can be stored in long-term memory.

Construction Holes

Construction holes occur when the subject gives an inconsistent answer, but after further exploration, it turns out the reason or reasoning in answering the question is not in accordance with scientific concepts. Fragmentation due to construction holes can be seen from the answers to the following subject.

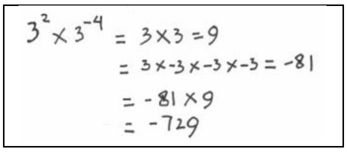

Figure 5. The subject's written answers to power of number operations

In the subject's answer in Figure 5 above, the subject uses an inconsistent concept, which is when determining $3^2 = 3 \times 3$ but $3^{-4} = 3 \times -3 \times -3 \times -3 = -81$ The results show inconsistencies in using the concept of power of number.

To find out more, interviews were conducted with the following summary of interviews. In the interview the researcher asked why the answer to the research subject was $3^2 = 3 \times 3$ but $3^{-4} = 3 \times -3 \times -3 \times -3 = -81$?. For $3^2 = 3 \times 3$, the subject can explain correctly, but in answer $3^{-4} = 3 \times -3 \times -3 \times -3 = -81$ the subject cannot explain correctly. The subject states that because 3 to the rank of min 4 means that 3 is multiplied by -3 until the number is 4. If asked $-3^4 = \cdots$,, the subject can answer correctly, that is $-3^4 = -3 \times -3 \times -3 \times -3 = 81$.

From the result of the interview, it shows that the subject actually already knows the concept of power of numbers. However, in the answer to questions the power of is a negative number, the subject does not use the concept that is owned, but the subject makes his own statement that is used in solving the problem. This shows the inconsistency of the subject in using the concepts that have been owned. As a result, the subject made a mistake in answering the problem. In addition, there are unstructured schemes in the construction process of problem solving carried out by the subject, causing a construction holes. The concept is not properly constructed due to an error in the process of assimilation and accommodation in the thought process. These results are in line with the results of his research (Hidayanto et al., 2017) explained that the holes construction occurs when the subject assimilates and also accommodates information about a mathematical concept so that the answer given is wrong. This is also consistent with (Hidayanto & Subanji, 2015) statement that the construction holes occurs because there is a scheme in constructing problem solving that is incomplete.

Mis Logical Construction

Mis logical construction, occurs when the subject cannot properly explain whether the rules used are true or false. Fragmentation due to mis logical construction can be seen from the answers to the following subject.

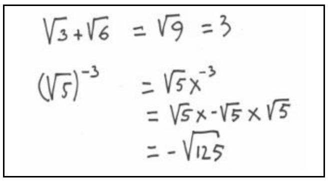

Figure 6. The subject's written answers to root form

From the answers of the subjects in Figure 6, it shows that the subject experienced an error in answering questions about power of numbers and square root. From these answers the subject allegedly used the wrong rules but the subject did not know that the rules used were wrong. This is evident from the summary of the results of the following interview. In the interview the researcher asked why $\sqrt{3}+\sqrt{6}$ = $\sqrt{9} = 3$? The research subjects explained that $\sqrt{3}+\sqrt{6}$ by adding the number 3 to the number 6, the result becomes $\sqrt{9}$. Because 9 is 3^2 then $\sqrt{9} = 3$. Next the researcher asked why $\left(\sqrt{5}\right)^{-3} = -\sqrt{125}$? The research subjects explained that $(\sqrt{5})^{-3} = \sqrt{5} \times -\sqrt{5} \times -\sqrt{5} = -\sqrt{125}$. Researchers ask why the results are negative? The research subject answered, because the rank is negative then the results must be negative. Actually, I'm not sure, but because of the power of negative number means the number multiplied to negative number

From the interview the subject uses the wrong rule. The subject uses the rules because that the power of number is a negative number so that it multiplies by -3. So, $\left(\sqrt{5}\right)^{-3}$ = $\sqrt{5} \times -\sqrt{5} \times -\sqrt{5}$ = $\sqrt{125}$. This is as expressed by Hidayanto et al. (2017) which states that misconceptions are inaccurate assumptions caused by wrong thinking or understanding. The statement was also supported by Glatzeder et al. (2010) who stated that misconceptions can cause errors in the subject in solving problems. If this misconception continues, it will harm learning mathematics. Students will need help understanding advanced concepts; in this case, misconceptions that are not corrected can cause students to struggle to understand more complex mathematical concepts, resulting in subsequent mathematics learning being disrupted and causing ongoing problems (Muhaimin & Kholid, 2023; Swidan et al., 2018). Then students will also make repeated mistakes; unresolved misconceptions tend to push them to make repeated mistakes in working on math problems; they may apply a wrong understanding of mathematical concepts when working on questions, leading to consistently wrong answers (Ghani & Maat, 2018). Students will also need help connecting concepts. This can hinder their ability to solve problems that require a deep understanding of these concepts (Suprapto, 2020). Lastly is a change in negative attitudes towards mathematics; this can affect their motivation to learn and contribute to non-participation in learning mathematics (Koutselini, 2008). Identifying and correcting misconceptions early on is essential to address the implications of mathematical fallacies. Teachers must engage students in active discussion, provide appropriate feedback, and use learning strategies focusing on a deep understanding of concepts. Correctly understanding mathematical concepts is essential to build a strong foundation and ensure success in learning mathematics.

CONCLUSIONS

Fragmentation of thinking structure and its impact on the concept structure and problem solving of numbers is, in constructing concepts and problem solving of number material. The fragmentation of thinking structures are: (1) Pseudo-construction, the subject tends to make a statement that does not know the basis of truth. The subject also does not have the correct concept of a number symbol with a number operation. The impact of the fragmentation is an error in the construction of the next concept. (2) Analogy errors, namely the subject uses the method considered correct to solve almost the same problem, but the subject is not appropriate in using this method. Supposedly what is reversed is the fraction as a divisor, but the subject considers what is reversed is a fraction, not necessarily the divisor. (3) construction holes, the subject shows the inconsistency of the subject in using the concepts that have been owned. The subject actually already knows the concept of power of numbers. (4) Mis logical construction, the subject uses the wrong rule but the subject does not know that the rule used is wrong, that is, when the subject uses rules on negative numbers with wrong rules. The impact of mis logical construction is the occurrence of misconceptions.

One of the key implications of this research lies in the development of educational interventions. The findings provide guidance for educators in designing targeted interventions to address cognitive fragmentation in junior high school students. Through the implementation of scaffolded learning activities and explicit instruction on cognitive strategies, educators can support students in developing integrated thinking and achieving a deeper understanding of concepts. Moreover, fostering metacognitive awareness among students can enable them to monitor and regulate their thinking processes, further enhancing their problem-solving abilities. The research also highlights the importance of curriculum design. By emphasizing the interconnectedness of different concepts and providing opportunities for students to make meaningful connections, curriculum designers can help alleviate cognitive fragmentation and promote coherent learning experiences. By creating a curriculum that fosters conceptual understanding and encourages students to see the relationships between various topics, educators can enhance students' ability to transfer knowledge and apply it to real-world problem-solving situations. Despite its significance, the research does have limitations that should be acknowledged. Firstly, the findings may not be generalizable to all student populations, as the study focused solely on junior high school students. Additionally, being a qualitative study, the interpretation of the data is subject to the researchers' biases and perspectives. However, steps were taken to mitigate subjectivity through rigorous data analysis and the use of multiple data sources. Moreover, the selection of participants may introduce certain biases into the findings. It is crucial to consider the sample's characteristics and experiences when interpreting the results. Additionally, conducting in-depth interviews and FGD can be time-consuming and resource-intensive, which may limit the number of participants or the depth of analysis that can be undertaken.

ACKNOWLEDGMENT

We would like to express our sincere gratitude to all individuals and institutions who have contributed to the completion of this research. *First and foremost*, we extend our heartfelt appreciation to the participating junior high school students who generously shared their time, thoughts, and experiences with us. Their active involvement and willingness to participate in the study were instrumental in obtaining valuable insights and data.

We would also like to thank the teachers who facilitated the data collection process and provided valuable inputs and observations throughout the study. Their expertise and dedication greatly contributed to the quality and depth of the research findings. We are grateful to the educational institutions that supported and approved this research, allowing us to access the student participants and conduct the necessary data collection procedures. Their cooperation and assistance were essential in the successful execution of the study.

Additionally, we extend our thanks to the Universitas Sebelas Maret (UNS) through Funding by PNBP UNS 2023 in a competitive grant scheme (Capacity Building of Research Groups for Scientific Publications).

AUTHOR CONTRIBUTIONS STATEMENT

In conducting this study each author made significant contributions to different aspects of the research. The author contributions statement is as follows:

BU played a key role in the conceptualization and design of the study. They conducted a comprehensive literature review, identified the research gap, and formulated the research questions. Their expertise in educational psychology and qualitative research methods greatly influenced the overall study design. **HEC** and **YK** was responsible for the data collection process. They developed the interview protocols, coordinated with the participating schools, and conducted in-depth interviews with the junior high school students. Their excellent communication skills and rapport-building abilities facilitated a comfortable and open environment for the participants to share their thoughts and experiences.

AH contributed to the data analysis and interpretation phase. They transcribed and coded the interview data, identified emerging themes, and conducted thematic analysis. Their expertise in qualitative data analysis techniques provided valuable insights into the cognitive fragmentation patterns observed in the students' conceptualization and problem-solving abilities. S focused on the integration of the findings and the development of the discussion section. They synthesized the results, critically examined their implications, and related them to the existing literature. Their knowledge of educational theories and their ability to critically analyze research findings contributed to the overall coherence and depth of the discussion. FN supervised the research project and provided guidance throughout the entire process. They ensured methodological rigor, reviewed the research instruments, and provided valuable feedback on the data analysis and interpretation. Their experience in educational research and qualitative inquiry significantly enhanced the overall quality of the study.

All authors were actively involved in reviewing and revising the manuscript, ensuring that it accurately represented the research findings and addressed the research objectives. They collaborated closely, sharing their expertise and perspectives, to produce a comprehensive and well-rounded research article. It is important to note that all authors have read and approved the final version of the manuscript, and they take collective responsibility for the integrity and accuracy of the study.

Overall, this research study was a collaborative effort that capitalized on the diverse expertise and contributions of each author. Their individual roles and collective efforts ensured a rigorous and insightful investigation into the impact of cognitive fragmentation on junior high school students' conceptualization and problem-solving abilities.

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