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Mathematical beliefs: What topics appear and how has this changed over time?

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Abstract

Background: The growing understanding of beliefs in mathematics education, especially as they relate to the nature of mathematics as well as the learning process, indicates that this topic has become a recent research trend. Specifically, in the context of mathematics educators, it is important to identify and explore the development of this theme in the academic literature.
Aim: The goal is to identify and analyze the prevailing beliefs in the field of mathematics education, particularly focusing on pre-service teachers, and to determine the thematic trends and gaps in current research.
Method: The Systematic Mapping Study (SMS) method was employed as the initial stage for selecting primary sources in a literature study. The Scopus database was used for SMS, leading to the identification of 171 relevant articles.
Result: The analysis of these articles reveals that mathematical belief has become a trending research topic. The thematic map formed from these studies identifies thirteen clusters, with key areas being motivation, pre-service mathematics teachers, mathematics education, beliefs, and attitudes. These clusters are currently considered global challenges in the field.
Conclusion: There are significant opportunities for further research in this area to complete the knowledge puzzle. Out of the numerous articles analyzed, 41 have been classified as primary articles on this theme, with three identified as primary references, highlighting the evolving trend of focusing on pre-service teachers' beliefs in mathematics education.

INTRODUCTION

Teachers play an essential role in learning (Doño & Mangila, 2021). Effectiveness in conveying lesson content depends on the beliefs and knowledge of the teacher's pedagogical content (Muhtarom et al., 2019). This includes content mastery of knowledge about students' learning styles and teaching and learning strategies (Amirali & Halai, 2010; Beswick, 2012), knowledge of how to make teaching simple and using appropriate methods to understand concepts is related to teachers' beliefs and commitment to pedagogical practice in the classroom (Muhtarom et al., 2017, 2019).

Mathematical beliefs are a discipline about how mathematics is learned and taught (Siswono, Kohar, & Hartono, 2017). Initial beliefs are formed based on the teacher's experience as a student and the teacher's influence at school. Beliefs can be categorized into beliefs in mathematics as a discipline, beliefs about teaching, and beliefs about learning mathematics (Beswick, 2012). Ernest, as quoted (Siswono, Kohar, & Hartono, 2017; Siswono, Kohar, Rosyidi, et al., 2017), states three views related to mathematics. First, the view that mathematics

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is a collection of facts, rules, and skills is called Instrumental. Second, is the Platonic view of mathematics as static and unifying several other types of knowledge. Mathematics was invented, not created. Third, mathematics is dynamic, namely the continuous development process resulting from human creation. Mathematics is a constant process of knowing, and there is no end.

Furthermore, there are four categories of teachers' mathematical beliefs (Buehl & Fives, 2009). First, beliefs about mathematics include that mathematics is complex or bound by rules. Second, self-confidence is confidence in learning and the ability to succeed or fail in mathematics. Third, beliefs about teaching include beliefs about what teachers should do to help students learn mathematics. Fourth is belief in a social context.

Several previous studies have proven that there is a significant focus on the relationship between teachers' beliefs and their practices in mathematics education. The relationship between beliefs in teaching and practice among teachers is often studied by researchers (Muhtarom et al., 2019). A group of researchers suggests that teachers' beliefs and teaching and learning values influence teachers' teaching practices (Bal, 2015; Beswick, 2012; Muhtarom et al., 2019; Siswono, Kohar, Rosyidi, et al., 2017). Tamba et al., (2020) highlighted the influence of teachers' epistemological beliefs on their mathematics teaching practices. Likewise, Sari et al., (2019) emphasized the significant relationship between teachers' epistemological beliefs and their teaching success. These findings underscore the importance of understanding teachers' beliefs in shaping effective mathematics teaching. Apart from teachers' beliefs about mathematics, mathematics learning also plays an important role in determining how teachers make students learn mathematics.

The contributions of mathematics education researchers in understanding teachers' beliefs and knowledge have been extensive, especially as this topic has only received intensive attention in the last decade. However, unfortunately until now there is no comprehensive factual data regarding the number of publications and mapping of the topics studied, thus indicating the need for annual mapping of the development of studies on this topic. This research offers a new perspective by tracing the evolution of mathematical beliefs over time, in contrast to previous studies that predominantly focused on static relationships between teachers' beliefs and teaching practices. Therefore, this research aims to identify and analyze emerging trends and changes in mathematical beliefs in the context of mathematics education. It is hoped that this will provide insight into the dynamics of mathematical beliefs and their changing influence on mathematics teaching practices, thereby opening up opportunities for the development of educational strategies that are more effective and relevant to the current changing educational paradigm.

METHODS

This study used the Systematic Mapping Study (SMS) method with bibliometric analysis (Aria & Cuccurullo, 2017; Borgman & Furner, 2005; Gupta & Bhattacharya, 2004; van Eck et al., 2010). This method can identify research trends on beliefs about the nature of mathematics, mathematics teaching, and mathematics learning and identify new development opportunities for further research (Armitage & Keeble-Allen, 2008; Borgman & Furner, 2005; Gupta & Bhattacharya, 2004; Van Eck et al., 2010). There are five research stages: determining keywords, searching for data, selecting articles, validating, and analyzing data (Figure 1).

Before entering the research phase, the Scopus database was chosen because of its broad coverage, credibility, and access to a collection of essential information for this study, including title, abstract, and keywords (Chadegani et al., 2013; Falagas et al., 2008).

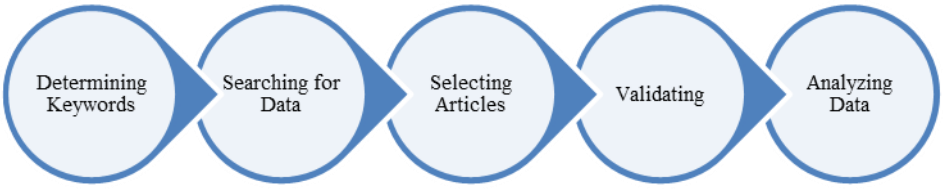


Figure 1. Research Stages

The Scopus database search used the keywords "nature of mathematics," OR "learning of mathematics," OR "teaching of mathematics," AND "beliefs". This search resulted in 222 documents, which were then reduced with various considerations, namely: (1) only English documents were selected to facilitate content analysis, (2) only documents in the form of articles of journals or proceedings were selected because they went through a rigorous review process and had been empirically validated. Based on the reduction, 178 documents were obtained in 168 journal articles and ten proceeding articles in English.

Furthermore, the examination and selection of titles and abstracts are carried out following the topics studied. It was found that seven documents did not match the topic, so they were removed from the research database, leaving 171 documents in the form of 163 journal articles or eight articles of proceedings. Table 1 shows the document selection stage. The data search results are then selected and validated to read and analyze the data. The data search results are then presented in the form of diagrams and data tables using Biblioshiny. Primary sources selected based on the SMS method can be used for further research using the SLR method, which is not discussed in this article.

Table 1. Document Selection Stage

Phase	Description	Results
Phase 1	Select database: Scopus.	-
Phase 2	Search the database with four keywords: "nature of mathematics," OR "learning of mathematics," OR "teaching of mathematics," AND "beliefs."	222 documents
Phase 3	Select only English article publications.	205 documents
Phase 4	Select only publications in the form of journal articles or proceedings.	178 documents
Phase 5	Check the title and abstract according to the topic being studied.	171 documents

RESULTS AND DISCUSSION

Publication Growth

In this first part, we discuss the growth of documents from year to year since this topic was first published. This research does not limit the year because it wants to see in detail the growth of documents related to this topic and how this topic develops. Figure 2 shows the primary information related to this research database. Research on mathematical beliefs began in 1901, and until now, there have been 171 published articles in 93 sources, either in journals or proceedings. There were 163 journal articles and eight proceedings articles. The document's annual growth rate reached 1.62%, indicating that this topic is still in demand today. The research database contains selected articles with an average age of 11.1 years using 367 keywords and 6991 references. This shows that for more than a century (1901-2022), it is only

in the last decade that the topic of research on mathematical beliefs has increased significantly. A total of 359 authors were involved in the publication of these articles. Of the 171 articles, 49 articles were written by a single author. One hundred twenty-two articles were written collaboratively by 2-3 authors for each article. 11.7% of the 171 selected articles in this research database, or as many as 20 articles, result from international collaboration. This can also mean that out of 122 collaborative author articles, there are 20 articles resulting from cross-country research (16.39%). This statement is also strengthened by looking at each document's average citation, which reaches 15.43. Based on these various descriptions, it can be concluded that the research topic on mathematical beliefs about the nature of mathematics, teaching mathematics, and learning mathematics is a current research trend and is very likely to continue. The movement of this research can be seen clearly in Figure 3.

The first articles about mathematical beliefs (Gillette, 1901) were written, linking them to one's emotions, but this research was not a fad at the time. No other publications on the topic were published until 1988. So, over eight decades, the subject was not in vogue (Figure 3). The number of articles on mathematical confidence increased significantly in 1989, with a peak in 2021 of 16 articles. Research on mathematical beliefs began to be re-initiated by Ernest (1989) those who examined the philosophy of mathematics and education. He argues that views of the nature of mathematics are fundamental in teaching mathematics, where they can significantly influence the mathematics curriculum as it is prepared to students. However, a distinction must be drawn between beliefs expressed as a nature of mathematics and views inferred from actual classroom practice (Ernest, 1989). Starting from the results of this study, in the following year, this topic became attractive to researchers (for example, Ernest, 1991; Julie, 1991; Whitman & Lai, 1990; and Willmoth, 1991).

Figure 2. Main information

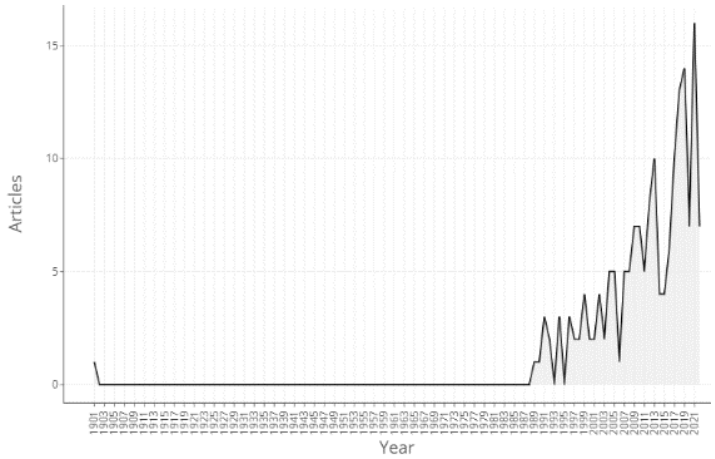


Figure 3. Number of articles each years

Whitman & Lai (1990) discuss the similarities and differences in beliefs about effective mathematics teaching held by teachers from different socio-cultural backgrounds: Tokyo, Japan, and Hawaii, USA. Although there are some similarities in beliefs about what constitutes effective teaching, the differences are more significant. These differences seem to reflect differences in the socio-cultural environment of teachers. In particular, differences emerge in classroom management, "saving face" strategies, and providing for individual needs and differences. Ernest (1991) then presents a model of belief systems related to teaching mathematics and the issue of the contrast between espoused and enforced beliefs. It is similarly argued by Willmoth (1991) that the inseparability of mathematical practice from theory means putting specific knowledge into practical effect.

Meanwhile, Julie (1991) stated that there is a widespread belief that computers should be used for teaching and learning mathematics. Computers are used in mathematics classes to (1) reinforce previously taught concepts, (2) enable students to build computer programs to simulate mathematical techniques known to students, and (3) explore the micro-world of mathematics that includes familiar mathematical ideas and concepts known to students. Much recent research has been conducted on an increasingly diverse range of mathematical beliefs (Bicer et al., 2022; Dobie & MacArthur, 2022; Fowler et al., 2022; Livers, 2022; Riard & Kaur, 2022; Safrudiannur et al., 2022; Vesga-Bravo et al., 2022).

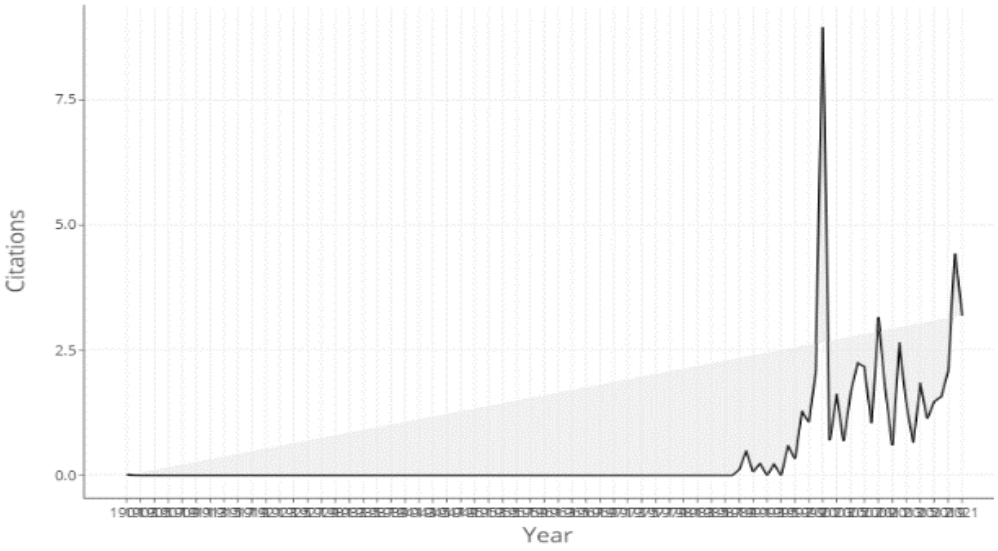


Figure 4. Average document citations each year

Based on Figure 4, it is known that the average document citations each year increased significantly from 1989. An interesting thing occurred in 2001, when there was the highest increase in average document citations each year, reaching 8.95. The following year continues to show a positive trend, with an increase in the average document citation every year compared to the previous year, although it has not yet exceeded 2021. If you look at the citation trends in Figure 4, in the coming years, it may exceed the citation achievements in 2021. The topic of beliefs about the nature of mathematics, teaching mathematics, and learning mathematics is a current research trend and is very likely to continue to develop.

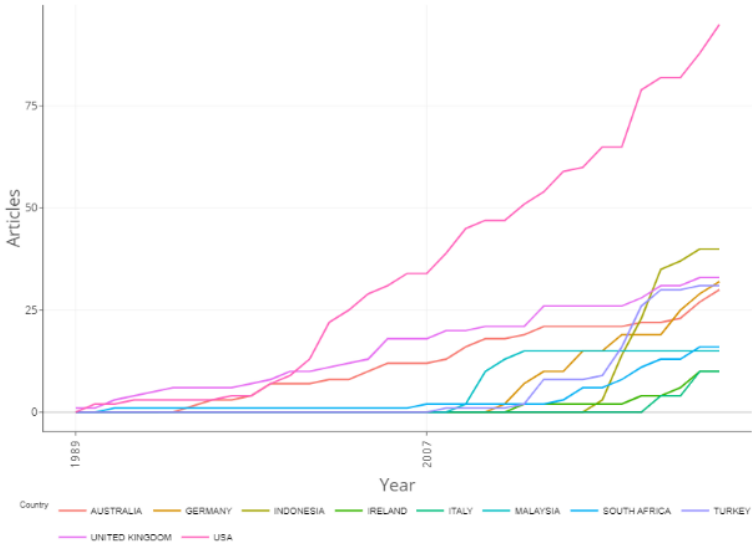
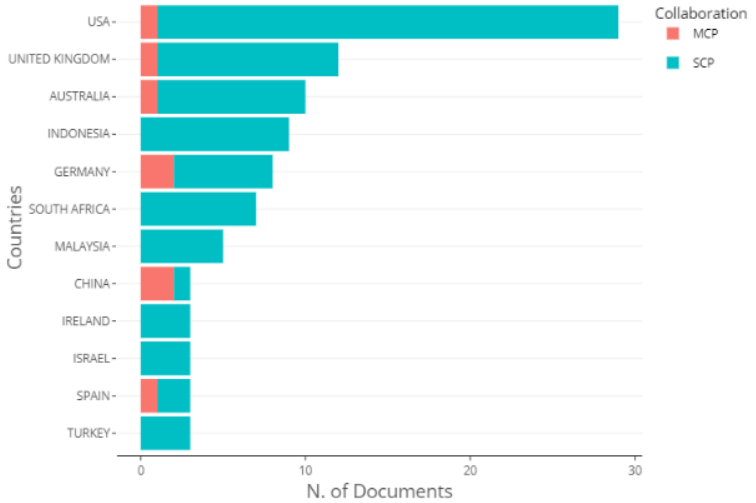


Figure 5. Document growth from countries with the most publications

Figure 5 shows the growth of articles in the ten countries that contributed the most articles (minimum ten articles) to the Scopus database related to beliefs about the nature of mathematics, teaching mathematics, and learning mathematics. The United States, with 95 articles, has the most publications in recent years, followed by Indonesia and the United Kingdom, with 40 articles and 33 articles, respectively. It can be seen that the United States is the most concerned country regarding this research topic. From year to year, there is always a very significant increase in document publications. The United States was the first to initiate this topic (Gillette, 1901). An interesting thing happened in Indonesia, which was the first time researching this topic (Purnomo et al., 2016), and there has been an astonishing increase until now in second place. Meanwhile, after over eight decades of disinterest, the UK was a driving force for the topic's revival (Ernest, 1989) and is currently in third place.



MCP: *Multiple Countries Publication*
SCP: *Single Multiple Countries Publication*

Figure 6. Countries with correspondence authors of at least three articles

Figure 6 shows the countries with the most correspondence authors. Correspondence authors are responsible for the article's content and the legality of article submission and repair. The country of origin of the correspondent author who publishes the most articles related to beliefs about the nature of mathematics, mathematics teaching, and mathematics learning is the United States, with 29 articles (17%). The UK and Australia occupy the following positions: 12 articles (7%) and ten articles (5.8%). Indonesia is in fourth place with nine articles (5.3%) as correspondent authors related to this topic. Figure 6 also shows the intensity of international research collaboration from a country. It can be seen that only six countries carry out global research collaborations. These countries are Spain, China, Germany, Australia, the UK, and the United States. Nine articles with correspondent authors from Indonesia, none of which are the result of international research collaborations.

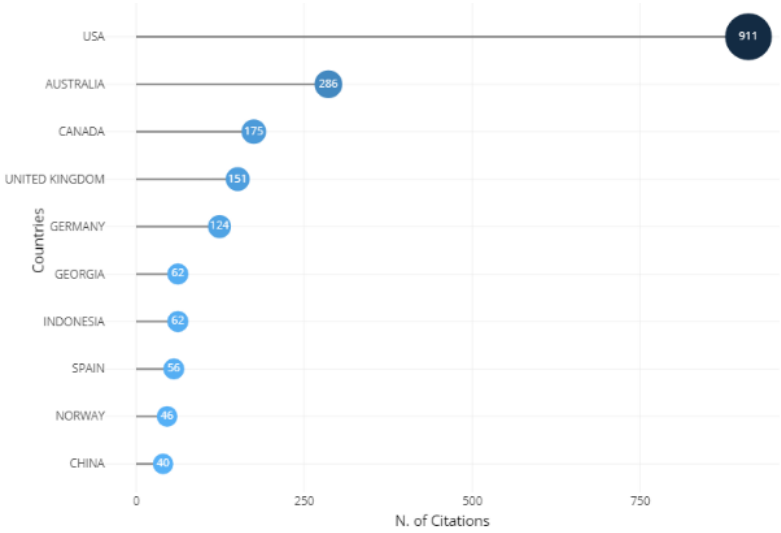


Figure 7. Ten countries with the most citations

Figure 7 shows the ten countries with the most significant scientific impact (at least 40 citations) on the literature in the Scopus database. The United States is the document-producing country with the highest scientific impact. Its publications have been cited 911 times with an average citation of 31.41. Australia occupies the second position with 286 citations, an average citation of 28.60. Canada occupies the third position with 175 citations, an average of 87.50. Although Canada is in third place for most citations, it has the highest citation rate of any other country. Indonesia ranks seventh with 62 citations and an average citation of 6.89.

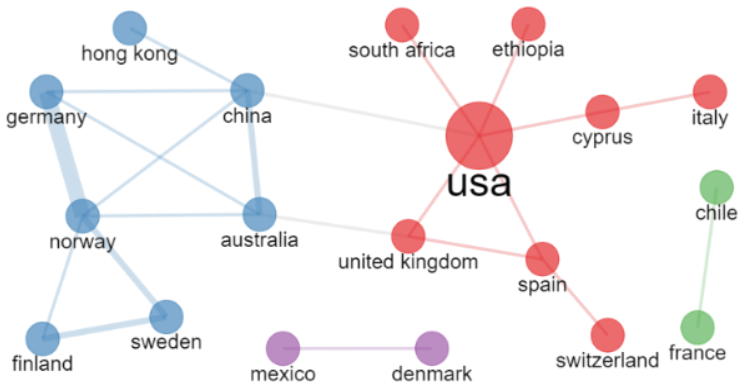


Figure 8. Network of state collaboration in publication

Figure 8 shows four clusters of country collaboration networks in research. The first cluster includes Germany, China, Norway, Australia, Finland, Sweden, and Hong Kong. The second cluster comprises the United States, United Kingdom, Spain, Switzerland, Cyprus, Italy, Ethiopia, and South Africa. In this second cluster, the United States is central to research collaboration between countries. The third cluster includes Mexico and Denmark, while the fourth includes France and Chile. The first and second clusters are interrelated. In particular, there is a collaboration between China, the United States, Australia, and the United Kingdom. The most intense cooperation between countries is between Germany and Norway, followed by Norway with Switzerland, China, and Australia.

Key Authors and Affiliates

Most Productive Author

Regarding the authors, of the 171 articles selected in this research database, the results show that 359 authors contributed to developing research on beliefs about the nature of mathematics, mathematics teaching, and mathematics learning. Frequency distribution of scientific productivity in Figure 9.

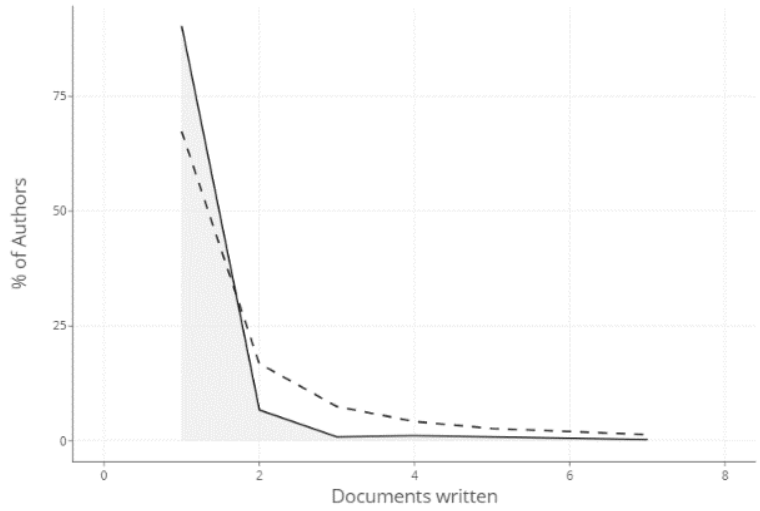


Figure 9. Frequency distribution of scientific productivity

Lotka's law of the frequency distribution of scientific productivity, presented in Figure 9, was used to determine the primary author on this topic. Core authors have written at least three articles about this topic, so 11 out of 359 authors (3%). Meanwhile, 324 authors (90.3%) wrote only one document, and 24 authors (6.7%) reported two articles. These 348 authors (97%) were not the core authors on this topic. The eleven core authors, namely Siswono TYE, Hartono S, Kohar AW, Zakaria E, Blömeke S, Ernest P, Kaiser G, Maat SM, Goos M, Gürsoy K, and König J.

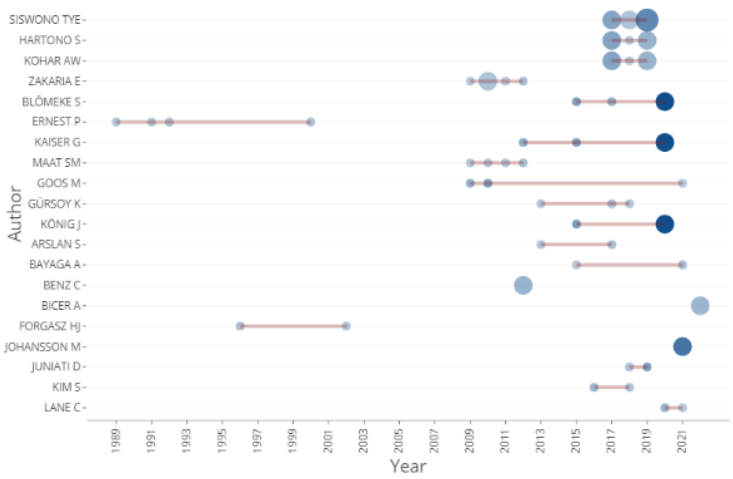


Figure 10. Track record of the twenty most prolific authors

After knowing the core authors on this topic, the track record of their productivity is explored, as shown in Figure 10. It can be seen that four of the twenty world's most productive authors on this topic are Indonesian authors. The authors are Siswono TYE, Hartono S, Kohar AW, and Juniati D. Siswono TYE published the most articles (7 articles in 2017-2019) with an h-index of 4, global citations 52 times and local citations ten times. Global citations are citations to all articles in the Scopus database, while local citations are citations to all selected articles (171 articles) in this research database. The exciting thing is presented in Figure 10; three authors have many citations even though fewer published articles are around 3-4. The frequency of citations indicates a document transmitting crucial scientific knowledge, which is then used as the basis for other research (Acedo & Casillas, 2005). These authors are Blömeke S, Kaiser G, and König J. Considering this, the leading authors on this topic, which were identified initially as eleven authors, can be reduced to four authors, namely Siswono TYE, Blömeke S, Kaiser G, and König J. The four authors made a significant contribution to the development of this topic and became the primary reference, without neglecting other authors who also contributed significantly.

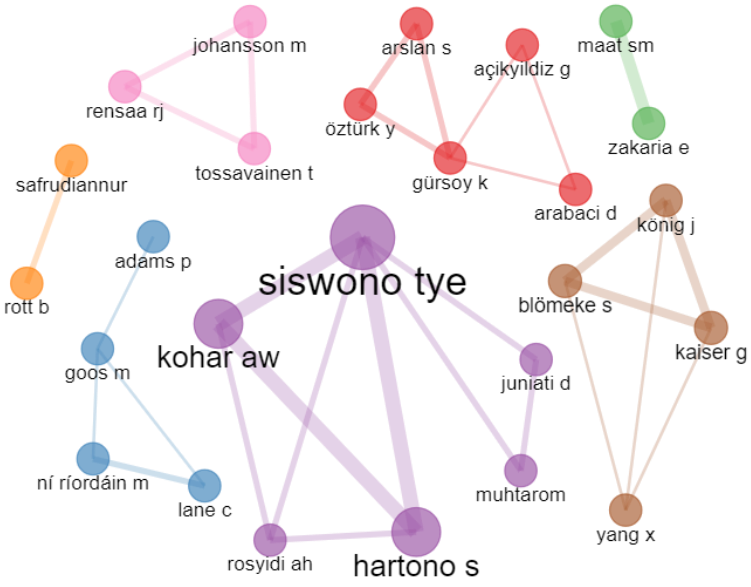


Figure 11. Research collaboration network in publications

Figure 11 shows seven clusters of research collaboration networks in the publication of articles on this theme. The first cluster includes Gürsoy K, ztürk Y, Açikyildiz G, and Arabaci D. The second cluster comprises Goos M, Lane C, Ní Ríordáin M, and Adams P. The third cluster includes Zakaria E and Maat SM. The fourth cluster includes Siswono TYE, Hartono S, Kohar AW, Juniati D, Muhtarom, and Rosyidi AH. The fifth cluster includes Rott B and Safrudiannur. The sixth cluster includes Blömeke S, Kaiser G, König J, and Yang X. The seventh cluster has Johansson M, Rensaa RJ, and Tossavainen T.

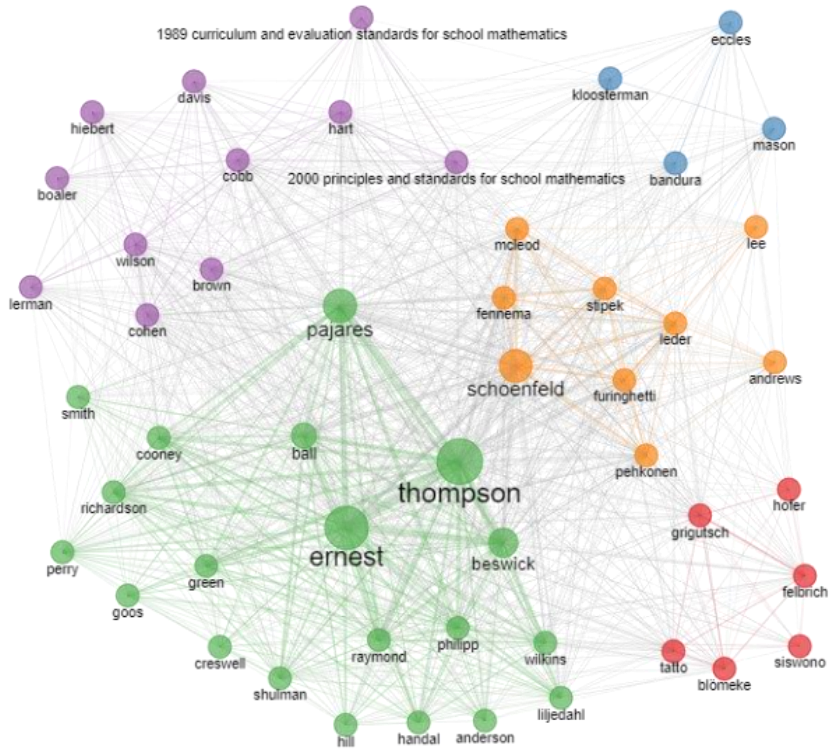


Figure 12. Co-citation network between authors

Figure 12 shows four clusters of co-citation networks between authors. Quotations reflect the interconnections between authors and conjunctions between different scientific concepts in a subject area (Kraus et al., 2014). Co-citation is the frequency with which another document quotes two articles together. If at least one other paper cites the same two articles, these articles are said to be co-cited. The more co-citations two articles receive, the higher the strength of their co-quotes, and the more likely they are to be semantically related (Small, 1973). The first cluster includes Blömeke, Siswono, Grigutsch, Hofer, Tatto, and Felbrich. The second cluster comprises Bandura, Eccles, Kloosterman, and Mason. The third cluster includes Ernest, Thompson, Beswick, Ball, Pajares, Cooney, Philipp, Richardson, Shulman, Raymond, Liljedahl, Green, Reliable, Wilkins, Smith, Goos, Hill, Perry, Anderson, and Creswell. The fourth cluster includes Brown, Cobb, Cohen, 2000 Principles and Standards for School Mathematics, Wilson, Boaler, Hiebert, Lerman, Hart, 1989 Curriculum and Evaluation Standards for School Mathematics, and Davis. The fifth cluster includes Schoenfeld, Leder, Fennema, Mcleod, Pehkonen, Stipek, Furinghetti, Andrews, and Lee.

Most Productive Affiliate

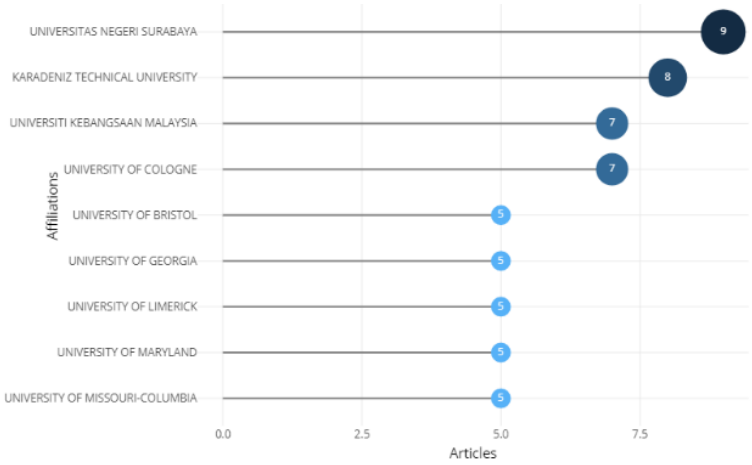


Figure 13. Most productive affiliates

Regarding author affiliation, Figure 13 shows the most productive affiliations that resulted in at least five articles on beliefs about the nature of mathematics, mathematics teaching, and mathematics learning. The Universitas Negeri Surabaya in Indonesia focuses on researching this topic. Nine articles have been published, followed by Karadeniz Technical University in Turkey with eight articles, Universiti Kebangsaan Malaysia in Malaysia, and the University of Cologne in Germany with seven articles each.

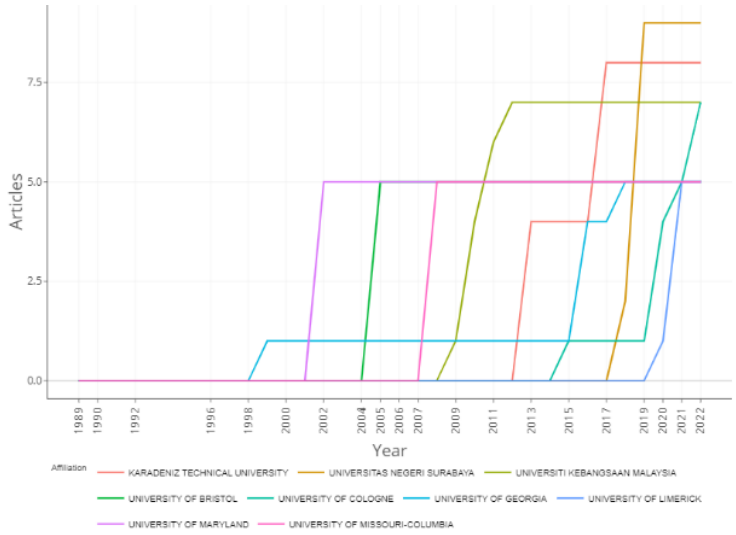


Figure 14. Document growth in the most productive affiliates

Furthermore, Figure 14 shows the growth of articles at each of these universities. The exciting thing from Figure 14 is that the Universitas Negeri Surabaya has only researched this topic for the first time in 2018, with as many as two articles. It increased significantly with the addition of 7 articles in 2019, so it has become nine articles until now. Since 2019, the university has not republished articles related to this topic. Even though the Universitas Negeri Surabaya has produced many articles related to this topic, looking at the trend in the last three years, this university is not showing any more productivity. The same happened to most other universities, such as the previous Karadeniz Technical University in 2017, the last Universiti Kebangsaan Malaysia in 2012, the last University of Missouri-Columbia in 2008, the last University of

Bristol in 2005, and the last University of Maryland in 2002. However, in contrast to the University of Cologne, which first published on this topic in 2015, then added three articles in 2020, 1 document in 2021, and 2 in 2022. The University of Cologne has seven articles related to this topic. That is, this topic is the focus of research at the University of Cologne in Germany at this time.

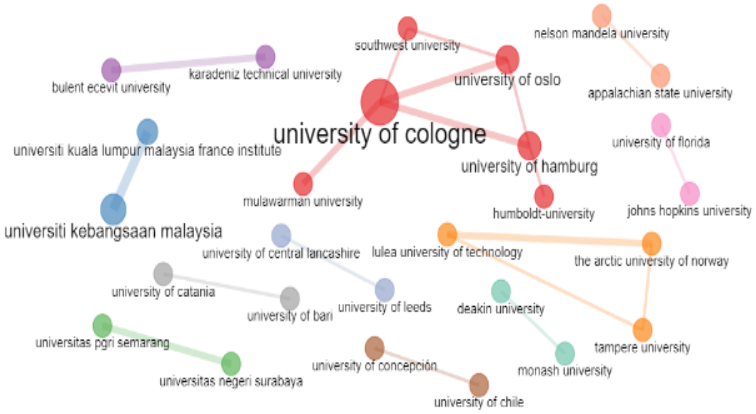


Figure 15. Affiliate collaborative network in publications

Figure 15 shows eleven affiliate collaboration network clusters in the article publication. The first cluster includes the University of Cologne, the University of Hamburg, the University of Oslo, Humboldt University, Mulawarman University, and Southwest University. The second cluster comprises Universiti Kebangsaan Malaysia and Universiti Kuala Lumpur Malaysia France Institute. The third cluster includes Universitas Negeri Surabaya and Universitas PGRI Semarang. The fourth cluster includes Karadeniz Technical University and Bulent Ecevit University. The fifth cluster includes Lulea University of Technology, Tampere University, and The Arctic University of Norway. The sixth cluster comprises the University of Chile and the University of Concepción. The seventh cluster includes the University of Florida and Johns Hopkins University. The eighth cluster comprises the University of Catania and the University of Bari. The ninth cluster includes Deakin University and Monash University. The tenth cluster includes Appalachian State University and Nelson Mandela University. The eleventh cluster consists of the University of Central Lancashire and the University of Leeds.

Keyword Trends

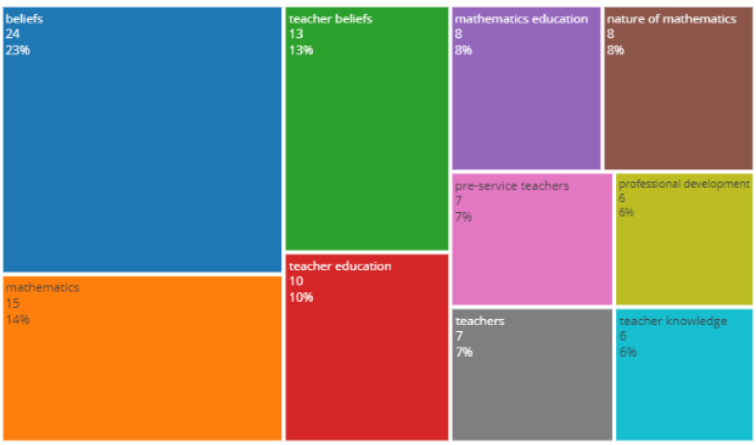


Figure 16. Treemap of the most relevant keywords

Based on Figure 16, it can be seen that ten keywords are often used in the selected articles in this research database. The most relevant keywords are "beliefs" 24 times (16%), "mathematics" 15 times (10%), and "teacher beliefs" 13 times (9%). In addition, several other keywords such as "teacher education", "mathematics education", "nature of mathematics", "pre-service teachers", "teachers", "professional development", and "teacher knowledge".

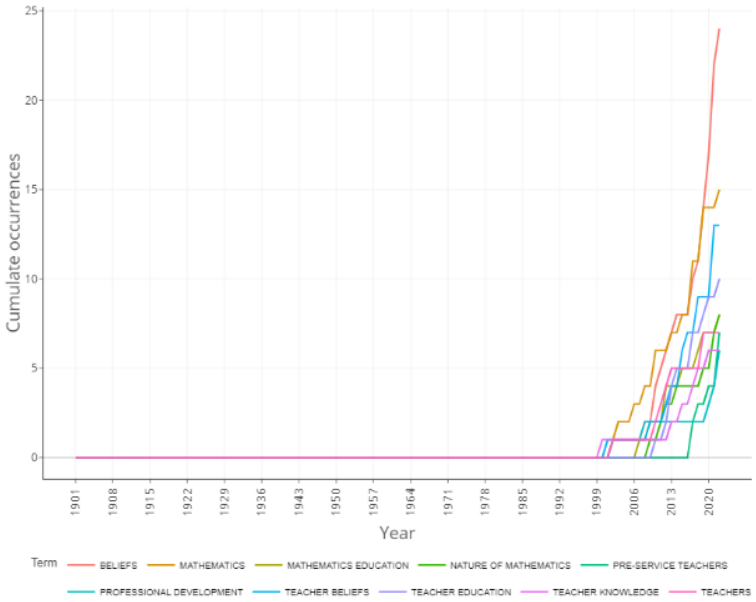


Figure 17. Development of the most relevant keywords

Figure 17 shows the development of the ten most relevant keywords in the article's publication. It can be seen that the keyword "beliefs" began to appear in 2002, but only in 2008 experienced a very significant increase. The same thing happened to the keyword "teacher beliefs," which started to appear in 2001 but only increased in 2009. The keyword "beliefs" was used more than "teacher beliefs" in the third position. The keyword "mathematics" began to appear in 2003, taking second place. Long before that, the keyword "teacher knowledge" had appeared in 2000, but it was only in 2013 that this keyword experienced an increase. The exciting thing is the emergence of the keyword "pre-service teachers" in 2017, which has rapidly increased. This means that research related to prospective teachers' beliefs about the nature of mathematics, mathematics teaching, and mathematics learning is in great demand.

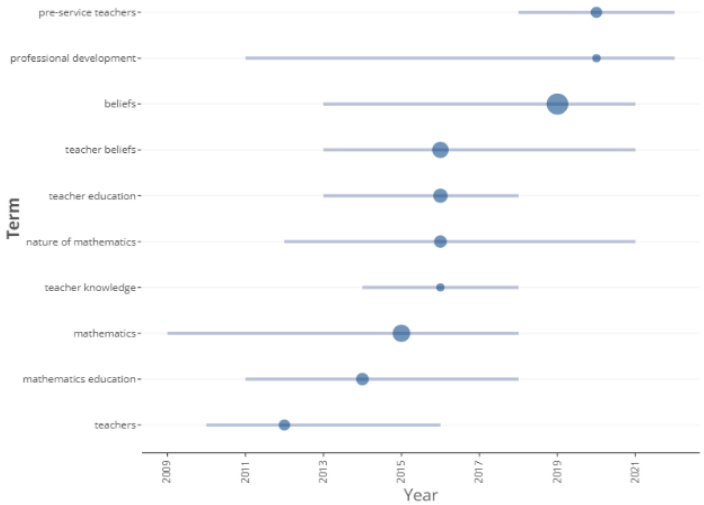


Figure 18. Trends in research topics based on the most relevant keywords

Furthermore, the trend of research topics based on the most relevant keywords is presented in Figure 18. The newest keywords are "pre-service teachers," followed by "professional development," "beliefs," "teacher beliefs," "teacher education," "nature of mathematics," "teacher knowledge," "mathematics," "mathematics education," and "teachers." However, "beliefs" is the most dominant and current research trend among these keywords.

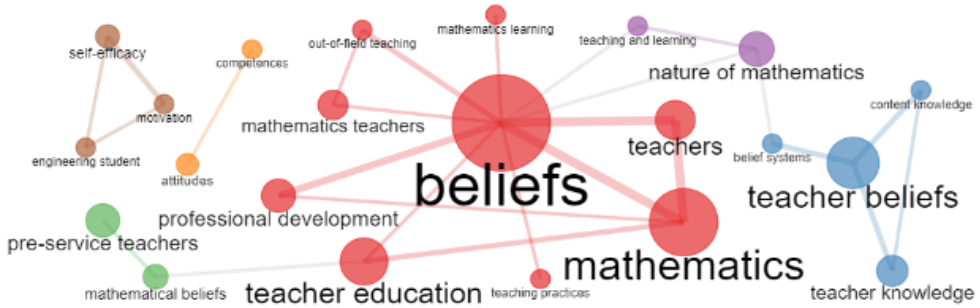


Figure 19. Keyword co-occurrence network

Based on Figure 19, it can be seen that six keyword network clusters are used together in an article. The first cluster contains "beliefs," "mathematics," "teacher education," "teachers," "professional development," "mathematics teachers," "mathematics learning," "teaching practices," and "out-of-field teaching." The second cluster contains "teacher beliefs," "teacher knowledge," "belief systems," and "content knowledge." The third cluster has "pre-service teachers" and "mathematical beliefs." The fourth cluster contains the "nature of mathematics" and "teaching and learning." The fifth cluster contains "attitudes" and "competencies." The sixth cluster has "motivation," "self-efficacy," and "engineering students."

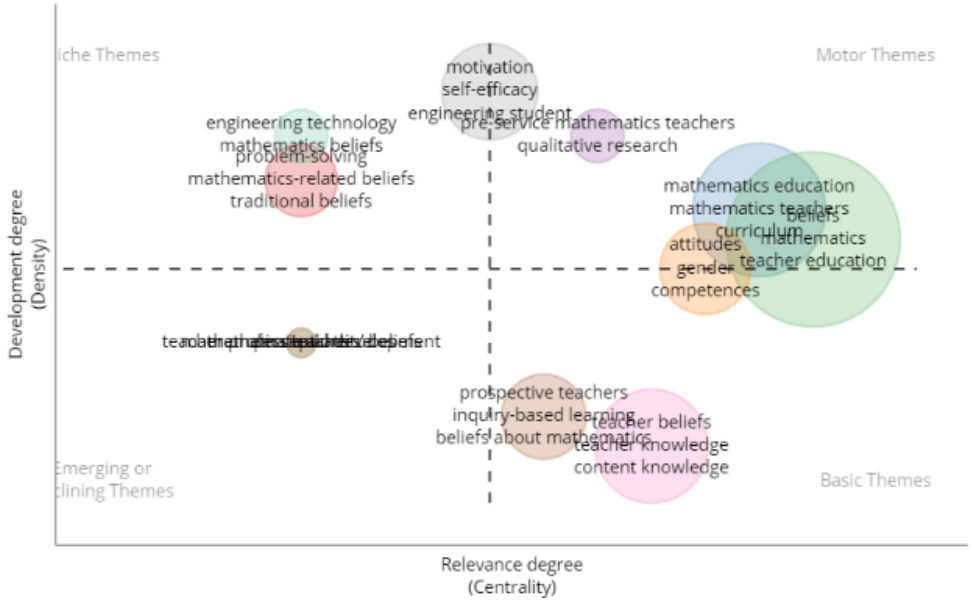


Figure 20. Thematic map

Figure 20 shows the thematic map of the research topics. There are thirteen clusters: problem-solving, mathematics education, beliefs, pre-service mathematics teachers, attitudes, prospective teachers, teacher beliefs, motivation, engineering technology, teacher professional development, mathematical attitudes, perceptions, and mathematics teachers' beliefs. Thematic maps are very intuitive plots, and we can analyze themes according to the quadrants in which

they are placed: (1) the themes in the upper right quadrant are known as motor themes, characterized by high centrality and high density, meaning they are developed and essential for the research field; (2) the themes in the lower right quadrant are known as basic themes, characterized by high centrality and low density, meaning that these themes are essential for a domain and involve general topics transversal to different research areas in the field; (3) the themes in the lower left quadrant are known as emerging or declining themes, with low centrality and low density, meaning that they are developing weakly and marginally; and (4) themes in the upper left quadrant are known as niche themes, with well-developed internal links (high density) but unimportant external links (low centrality), meaning that the theme is not particularly important for the field (Aria et al., 2020).

Motor themes include motivation, pre-service mathematics teachers, mathematics education, beliefs, and attitudes cluster. These themes are currently being developed and are essential for the field of research. However, if you look back, the motivation cluster tends to focus on niche themes. The cluster of pre-service teachers and teacher beliefs is the basic theme. These themes are essential for a domain and involve general topics that cut across different research areas in the field. The cluster of teacher professional development, mathematical attitudes, perceptions, and mathematics teachers' beliefs are emerging or declining themes. The development of these themes is weak and marginal. Engineering technology and problem-solving clusters are niche themes. These themes are not particularly important to the field.

Primary Source of Study

Primary Source of Study

Based on Bradford's Law, it is known that there are 6 out of 93 sources (journals and proceedings) that contribute the most publications on the research topic of beliefs about the nature of mathematics, teaching mathematics, and learning mathematics (Figure 21). These six sources can be used as the primary source for the study of the topic of mathematical trust, and it turns out that all of them are in the form of journals, none of which are in the record of proceedings. The journals are Mathematics Education Research Journal; Educational Studies in Mathematics; International Journal of Mathematical Education in Science and Technology; International Journal of Science and Mathematics Education; Eurasia Journal of Mathematics, Science and Technology Education; and ZDM - International Journal on Mathematics Education.

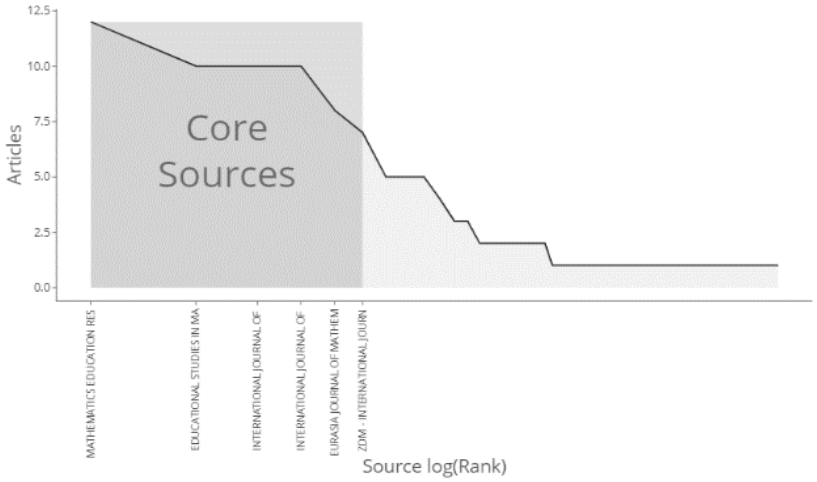


Figure 21. Primary sources based on Bradford's law

In Table 2, the six journals have been fully identified as the primary sources of study on the research topic of beliefs about the nature of mathematics, mathematics teaching, and mathematics learning.

Table 2. Journals as the Primary Source of Study

No	Journal	Number of Publications	First Publication	Number of Local Citation	H-Local Index	SJR	SNIP	Subject Areas and Categories
1	Mathematics Education Research Journal	12	1989	75	7	0.779	1,755	Mathematics, Education
2	Educational Studies in Mathematics	10	1968	185	8	1,543	2,571	Mathematics, Education
3	International Journal of Mathematical Education in Science and Technology	10	1970	38	6	0.479	1.326	Mathematics, Education, Applied Mathematics
4	International Journal of Science and Mathematics Education	10	2003	43	5	1,149	2,119	Mathematics, Education
5	Eurasia Journal of Mathematics, Science and Technology Education	8	2006	27	5	0.569	1,424	Education, Applied Mathematics
6	ZDM - International Journal on Mathematics Education	7	1997	84	6	1,368	2,255	Mathematics, Education

The number of local citations is the number of citations obtained from selected articles in the research database. Likewise, the local h-index is also in the same scope. Apart from revealing the vital interest of these journals in these areas of study, mathematics and education can be seen as strategic tools in research topics of beliefs about the nature of mathematics, mathematics teaching, and mathematics learning.

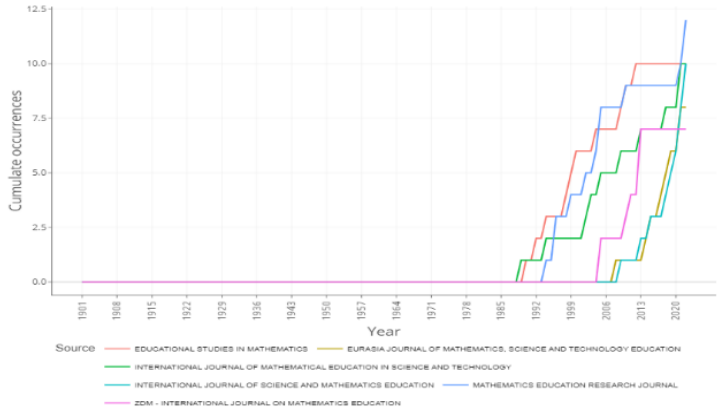


Figure 22. Primary source development

Figure 22 shows the growth of articles on the primary research sources on beliefs about the nature of mathematics, teaching mathematics, and learning mathematics. The exciting thing is that the International Journal of Science and Mathematics Education was first published in 2003 and only published articles related to this topic in 2009, quickly growing to have the fourth most articles. The Eurasia Journal of Mathematics, Science, and Technology Education also experienced similar growth. Meanwhile, journals with a long track record of publishing articles related to this topic are the International Journal of Mathematical Education in Science and Technology, which started in 1989, Educational Studies in Mathematics, which started in 1990, and Mathematics Education Research Journal, which began in 1993. Another exciting thing is that Educational Studies in Mathematics and ZDM - International Journal on Mathematics Education have not published articles on this topic since 2013 and 2014, respectively. The same thing happened with the Mathematics Education Research Journal, which did not publish articles on this topic during 2011-2020, but in 2021 added 1 document. In 2022, it published two articles related to this topic.

Study Primary Document

After looking at the primary sources, you can see the prior articles on this theme—selecting the primary document by considering the number of global and local citations. Global citations are citations to all articles in the Scopus database, while local citations are citations to all selected articles (171 articles) in this research database. Therefore, every document with local citations must also get global citations, not vice versa. Forty-one articles have received local citations and global citations. Articles Stipek et al. (2001) on Teach Teach Education received the most local citations ten times and the most globally, 375 times. Therefore, this article becomes the primary reference on this topic. The article Beswick (2012) on Educ Stud Math was followed by seven local and 99 global citations. The third position, the article Felbrich et al. (2012) on ZDM Internat J Math Edu, received four local and 33 global citations. Besides these three articles, 38 other articles can be considered primary articles. These prior articles, such as the Systematic Literature Review (SLR), can be used for further studies. SLR is exploratory and exploitative, providing adequate transparency and replication as a research method (Armitage & Keeble-Allen, 2008; Tranfield et al., 2003).

CONCLUSIONS

This research found that the research topic of beliefs related to the nature of mathematics, teaching, and learning mathematics has attracted global attention. Since 1989, there has been a significant increase in the number of publications and citations. The United States is the most productive country, followed by Indonesia and the United Kingdom. Indonesia showed remarkable improvement to occupy second place, while the UK sparked a revival of this topic after several decades. Prolific authors on this topic include Siswono TYE, Blömeke S, Kaiser G, and König J. In terms of institutions, Surabaya State University in Indonesia initially led the way in research on this topic, but recent trends point to the University of Cologne in Germany as the leader. The main keyword in these articles is “belief,” with “pre-service teacher” as the new trending keyword. There are thirteen thematic groups, with motivation, pre-service mathematics teachers, mathematics education, beliefs, and attitudes as the main themes. These results indicate great opportunities for further research in this theme. This research identified

41 primary articles on this topic, of which three are primary references, providing an exploratory view of this research topic in scientific publications and providing important information for researchers and institutions for future research considerations.

AUTHOR CONTRIBUTIONS STATEMENT

MM was a research leader who contributed to developing ideas and searching for and selecting literature. NN and YH contributed to coding data from each document, and SS was responsible for analyzing research data. Meanwhile, MB helps in finalizing the writing of research articles. All authors collectively wrote this manuscript.

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