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How are the classfication of students' mathematical connections in solving non-routine problems?

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Abstract

Background: Mathematics is not just a science for its own sake; it is a science that proves to be useful for most other sciences. Mathematical connections play a crucial role in helping students develop a deeper understanding and enhance their thinking about mathematics. These connections represent relationships between different mathematical ideas, between mathematics and other subjects, and between mathematics and everyday life.
Aim: This study aims to classify students' mathematical connection abilities when solving non-routine mathematical problems.
Method: This research employs a qualitative approach using descriptive and exploratory methods. The study involved 23 students who exhibited mathematical connection skills in problem-solving. Data collection methods included tests, observations, and interviews. The data analysis process consisted of two main stages: data reduction and data presentation. To ensure data validation, the triangulation method was used by comparing the results from the subject tests (answer sheets), observations, and interviews.
Results: The findings of this study revealed three distinct classifications of students' mathematical connections when solving non-routine problems: 1) Patterns; 2) Variables; and 3) Arguments.
Conclusion: Findings suggest that teachers need to be aware of the different classifications of students' mathematical connections. This awareness will enable them to implement mathematics learning strategies that cater to the diverse mathematical connection classifications found in their classrooms. For instance, teachers can utilize different mathematics learning media tailored to students' learning styles, such as mind, incubation, and visual learning. By doing so, students can better optimize their mathematical connections and achieve improved outcomes in their mathematics learning.

INTRODUCTION

Mathematics is not a science for its own sake, but a science that is useful for most other sciences (Kilpatrick, 2020). Mathematics is considered a branch of science needed because, in everyday life, in general, many things are related to mathematics (Simanjuntak et al., 2021). In mathematics, many things are studied with the aim of solving academic and non-academic problems. It follows one of the objectives of learning mathematics in schools: understanding mathematical concepts, explaining the relationship between concepts, and applying concepts or algorithms accurately, flexibly, and efficiently in solving problems (Ministry of National Education, NCTM).

According to the National Council of Teachers of Mathematics (2000), mathematical ability is defined as mathematical power includes the ability to explore, conjecture and reason logically to solve non-routine problems, to communicate about and through mathematics and

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to connect ideas within mathematics and between mathematics and other intellectual activity. National Council of Teachers of Mathematics (2000) establishes basic mathematical abilities that students need to have, including: (1) mathematical problem-solving, (2) mathematical reasoning and proof, (3) mathematical communication, (4) mathematical connections, and (5) mathematical representation. These abilities are higher-order mathematical thinking that students in learning mathematics must develop. Based on this description, mathematical connections are one of the most important abilities and become one of the focuses in education.

Mathematical connections are used as one of the curriculum standards to help shape students' consciousness (Prawira & Aripin, 2022). The goal is that students consider mathematics as an independent document and understand the relevance and benefits of mathematics inside and outside the school environment. Mathematical connections are grouped into internal and external connections (Islami et al., 2018a; Zengin, 2019). Internal connections are relationships between mathematical concepts, while external connections link mathematics with other sciences or everyday life. By having mathematical connections, students can see mathematics as integrated learning (Wahyudin, 2008). By studying mathematics in integrated learning, the learning material becomes close to students' lives so that students can easily understand and do it. Students can also easily relate the relationship between subject matter in one subject and other subjects. Integrated learning accommodates students' types of intelligence. In addition, mathematical connection abilities help students solve mathematical problems creatively (Eli et al., 2013). In other words, students with good mathematical connections also have good creativity.

National Council of Teachers of Mathematics (NCTM, 2000) explained that there are indicators of mathematical connections, including: (1) the relationship between mathematical concepts, (2) the relationship between mathematical concepts and other disciplines, and (3) the relationship between mathematical concepts and everyday life. The first indicator of relationships between mathematical concepts is transverse relationships, relating to specific mathematical concepts appearing in different mathematical contexts (Gamboa et al., 2016). Mathematical contexts can be contextual problems, Realistic Mathematical Education (RME), and ethnomathematics. The second indicator of the relationship between mathematical concepts and other disciplines is the relationship between other disciplines and mathematics that occurs in the practice of each field and the overall student experience (Frykholm & Glasson, 2005). Because in mathematic problem solving, students need to connect on concept to other math concepts. The relationship between mathematics and other disciplines emerges from a foundation of pre-existing knowledge and experience. The third indicator of the relationship between mathematical concepts and everyday life is the cognitive process by which a person uses two or more ideas, concepts, definitions, theorems, procedures, representations, and meanings of each other in everyday life (García-García & Dolores-Flores, 2018, 2021a).

A problem solver is said to have a mathematical connection if it has all three indicators. The better the mathematical connection ability of the problem solver, the better the performance. During the educational process, achievement becomes a crucial indicator because achievement is evidence of the achievement of an effort (Winkel, 1996). According to Anecdote, unfortunately, many students still have a low level of mathematical connection ability. This is in accordance with the results of analysis carried out by other research. This research shows that the mathematical connection ability for each indicator has not yet been achieved, so that students' mathematical connection ability is still relatively low (Dinata et al., 2023). The low level of mathematical connection ability is characterized by the inability of students to solve mathematical problems that exploye mathematical connect (Kusuma, 2003).

The problem-solving process needs to connect problem-solving steps to find solutions based on the capabilities possessed (Fadilah & Hakim, 2022). Students can develop conceptual understanding through mathematical connections to use inter-mathematical concepts, mathematical concepts, and other disciplines or everyday life in solving problems (Anthony & Walshaw, 2009; Rohendi & Dulpaja, 2013).

One attempt to see the extent of a problem-solver's mathematical connection capabilities is to provide a mathematical problem (Rafidah et al., 2020). Mathematical problems emphasize non-routine things (Musser et al., 2011). It means the solution of non-routine problems can not immediately known (J. Sternberg & Sternberg, 2012). Non-routine math problems are problems that require problem solvers to think and reason in more and different ways (Lee & Chen, 2009). For example, in solving problems with Systems of Linear Equations in Two Variables, students can employ various methods: elimination, substitution, mixture, and graphing. Giving non-routine problems to students means training students to apply various mathematical concepts so that, in the end, they can use various scientific concepts that students have learned to solve problems about the relationship between concepts and one another and the relationship of a problem with mathematical concepts. Non-routine problems that can test students' mathematical connection skills are problems on geometry row material. To solve the problem, students need to understand it accurately (Aytekin et al., 2016), because geometry sequences learn how to solve the sequence problem by using exponent numbers.

Several experts have carried out research related to mathematical connections. Campo-Meneses et al. (2021) conducted research with new findings, namely students lack the establishment of procedural mathematical connections in symbolic registers, due to the incompetence of forming mathematical connections of reversibility and the absence of procedural strategies for solving equations. Quilang & Lazaro (2022a) conducted research with the finding that mathematical connection ability is a powerful tool in increasing mathematical understanding among prospective mathematics teachers. Rafiepour & Faramarzpour (2023) conducted research with new findings, namely the qualifications of students on the indicators of the relationship between mathematical concepts are pretty good and on the indicators of the relationship between mathematics and other sciences are also quite good. However, the indicator of connectedness between mathematics and everyday life is still low, so students' mathematical connections still have to be improved. Gamboa et al. (2023) conducted research with new findings, namely connections arise as networks of relationships resulting from interactions between teachers and students.

As for the urgent need for research on mathematical connections, the findings can be compiled into theoretical guidelines to describe mathematical connections in problem-solving. In addition, conclusions can be used as guidelines for teachers to improve students' mathematical connections. Another urgency is that it can help students develop critical, logical thinking skills and increase students' understanding, leading to innovation and better application of technology in various fields in everyday life.

This study will classify students' mathematical connection ability in solving non-routine mathematical problems. This research will contribute new theories for classifying mathematical connections in solving non-routine problems. The results of further research can be used as a guideline to describe the ability of mathematical connections in solving mathematical problems and a theory to improve students' mathematical connection abilities.

Research Position

Researchers have reviewed several previous studies related to mathematical connections and grouped them into three categories, including (1) mathematical connection relationships and other skills, (2) mathematical connections and other variables, and (3) mathematical connections in learning.

First, research related to the relationship between mathematical connections and other skills concluded that (1) students with high mathematical logical intelligence in solving geometry problems made internal and external connections based on polya problem-solving steps (Islami et al., 2018b); (2) to complete graphic tasks, Mexican students rarely use mathematical connections of visualization so that the level of mathematical understanding of the Fundamental Theorem of Calculus (FTC) is still relatively low (García-García & Dolores-Flores, 2021b); (3) in solving mathematical problems elementary school students still have low mathematical connection ability (Kenedi et al., 2019); (4) The qualifications of students on the indicators of the relationship between mathematical concepts are pretty good and on the indicators of the relationship between mathematics and other sciences are also quite good. However, the indicator of connectedness between mathematics and everyday life is still low, so students' mathematical connections still have to be improved (Rafiepour & Faramarzpour, 2023).

Second, research related to mathematical relationships and other variables concluded that (1) the process of mathematical connections of male and female students in solving problems with polya stages has similarities and differences (Baiduri et al., 2020); (2) students lack the establishment of procedural mathematical connections in symbolic registers, due to the incompetence of forming mathematical connections of reversibility and the absence of procedural strategies for solving equations (Campo-Meneses et al., 2021); (3) some students lack mathematical connection skills and fail to solve problems in solving problems on 2D trigonometric material in an ABL environment (Maphutha et al., 2023); (4) when marking connections, prospective teachers (PSTs) recognize various pedagogical considerations and they can also consider different types of mathematical connections in their field experience (Foster & Lee, 2021).

Third, research related to mathematical connections in learning concluded that (1) students make unexpected mathematical connections, they use naturally the reversibility relationship between the concept of positional velocity associated with the notion of derivative and integral (García-García & Dolores-Flores, 2021b); (2) there are differences in the improvement and achievement of mathematical connection skills between students who carry out prezi-based scientific learning compared to lectora inspiration-based learning and conventional learning (Susilawati & Nuraida, 2021); (3) the improvement and achievement of mathematical connections of students learning through the CORE RME model is better than student learning through conventional models (Son, 2022); (4) Cross-case analysis of model-eliciting activities revealed that four types of mathematical connections were identified, connections to real-world situations, connections between mathematical concepts, connections between different modes of representation, and connections to other disciplines (Genc, 2023); (5) connections arise as networks of relationships resulting from interactions between teachers and students (Gamboa et al., 2023); (6) teacher candidates build mathematical connections of different types: different characteristics, meanings, procedurals, representations and implications (Hernández-Yañez et al., 2023); (7) teacher candidates perform best in making teaching-oriented connections but have difficulty in making implication connections (Quilang & Lazaro, 2022).

Researchers found potential research opportunities after reviewing several previous studies on mathematical connections and grouping them into three main categories. Namely (1) classification of students' mathematical connections in solving mathematical problems, (2) development of sICT-based learning media to improve mathematical connections, (3) factors that affect mathematical connection abilities, and (4) development of learning models to improve students' mathematical connections. This research focuses on students' mathematical connections in solving math problems. Figure 1 illustrates this study, including previous research, potential research, and research carried out.

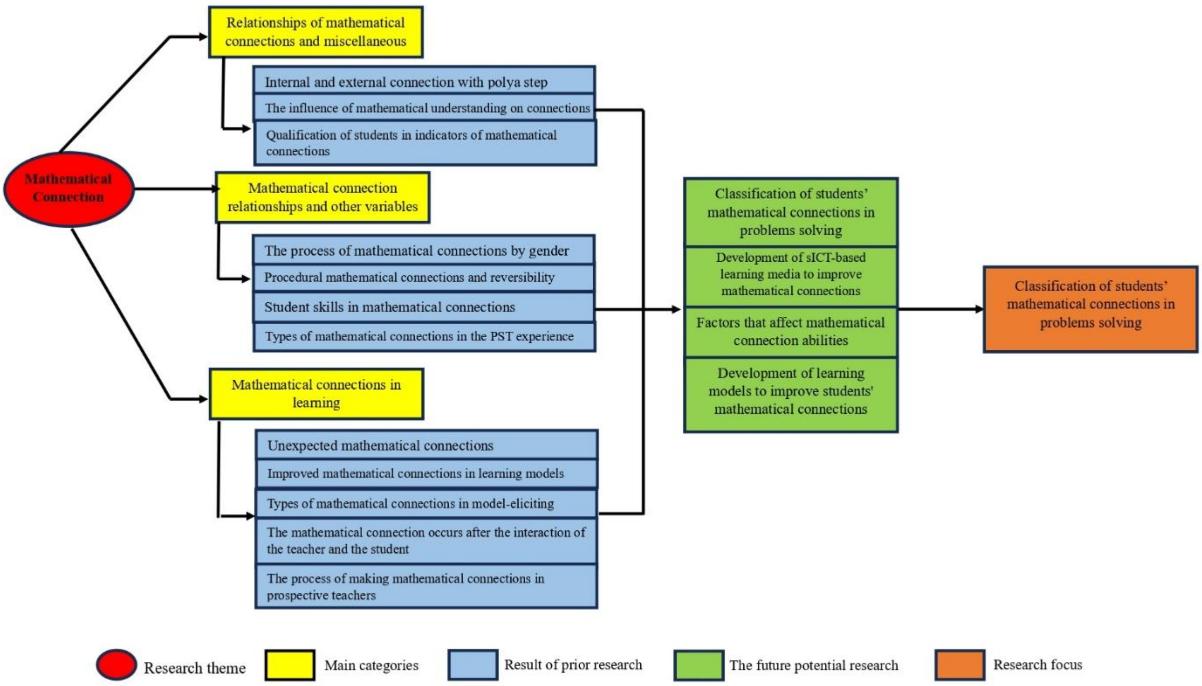


Figure 1. The research position

METHODS

Design

This type of research employed qualitative research (Creswell, 2014). The method used is descriptive and exploratory, using tests, observations, and interviews. This research uses a qualitative approach that explores a phenomenon or social process (Creswell, 2014) by all facts in the field without manipulation (Sagala et al., 2019). It describes the classification of students' mathematical connections in solving non-routine problems based on indicators of mathematical connections and student activity.

Participants

The subjects in this study were 10th-grade high school students in Surakarta City. The student is a problem solver with mathematical connection skills and is willing to volunteer as a subject. Subject selection is carried out until the researcher obtains representative data. The subjects of this study numbered 23 students.

Instruments

The main instrument in this study is the researcher as a planner and the implementer of data collection and analysis, concluding, and the researcher as a compiler of reports from research results. Companion instruments include geometry row problem-solving tests, observation sheets, interview guidelines, and audio-visual recording devices. The companion instrument was developed following research objectives, problem construction (Yorulmaz et al., 2021), and language suitability. Before data collection, both instruments are validated first (Tutticci et al., 2017). Validation is carried out by experts in the field, namely experts in the cognitive field of mathematical problems and experts in the field of qualitative research in mathematics education.

Data Collection

Data collection was carried out using test, observation, and interview methods. This instrument is used to understand students' mathematical connections in solving problems. This has been adjusted for indicators of mathematical connections and student activity (Table 1). Students are asked to solve non-routine problems about geometric series (see Figure 2) in a different way according to the student's thinking and reasoning in solving the problem.

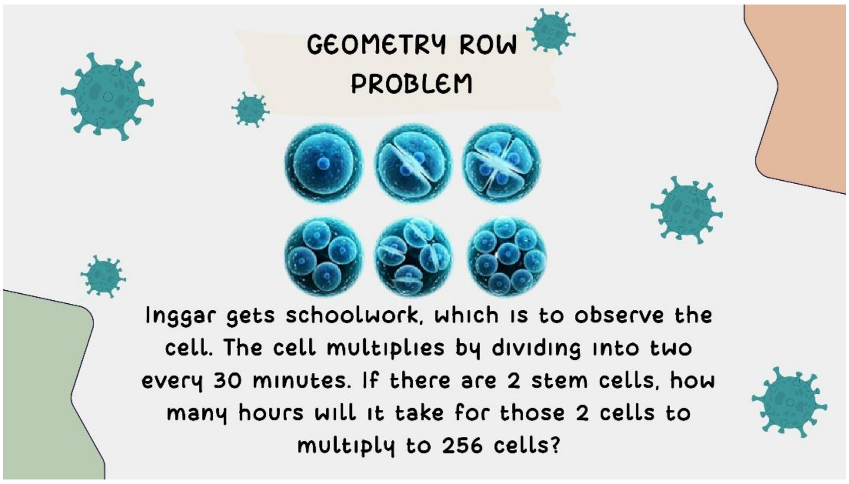


Figure 2. Non-routine Problems of Geometric Row Material

The researcher observes the subject during the process of solving the problem. Researchers recorded all actions taken by subjects in solving non-routine problems based on observation sheets to classify students' mathematical connections. In-depth interviews of subjects using audio-visual recording devices were also used for data collection.

The indicators of connection mathematical developed by the author in preliminary research. Table 1 lists mathematical connection indicators. In the first indicator, linking concept representation to the procedure, five student activities are coded R1, R2, R3, R4, and R5. In the second indicator, the relationship between mathematical concepts and between mathematical concepts and other sciences includes four student activities coded D1, D2, D3, and D4. In the last indicator, applying mathematics in everyday life, there is one student activity with the code L1. These codes are structured to assist researchers in conducting data analysis.

Data validation is performed using the triangulation method. The triangulation method compares the results of subject tests (answer sheets), observations, and interviews (Al-Moghrabi et al., 2020). Triangulation results are analyzed to determine students' mathematical connections' tendency in solving non-routine geometric row material problems based on indicators with valid conclusions.

Table 1. Mathematical Connection Indicators and Student Activities

Mathematical Connection Indicators	Student Activities	Code
Linking concept representations to procedures	1. Understand the meaning of the question	R1
	2. Understand the data provided	R2
	3. Summarize the solutions that can be used	R3
	4. Plan a problem-solving strategy	R4
	5. Understand how to prevent difficulties that may be experienced	R5
Linking between mathematical concepts and between mathematical concepts with other sciences	1. Doing calculations	D1
	2. Uses mathematical concepts relevant to problem-solving	D2
	3. Using other concepts in solving problems	D3
	4. Using arguments in solving problems	D4
Applying mathematics in everyday life	1. Representing or translating story problems into mathematical models	L1

The flowchart at the research procedure illustrated in figure 3.

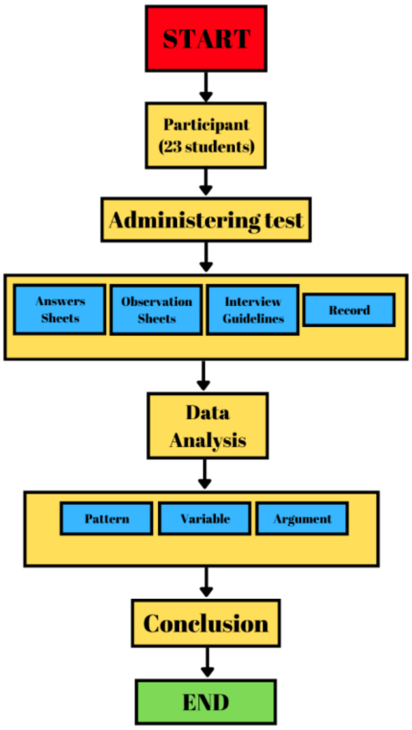


Figure 3. Research Procedure

Data Analysis

Data is analyzed through two stages, namely data reduction and data presentation. At the reduction stage, the data is reduced and used as a reference for research objectives. At the presentation stage, data on the results of students' mathematical connections were presented in solving non-routine geometric row problems based on indicators of mathematical connections and student activities.

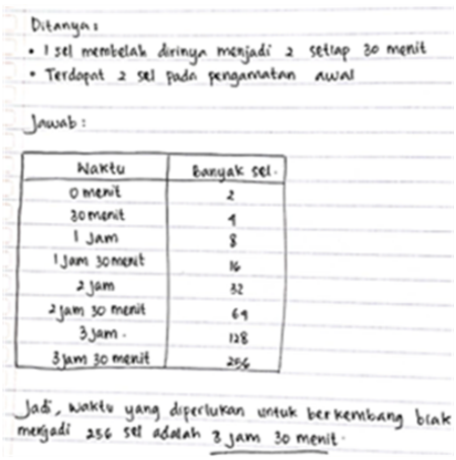
RESULTS AND DISCUSSION

Based on the data analysis, three types of classification of students' mathematical connections were found in solving non-routine problems of geometric row material. The three types of classifications are (1) patterns, (2) variables, and (3) arguments and their description.

Classification I: Pattern

Figure 4 reports the solver's answer with a pattern-type mathematical connection classification. The problem solver understands the intent of the question and the data provided in-depth related to the given problem (R1, R2). The problem solver needs to read the question more than once to understand the intent of the question and the available data. Then, the problem solver records all the information: one cell divides into two every 30 minutes, and there are two cells on initial observation (L1). The initial observation cell in question is a parent cell that will be observed sometime in the future.

Original Version



Ditanya:
• 1 sel membelah dirinya menjadi 2 setiap 30 menit
• Terdapat 2 sel pada pengamatan awal

Jawab:

Waktu	Banyak sel
0 menit	2
30 menit	4
1 Jam	8
1 Jam 30 menit	16
2 Jam	32
2 Jam 30 menit	64
3 Jam	128
3 Jam 30 menit	256

Jadi, waktu yang diperlukan untuk berkembang biak menjadi 256 sel adalah 3 jam 30 menit.

Translated Version

Asked:
1 cell divides into two every 30 minutes
There are two cells on initial observation
Answer:

Time	Multiple Cells
0 minutes	2
30 minutes	4
1 hour	8
1 hour and a half	16
2 hours	32
2 hours 30 minutes	64
3 hours	128
3 hours 30 minutes	256

So, the timerequired to multiply to 256 cells is 3 hours 30 minutes

Figure 4. Problem Solver 1

Students solve problems using raw data and determine problem-solving strategies (R3, R4). The strategy students use is to look at the pattern of multiples of 2 according to the problem "The cell multiplies by dividing into two every 30 minutes". That is, the number of cells will be double the previous number. Therefore, each number in the previous operation will be multiplied by the number 2 until it gets the sum according to the question. In this type, students only think briefly based on their knowledge (R5). In addition to knowledge, problem solvers also rely on previous experience in solving similar problems (Khoyimah & Susanah, 2021).

Next, the troubleshooter writes that at zero minutes, there are 2 parent or initial cells. Then, the problem solver multiplies the previous number of cells by the number 2 so that from

2 stem cells to 4 cells is obtained from $2\ cells\ x\ 2$ because each cell divides into two for 30 minutes. In the next 30 minutes, namely in the first 1 hour, each cell will divide again into two so that the 4 cells become 8 cells obtained from $4\ cells\ x\ 2$. Within 1 hour 30 minutes, the cell divides from 8 to 16 cells obtained from $8\ cells\ x\ 2$. In the following 30 minutes, the cell becomes 32 because the previous cell was 16, then $16\ cells\ x\ 2\ =\ 32\ cells$. From 32 cells dividing again into two each cell, then $32\ cells\ x\ 2\ =\ 64\ cells$ within 2 hours 30 minutes after observation. Then, after 3 hours of observation, the cells divide from 64 cells to $64\ cells\ x\ 2$, which is 128 cells. Of the 128 cells, each cell will divide into 2 again after 30 minutes. Then, on observation for 3 hours 30 minutes, the resulting cell is $128\ cells\ x\ 2\ =\ 256\ cells$ (D1). This follows the question, "How many hours does it take for the 2 cells to multiply to 256 cells?".

The troubleshooter reviews the process or troubleshooting steps. Students scrutinize each step to make sure there are no errors during calculations. In addition, the problem solver also corrects the results of the answers that have been obtained and is ready to correct if there are errors. As it says on the answer sheet, "So, the time required to multiply to 256 cells is 3 hours 30 minutes". Problem solvers include having mathematical connections and those that do include pattern-type classification. This is because problem solvers tend to look at patterns or arrangements of numbers when solving a given mathematical problem (D3). Based on the answer sheet, students solve the problem by looking at the pattern of cell division.

Students with pattern classification are categorized as students who can make mathematical connections with pattern tendencies or solve problems using strategies by looking at the number of patterns formed. Pattern-related activities are important for making mathematical connections, understanding systems, and mathematical logic (Burns, 2000). In pattern-type classification, problem solvers will solve problems using number patterns. In this study, the mathematical problems presented formed a pattern of multiples of two. Decision-making on how to solve problems is taken when students have understood the meaning of the problem. In addition, problem solvers also use previous knowledge and experience to solve problems. It solves a given non-routine problem using a pattern formulated like the problem provided (Rodgers, 2002). Mulligan & Mitchelmore (2009) argues that most mathematics is based on patterns and structures. Meanwhile, according to Steen (1988), mathematics is considered the science of patterns and order, and according Orton (1999), looking for patterns is one of the actions carried out in mathematics as a whole. So, in mathematics, patterns become one of the good solutions for students in solving problems. This follows Agustina & Munandar (2022), who explain that number patterns are mathematical sciences that help students solve problems in everyday life. So, by seeing patterns, students can solve problems easily. In addition, patterns also help students to develop counting skills and strategize thinking (Reys et al., 1998) and play an important role in improving reasoning, communication, association and problem-solving skills (Tanişli & Özdaş, 2009). The picture of student when conduct mathematical problem solving is presented in Figure 5.

Figure 5. Picture of Student 1

Classification II: Variable

In variable types, the problem solver understands the intent of the question and all the data provided in-depth and repeatedly (R1, R2). Problem solvers tend to infer more information from the problems available on the problem. He recorded the known data on the question "initial 2 cells" and the essence of the question "How long did it take for the cells to be 256 cells?". Then, students define and devise problem-solving strategies (R4). The strategy is to suppose the available data as variables (L1). The problem solver records directly the example of the initial number of cells or stem cells with the letter a, the example of the cell dividing into two with the letter r, the example of the number of cells asked is 256 cells with the letter Un, and the example of the number of cell divisions with the letter n (Figure 6). This corresponds to how to solve the geometry row problem.

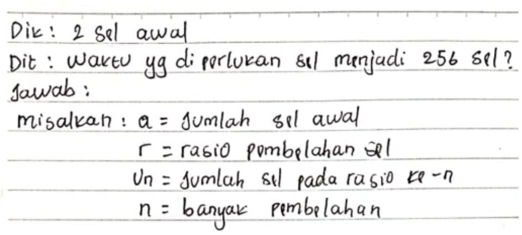
<table>
<tr><th>Original Version</th><th>Translated Version</th></tr>
<tr><td>
</td><td>Known: 2 starting cells
Asked: How long does it take for a cell to become 256 cells?
Answer:
Suppose: a = number of starting cells
r = cell division ratio
Un = number of cells at the n-th ratio
n = number of division</td></tr>
</table>

Figure 6. Problem Solver 2 part 1

Next, the problem solver writes down the geometric sequence formula "$Un = ar^{n-1}$" (R3, R5). He substituted whatever was known into geometric sequence formulas (Figure 7). The problem solver subtilises many of the cells in question: 256 into Un (Un in the geometry row is the value of the nth term), the number of initial cells or stem cells is 2 into a (a in the geometric row is the first term), and the cell divides into 2 into r (r in the geometric row is the ratio). The troubleshooter then moves the value a=2 in the right field to the left so that the value of a becomes a divisor of the number in the left field "$\frac{256}{2} = 2^{n-1}$. This follows the applicable rules: if the previous operation was multiplication, then the field will become a division operation after moving. The operation obtained a result of 128 from 256 divided by 2 "$128 = 2^{n-1}$". Next, the problem solver looks for the exponent number of 128, namely $2x2x2x2x2x2x2$ or 2^7, so that the student's answer becomes "$2^7 = 2^{n-1}$". Because the value of the principal number or base number on the right and left segments is the same, namely 2, the base number can be omitted so that it is enough to operate the power only "$7 = n - 1$". In the last step to find the value of n, the troubleshooter moves the number 1 in the right field to the left field. Because the number operation is subtraction, when the field is moved to an addition operation, $n = 7 + 1$ obtains the value $n = 8$ (n in the geometric row is the number of terms in the geometric row). That is, the cell divides as many as 8 divisions.

Original Version	Translated Version
Dengan rumus baris geometri $Un = ar^{n-1}$ $256 = 2 \cdot 2^{n-1}$ $\frac{256}{2} = 2^{n-1}$ $128 = 2^{n-1}$ $2^7 = 2^{n-1}$ $7 = n-1$ $n = 8$	By geometry row formulas $Un = ar^{n-1}$ $256 = 2.2^{n-1}$ $\frac{256}{2} = 2^{n-1}$ $128 = 2^{n-1}$ $2^7 = 2^{n-1}$ $7 = n - 1$ $n = 8$

Figure 7. Problem Solver 2 part 2

After finding many cell divisions, the problem solver looked for the result of the problem given: "How many hours does it take for the 2 cells to multiply to 256 cells?". Cell division is obtained 8 times, but remember that the initial 2 cells are parent cells, so they do not include the division in question. Therefore, cell division occurs as much as $n - 1 = 7$ divisions. Because cleavage occurs every 30 minutes, the troubleshooter multiplies 30 minutes by the number of divisions "$7\ x\ 30\ minutes = 210\ minutes$". Then, the troubleshooter converts minutes into hours by dividing the number of minutes gained by 60 minutes "$\frac{210\ minutes}{60\ minutes} = 3{,}5\ hours$". Cells take 3.5 hours or 3 hours 30 minutes to divide into 256 cells (D1).

Original Version	Translated Version
karena sel awal bukan pembelahan sel tetapi induk sel, maka pembelahan terjadi $n-1 = 8-1$ $= 7\times$ pembelahan $= 7$ kali pembelahan Jadi 7×30 menit $= \frac{210 \text{ menit}}{60 \text{ menit}} = 3{,}5$ jam	Since the initial cell is not cell division, but the parent cell, then division occurs $n - 1 = 8 - 1$ $= 7\ x\ division$ $= 7\ times\ of\ division$ So, $7\ x\ 30\ minutes = \frac{210\ minutes}{60\ minutes} =$ $3{,}5\ hours$

Figure 8. Problem Solver 2 part 3

After finding results, the problem solver examines each step to make sure everything is noticed in both the calculation and the intent of the given problem and is ready to correct if there are any errors. Then, the troubleshooter also corrects the final answer results to ensure that the results are correct. Based on the answer sheet, problem solvers include having mathematical connections, and what is done includes classifying variable types. This is because problem solvers tend to emulate available information as variables. Then, he solved the problem by using the geometric row formula (D2).

Students with variable classification are categorized as students who can make mathematical connections with variable tendencies or solve problems using variable strategies by supposing known data. According to the Ministry of National Education in Akdoğan (2023), variables are defined as letters in algebraic expressions representing numbers. Meanwhile, according to Wardah et al. (2021), variables are substitute symbols or symbols for numbers whose values are not yet clearly known. So, in problem-solving, students record all data obtained by, for example, using letters or mathematical symbols to make it easier to find solutions to problems. Using variable strategies is an adaptive problem-solving behaviour (Sidney et al., 2018). The troubleshooter then uses formulas that may be usable. In this study, the formula that can be used is the geometric series formula in finding Un, namely $Un = ar^{n-1}$ or by using exponent numbers a^x. The picture of student when conduct mathematical problem solving is presented in Figure 9.

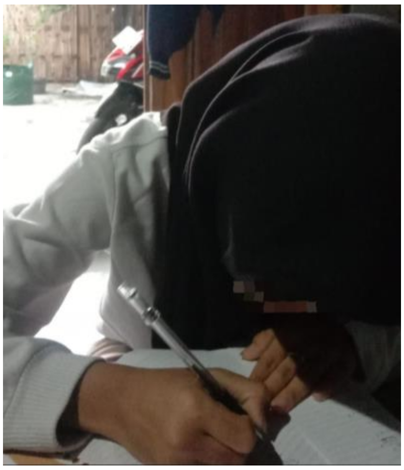


Figure 9. Picture of Student 2

Classification III: Argument

Problem solvers in the argument type tend to solve problems by expressing their thoughts in written form. Before solving the problem, students repeatedly read the problem and all available data so that there is no misunderstanding in understanding the problem (R1, R2). Then, he recorded what was known in the problem: "2 cells divide takes 30 minutes". What is meant by 2 cells is two stem cells or initial cells that will divide into 2 every 30 minutes. The troubleshooter understands that the number of cells will be 2 times more than the previous cell: "So after 30 minutes, the cell will divide into 2x its fold" (L1, R5).

In determining problem-solving strategies, students use raw data to find the results asked (R4). Problem solvers use strategies by writing or writing down what they think. So, what the problem solver has in mind will be recorded without taking the essence of what he is thinking (R3). This corresponds to the troubleshooter's answer sheet. In this strategy, researchers classify into argument types because in solving problems, students use long texts or elaboratory sentences.

Original Version	Translated Version
Diketahui = 2 sel membelah membutuhkan waktu 30 menit jadi setelah 30 menit sel akan membelah menjadi 2x lipatnya pada pengamatan pertama (2 sel x2) = 4 sel = 30 menit kemudian 30 menit berikutnya 4 sel membelah lagi menjadi (4 sel x2) = 8 sel = 1 jam kemudian 30 menit berikutnya (8 sel x2) = 16 sel = 1 jam 30 mnt kemudian 30 menit berikutnya (16 x2) = 32 sel = 2 jam kemudian 30 menit berikutnya (32x2) = 64 sel = 2 jam 30 mnt kemudian 30 menit berikutnya (64x2) = 128 sel = 3 jam kemudian 30 menit berikutnya (128 x2) = 256 = 3 jam 30 mnt jadi, waktu yang diperlukan 2 sel tersebut untuk berkembang biak menjadi 256 adalah 3jam 30 menit.	Known = 2 cells splitting takes 30 minutes So, after 30 minutes the cell will divide into 2x its fold. At first observation $(2\ cells\ x\ 2) = 4\ cells = 30\ minutes$ Then the next 30 minutes the next 4 cells divide again into $(4\ cells\ x\ 2) = 8\ cells = 1\ hour$ Then the next 30 minutes $(8\ cells\ x\ 2) = 16\ cells = 1\ hour\ 30\ minutes$ Then the next 30 minutes $(16\ cells\ x\ 2) = 32\ cells = 2\ hours$ Then the next 30 minutes $(32\ cells\ x\ 2) = 64\ cells = 2\ hours\ 30\ minutes$ Then the next 30 minutes $(64\ cells\ x\ 2) = 128\ cells = 3\ hours$ Then the next 30 minutes $(128\ cells\ x\ 2) = 256\ cells = 3\ hours\ 30\ minutes$ So, the time it takes for those 2 cells to multiply to 256 is 3 hours 30 minutes.

Figure 10. Problem Solver 3

The troubleshooter writes, "On first observation $(2\ cells\ x\ 2) = 4\ cells = 30\ minutes$", meaning that 2 stem cells divide into 2 cells each so that the cells become 4 cells in the first 30 minutes. Then, at the observation time for 1 hour, each cell divides again into 2 so that the number becomes 8 cells using $4\ cells\ x\ 2$. On observation for 1 hour 30 minutes, the cells numbered to 16 cells "Then the next 30 minutes $(8\ cells\ x\ 2) = 16\ cells = 1\ hours\ 30\ minutes$". At observation for 2 hours, there were 2 times more cells, where the previous cells numbered 16. In the next 30 minutes, the cells became 32 cells. Of the 32 cells, each cell will divide again into 2 so $32\ cells\ x\ 2$ obtained 64 cells on observation for 2 hours 30 minutes. After 3 hours of observation, the cells divide again into $64\ sel\ x\ 2 = 128$ cells. Then, from 128 cells, within 3 hours and 30 minutes, the cells divide again into 256 cells. "Then the next 30 minutes $(128\ cells\ x\ 2) = 256\ cells = 3\ hours\ 30\ minutes$" (D1).

Based on the student's answer sheet, problem solvers include having mathematical connections, and what is done includes the classification of argument types. This is because problem solvers tend to use long text or elaboration sentences in solving the presented problem (D4). In addition, problem solvers also correct the results of answers to avoid errors in writing and calculation. The troubleshooter remembers to double-check the troubleshooting process that he has done and is willing to fix it if there is an error. As it is written on the answer sheet, "So, the time it takes for those 2 cells to multiply to 256 is 3 hours 30 minutes."

Students with argument classification are categorized as students who can make mathematical connections with argument tendencies or solve problems using long texts or elaboratory sentences. In mathematics, arguments are often used to prove mathematical theorems. In this study, students use their arguments to find the results of a given problem. Students use the ability to express data, reason, writing skills, and discourse to solve problems and prove the truth of the answers. This aligns with the opinion of (Indrawatiningsih et al., 2020), which interprets arguments as activities that coincide with evidence or justification. In the mathematics education community, argument is an important form of discipline that must be carried out in all classrooms. Although important for mathematics learning, applying argument in mathematics learning is rare (Bieda, 2010; Bleiler et al., 2014; Staples et al., 2012).

In addition, arguments are a natural part of mathematical practice because mathematics is the science of proof, and mathematical arguments are central to proof (Ubuz et al., 2014). In this case, students believe using arguments can strengthen critical thinking skills, and proof is an important indicator of mathematical thinking. This is in line with the opinion (CCSSM, 2010) that students' ability to justify claims that are part of an argument is considered an important indicator of students' mathematical thinking, and (Elbrink & Stump, 2007) says argument skills should be honed to strengthen students' critical thinking. The picture of student when conduct mathematical problem solving is presented in Figure 11.

Figure 11. Picture of Student 3

CONCLUSIONS

In solving mathematical problems, there is a different way for each problem solver to make mathematical connections. Through the provision of non-routine math problems, problem solvers manage to find the right results using solving methods according to their abilities. Then, the researcher classifies it into three types of classification. Pattern-type classification indicates the problem-solver's ability to connect other concepts. Variable type classification indicates the problem-solver's ability to remember and apply relevant concepts. The classification of argument types shows the ability of problem solvers to express opinions to solve problems. Implication: teachers need to know the classification of students' mathematical connections so they can implement mathematics learning in classes with various mathematical connection classifications. For example, teachers use different mathematics learning media for students of a mind, incubation, and visual. By using different mathematics learning media, students can optimize their mathematical connections and mathematics learning outcomes. Further research: can focus on 1) development of siCT-based learning media to improve mathematical connection; 2) factors that affect mathematical connection abilities and; 3) development of learning models to improve students' mathematical connection.

AUTHOR CONTRIBUTIONS STATEMENT

MNK designed the study, formulated the research questions, collected data, analyzed and interpreted the results, and drafted the manuscript. RD assisted in developing the methodology, collected and organized data, validated findings, and provided critical revisions to the manuscript.

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