



# Training Egyptian undergraduate mathematics students to implement REACT strategies: An approach to strengthen their conceptual and procedural knowledge of rational numbers and capability to create contextual situations

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## Abstract

**Background:** The quality of instruction significantly influences students' understanding of school mathematics, highlighting the importance of initial teacher education. Concerns about how to enhance prospective teachers' pedagogical skills remain significant. This study addresses the scarcity of national investigations on employing REACT strategies in preparing pre-service mathematics teachers.

**Aim:** This study aims to strengthen the conceptual and procedural knowledge of rational numbers among undergraduate mathematics students and develop their abilities to create contextual situations that align with the five interpretations of rational numbers: part-whole, operator, quotient, ratio, and measure.

**Method:** An embedded design was employed, selecting a convenient sample of thirty undergraduates from the mathematics teacher preparation program at the Faculty of Education, Tanta University in Egypt, during the academic year 2022-2023. Data collection involved administering a test to assess participants' conceptual and procedural knowledge and a survey to explore their capabilities in creating contextual situations. The data were analyzed using descriptive and inferential statistics, coupled with qualitative analysis of participants' answers.

**Results:** The adapted training significantly enhanced participants' knowledge of rational numbers, evidenced by a large effect size (Cohen's  $d = 2.71$ ). Furthermore, participants' ability to generate contextual scenarios improved, demonstrated by the diversity of contexts and the inclusion of all interpretations of rational numbers in their scenarios.

**Conclusion:** The study demonstrates the effectiveness of REACT strategies in improving prospective mathematics teachers' knowledge and skills. Future research should evaluate the quality of contextual scenarios proposed by prospective teachers across various domains of mathematics.

## INTRODUCTION

In general, the public perceives mathematics as a challenging discipline that could be mastered by certain people (Laurens et al., 2017). The abstraction of mathematical concepts, which is reflected in the inability to establish a meaningful connection between mathematics and our day-to-day experiences, contributes to these challenges (Akperov et al., 2023; Samritin et al., 2023), notably concepts linked to rational numbers.

Concepts of rational numbers are widely encountered in students' daily situations (Yang & Wang, 2022). For instance, when students comprehend the part-whole concept and realize its validity in situations like fair-share distribution, they could assist their parents in cutting a

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cake into equal pieces (Selvianiresa & Prabawanto, 2017). Nonetheless, most school textbooks neglect this practical facet and diminish providing a variety of context-based tasks (Wijaya et al., 2015). This led many teachers to merely strengthen the procedural aspect (e.g., to divide two rational numbers, first invert then multiply both) and also directed them to sharpen one single meaning of rational numbers, which is part-whole (Putra, 2018). In that sense, assisting teachers in operating contextual scenarios of rational numbers is necessary.

The set of rational numbers, including relevant operations on this set, are fundamental topics in school mathematics, particularly in middle grades (Ministry of Education and Technical Education in Egypt, 2020; National Council of Teachers of Mathematics [NCTM], 2000); hence, the Common Core State Standards for Mathematics emphasized the topic of rational numbers and related concepts of ratios, rates, and proportions to be essential for elementary grades (Common Core State Standards Initiative, 2010). Yet, teaching and learning these topics remain challenging (Kashim, 2016; Spitzer et al., 2023). Siegler and Lortie-Forgues (2017) categorized the challenges in learning rational numbers into two main groups: inherent difficulties and culturally contingent sources. The former arises from the nature of concepts and related algorithms, such as relationships among arithmetic operations. On the other hand, the latter varies among diverse cultures, including factors like teachers' knowledge, which is the focal point of this study.

As research findings indicated, most pre-service teachers (including undergraduates) lack conceptual and procedural knowledge of rational numbers (Reitz-Koncebovski et al., 2022; Lenz & Wittmann, 2021; Kolar et al., 2018; Lin et al., 2013; Salifu, 2021; Van Steenbrugge et al., 2014; Zakaryan & Ribeiro, 2018). In particular, pre-service teachers' insufficient knowledge of rational numbers – a culturally contingent source- stays critical since they would be mathematics teachers, and one reason why pupils struggle when learning rational numbers is thought to stem from teachers who do not possess a solid understanding of relevant concepts and procedures (Chinnappan & Forrester, 2014; Putra et al., 2023).

This is summarized in Putra's (2018, 2019) studies and Olanoff et al.'s (2014) review wherein prospective teachers perform the procedures effectively; however, they cannot explain the meaning behind these procedures (why procedures work) due to lack of conceptual knowledge and inability to adapt rational numbers sense. Noting that conceptual knowledge, in this context, identifies understanding the core concepts and relationships among them, whereas procedural knowledge defines symbols, conditions, and techniques used to perform a specific mathematical task (Hiebert & Lefevre, 1986).

Teacher education has a crucial role in addressing this situation, especially in the Arabian context. For example, after Qetesh et al. (2020) exposed the low level of Jordanian teachers' conceptual and procedural knowledge and their tendency to memorize and apply procedures routinely without a deep understanding of the concepts, they attributed the reason for so to the lack of enthusiasm towards training programs of prospective teachers and the superficial courses offered to them. This explains why Kolar et al. (2018) called for broader studies on how teacher education programs could empower prospective teachers to deepen their understanding of basic mathematical concepts like rational numbers within the course of didactics of mathematics.

In addressing concerns of abstraction and deficient conceptual knowledge (Ma, 1999), contextual mathematics education could function by ascertaining that mathematics is not just

calculations; it has much to do with everyday situations. That is, the perception of mathematics as a remote body of knowledge could be overcome using contexts (Boaler, 1993), wherein context refers to the notion of settings “various physical sites of human activities” (Roth, 1996, p. 491) and situations “social, physical, historical, spatial, and temporal aspects” (ibid. P.491). Hence, the contextual mathematics education approach was described by Raslan (2018) as contemporary teaching/learning practices arising from the philosophy of realistic mathematics education to make learning more meaningful by transforming mathematical academic knowledge into practical experiences. Furthermore, the importance of contextualizing mathematical tasks to be relevant to students’ everyday situations was highlighted by the Organisation for Economic Co-operation and Development [OECD] (OECD, 2009) to develop students’ competencies (e.g., reasoning, argumentation, communication, representation). Adopting this, the efficacy of this approach that connects mathematical content to learners’ daily experiences to enhance conceptual understanding was acknowledged in multiple studies (e.g., Jazuli et al., 2017; Selvianiresa & Prabawanto, 2017).

One of the methods rooted in contextual education is REACT strategies, encompassing Relating, Experiencing, Applying, Cooperating, and Transferring. They are manipulated to enable students to interpret mathematical concepts through the lens of daily experiences. REACT strategies have gained widespread recognition due to their effectiveness in fulfilling learning outcomes across different settings. Put differently, research has shown that incorporating learners into situations that align with REACT has a positive influence on the development of mathematical representation, reasoning, and disposition (Sari & Darhim, 2020; Supandi et al., 2016), mathematical communication (Musyadad & Avip, 2020), mathematical connection (Khatimah & Fatmah, 2021), mathematical problem-solving (Widada et al., 2019), creative thinking (Marlan, 2017; Qadri et al., 2019), and higher-order skills (Herlina & Ilmadi, 2022). Yet, In the realm of Egyptian mathematics education research, there is a lack of studies that examine adapting contextual education, particularly REACT strategies, to prepare mathematics teachers. This scarcity hinders the achievement of desired outcomes for prospective teachers.

In detail, at the secondary level and to enhance processes of teaching and learning biology, while Saleh (2018) utilized REACT to promote the students’ deep understanding and academic self-efficacy, Gad (2021) operated it to improve achievement, genetic problem-solving skills, and motivation to learn biology. Likewise, Elfouly’s (2022) study highlighted the significance of REACT in building conceptual understanding and self-regulation skills among agricultural secondary school students. This is consistent with Abdul Karim’s (2017) investigation in which employing REACT was significant in fostering successful intelligence ability, conceptual understanding, and aspiration level of learning chemistry. In addition to secondary grades, researchers such as Abdou (2020), Mohamed (2019), and Nosehy (2021) have investigated the efficacy of implementing REACT to learn science in elementary grades. Their study findings indicated that cognitive factors like future thinking, imaginary thinking, and 21st-century skills, as well as non-cognitive aspects such as learning enjoyment and achievement motivation, were positively impacted. Regardless, to improve mathematics learning and to the best of our knowledge, merely Raslan’s (2018) study was found. It utilized contextual strategies REACT to develop mathematical problem-solving skills and engagement in the learning processes of primary school students.

On one side, the previously cited studies have recommended embedding contextual teaching and learning strategies within teaching methods courses during teacher education programs. They particularly highlighted qualifying pre-and in-service teachers to practice REACT to better engage their pupils in deep learning. More specifically, Raslan (2018) declared the necessity of training mathematics teachers on performing contextual strategies while teaching all mathematics content areas. The importance of so lies in Lee's (2012) argument, wherein although the utilization of real context would sustain pupils' deep understanding of concepts, this depends essentially on teachers' knowledge, perceptions, and interpretations of these contexts, which eventually hinder or facilitate fulfilling the curriculum goals. Nonetheless, the scarcity of research addressing this concern prompted the current study to investigate it further. More precisely, this study attempted to explain the processes of qualifying undergraduate mathematics students (pre-service mathematics teachers) to execute REACT strategies to strengthen conceptual and procedural knowledge and capability to create contextual situations of rational numbers.

## **STATEMENT OF THE PROBLEM**

Since mathematics is taught to pupils in classrooms through experiences offered to them by teachers, responsibility is placed on teacher preparation to ensure graduates' mastery of essential school mathematics concepts that will be taught. Among these concepts is the notion of rational numbers that, according to the Egyptian school curriculum, begin to be learned in grade 2 through the unit fraction concept and last until the middle grades, wherein it constitutes an essential topic of the grade 7 curriculum.

Indeed, except in a few instances, one of the concerns observed in local research on mathematics teacher education is that it often handles broad claims with no focus on specific content. For example, Mehawed (2021) conducted an exploratory developmental study to analyze the level of technological pedagogical content knowledge of pre-service mathematics teachers. As a part of his problem statement, he referred to several national studies that asserted pre-service teachers' lack of mathematical knowledge for teaching during their college years. Also, one rationale for Abd Elsaied's (2022) study, which attempted to develop student teachers' performance and professional self-efficacy, is their insufficient knowledge of school mathematics, which caused a lack of confidence to teach it. Commonly, such local investigations described undergraduates' knowledge of school mathematics as inadequate, which stays consistent with multinational research findings (Bowie et al., 2019).

Yet, Abd Elmalak's (2021) study represents one of the few exceptions since it precisely addressed undergraduates' knowledge of rational numbers. This study investigated the effectiveness of a program based on the Mathematical Knowledge for Teaching (MKT) framework in developing the undergraduates' professional noticing skills of pupils' thinking. As Abd Elmalak (2021) reported, she focused on the unit of rational numbers due to its connection to multiple real-life situations besides the challenges encountered while learning and teaching it.

Globally and until recently, not just in Egypt, mathematics teacher education research has long been concerned with knowledge of rational numbers and related concepts. For example, in the broader African context, Salifu (2021) examined pre-service teachers' conceptual and

procedural knowledge of rational numbers at the Evangelical Presbyterian College of Education in Ghana, wherein the study concluded that those teachers' knowledge was at average-low and high-average levels, respectively. Moreover, a significant difference between teachers' conceptual and procedural knowledge, in favor of procedural knowledge, was found with a very high effect size. Hence, the study suggested that mathematics teachers should concentrate on teaching for conceptual understanding of rational numbers, particularly considering the insufficient research in this arena. This is consistent with Kashim's (2016) recommendations regarding training mathematics teachers to implement suitable teaching methods that could grow conceptual understanding of rational numbers after his study revealed low conceptual knowledge of rational numbers among Nigerian primary teachers.

In addition to the above-listed concerns and two weeks before the main study, a pilot investigation was conducted to determine the undergraduate mathematics students' capabilities to create contextual situations of rational numbers. This was inspired by Lee's (2012) argument, wherein prospective teachers' capacity to construct suitable contextual scenarios gives insights into their knowledge and beliefs about real-life connections, which, in turn, influence the actual enactment of lessons. Moreover, teachers' awareness of diverse contexts through which mathematical concepts could be presented was emphasized by Altay et al. (2020) to foster students' enthusiasm and enhance their mathematical connection skills.

At first, when the undergraduates were invited to propose some examples to warm up the unit of rational numbers, about (70%) of their responses were decontextualized (context-free), like how to calculate  $\frac{1}{2} + \frac{1}{5}$ . They also declared that they should rely on pupils' prior knowledge of fractions to deliver the rational numbers concept. After (two weeks from the treatment), the researcher questioned them again to think of real-life situations relevant to concepts of rational numbers. As a result, it was evident that they predominantly offered buying scenarios that involved the quotient concept (see the Results) to deliver rational numbers. Yet, other interpretations of part-whole, operator, ratio, and measure (Behr et al., 1983) were lacking. This mirrors Ma's (1999) view, in which teachers with high procedural knowledge often lack creating authentic scenarios connected to these procedures.

To tackle such issues, experts have suggested adopting efficient approaches, in particular, contextual teaching and learning (Abebe et al., 2023; Al-Mutawah et al., 2019). Still, the current body of research does not provide sufficient support to assert that this approach has thus far improved students' conceptual knowledge. This might be happening because of several factors like the nature of the context, the employed instructional methods, and teachers' knowledge and perceptions of the context, which contributes to inconsistent findings (Abebe et al., 2023; Lee, 2012; Zakaryan & Ribeiro, 2018). Moreover, In Egypt, a lack of research on how to systematically incorporate contextual teaching and learning into mathematics teacher education was recognized (Raslan, 2018). Accordingly, the present study aims to address these gaps.

## **RESEARCH OBJECTIVE AND QUESTIONS**

Focusing on processes of preparing prospective mathematics teachers, the current study aimed to strengthen conceptual and procedural knowledge of rational numbers among undergraduate mathematics students and equip them with the skills required to create relevant contextual

situations by training them to implement REACT Strategies. In that sense, the study attempted to answer this principal research question:

How could undergraduate mathematics students be trained to implement REACT strategies so that their conceptual and procedural knowledge and capabilities to create contextual situations related to rational numbers could be strengthened?

Hence, the next sub-questions were discussed:

**RQ1.** What is the effect of training undergraduates to implement the REACT strategies on strengthening their conceptual knowledge of rational numbers?

**RQ2.** What is the effect of training undergraduates to implement the REACT strategies on strengthening their procedural knowledge of rational numbers?

**RQ3.** What is the relationship between undergraduates' conceptual and procedural knowledge of rational numbers?

**RQ4.** How does training undergraduates to implement the REACT strategies while teaching rational numbers affect their capability to create contextual situations?

## **LITERATURE REVIEW AND RELATED STUDIES**

### ***Conceptual and Procedural Knowledge of Rational Numbers***

The concept of rational numbers defines an essential construct within the content area of Numbers and Operations, particularly in the middle grades. As highlighted in the Principles and Standards for School Mathematics document issued by the NCTM, teaching rational numbers to middle-grade students should be built on their prior knowledge of concepts of whole numbers, fractions, decimals, and percentages learned in lower grades and grounded in everyday situations (NCTM, 2000). Thus, the standards for the preparation of middle-level mathematics teachers reflect such focus in which candidates should “apply understandings of major mathematics concepts, procedures, knowledge, and applications of number, including flexibly applying procedures, using real and rational numbers in contexts, attending to units, developing solution strategies and evaluating the correctness of conclusions.” (NCTM, 2020, p. 7). In that sense, prospective teachers should maintain sufficient knowledge of rational numbers and related concepts to enhance their future teaching and support pupils' understanding. This entails designing effective teaching-learning environments through which pupils' and teachers' knowledge could be promoted (Khashan, 2014), notably teachers' specialized content knowledge (Chinnappan & Forrester, 2014).

Exploring conceptual and procedural knowledge has significant value for mathematics educators and the research community. It is rooted in Skemp's (1976) identification of instrumental and relational understanding. While instrumental understanding involves knowing the rules and being capable of employing them, which indicates features of procedural knowledge, relational understanding demonstrates “knowing both what to do and why” (Skemp, 1976, p. 20), which reflects characteristics of conceptual knowledge that determines understanding the concepts and their interconnections.

A concept conveys “a mental representation that embodies all the essential features of an object, a situation, or an idea. Concepts enable us to classify phenomena as belonging, or not belonging, together in certain categories” (Westwood, 2008, p. 24). In mathematics, conceptual knowledge outlines a profound understanding of the fundamental mathematical concepts and

principles related to a specific domain. Such understanding emerges in the ability to express concepts in several equivalent representations and apply them with suitable procedures while solving mathematical problems (Chirove & Ogbonnaya, 2021; Star & Stylianides, 2013a). This is similar to Khashan's (2014) description of conceptual knowledge, which inspired the present study. According to Khashan (2014), conceptual knowledge defines understanding and applying interconnected mathematical concepts and ideas. It allows individuals to comprehend and clarify the rationale behind the utilization and functionality of precise mathematical procedures or formulas within a particular context.

Practically, maintaining solid conceptual knowledge enables learners to solve diverse mathematical problems, evaluate the suitability of certain procedures to a specific situation, explain why these procedures work, and promote procedural flexibility during the problem-solving process (Al-Mutawah et al., 2019; Schneider & Stern, 2010; Schneider et al., 2011). This is clarified by Salifu (2021) as conceptual knowledge is expressed in individuals' (a) interpretations of why/why not a mathematical claim is valid and (b) explanations of the required procedures to solve problems and why such procedures work, which, in case of teacher education, corresponds clarification of teachers' specialized content knowledge.

On the other side, Rittle-Johnson and Schneider (2015) defined a procedure as a "series of steps, or actions, done to accomplish a goal" (p. 1119); hence, procedural knowledge, according to them, specifies knowledge of the procedures. It describes "knowing how, or the knowledge of the steps required to attain various goals" (Rittle-Johnson & Schneider, 2015, p. 1119). This description aligns with Salifu's (2021) illustration of procedural knowledge as the ability to recall related mathematical concepts, processes, rules, and principles while solving problems. It is often defined by researchers as the application of formulas, procedures, and algorithms to solve specific mathematical problems (Chinnappan & Forrester, 2014; Hiebert, 2013; Schneider & Stern, 2010; Star & Stylianides, 2013a).

The above explanation of procedural knowledge reveals a limited focus on addressing only particular types of routine problems rather than real-life scenarios. As Zakaria and Zaini (2009) argued, procedural knowledge allows learners to apply certain procedures to solve specific problems and illustrate how the answer was obtained; however, learners might remain unable to explain why these procedures functioned in this situation. Like Al-Mutawah et al.'s (2019) description of procedural knowledge, practically, it frequently manifests in the capabilities to apply an algorithm correctly and understand the outcomes of this algorithm in a specific problem scenario.

Regarding the relationship between conceptual and procedural knowledge, three viewpoints were identified (Rittle-Johnson & Schneider, 2015; Schneider & Stern, 2010): (1) The uni-directional relationship (either procedures or concepts come first). It is expressed in researchers' views on procedural knowledge that works as a vehicle or a prerequisite to accessing conceptual knowledge, or others who believe that prior learning of concepts leads to acquiring procedures. (2) The null relationship (inactivation view), which claims that procedures and concepts are independently developed. (3) The bi-directional relationship (iterative view), which claims that procedures and concepts depend on each other (Hurrell, 2021; Lenz & Wittmann, 2021; Thurtell et al., 2019). This bi-directional relationship was acknowledged in several studies when the researchers asserted that conceptual and procedural knowledge are inseparable, impact each other, and constitute integral components of the

individual's cognitive schema (Chinnappan & Forrester, 2014; Chirove & Ogbonnaya, 2021; Hiebert & Lefevre, 1986; Kieran, 2013). For example, Rittle-Johnson et al. (2001, 2015) reported that conceptual and procedural knowledge lies on a continuum and cannot be completely separated.

In general, most teacher candidates enter university with inadequate knowledge of school mathematics, particularly when it comes to rational numbers (refer to the introduction). This is apparent in Putra's (2018, 2019) and Putra et al.'s (2023) studies, wherein insufficient knowledge of rational numbers among pre-service teachers was revealed. Likewise, similar concerns were exposed in the Arabian context, as evidenced in the research conducted by Khashan (2014) and Qetesh et al. (2020), which scrutinized Saudi and Jordanian teachers' knowledge of rational numbers, respectively. Further to this, it has been reported that teachers primarily possess procedural knowledge, yet their level of conceptual knowledge stays comparatively lower (Chinnappan & Forrester, 2014; Khashan, 2014; Putra et al., 2023; Reitz-Koncebovski et al., 2022; Salifu, 2021). In these circumstances, teachers are neither expected to help their students build deep conceptual connections nor eliminate emergent misconceptions (Ma, 1999). Hence, better consideration of undergraduates' courses is needed, notably because such courses taught in teacher education merely replicate traditional algorithms and fail to enhance prospective teachers' knowledge beyond what they have already acquired during their K-12 education (Tobias, 2009).

### ***Contextual Situations Related to Rational Numbers***

In the early stages, mathematical concepts should be approached through real-life contexts relevant to pupils' daily experiences to help them build meaningful understanding through realizing the connections between such abstract concepts and their utilization.

According to Meyer et al. (2001), context refers to verbal textual problems, pictures, graphics, and tables. In this regard, contextual situations are anticipated to assist learners in connecting their informal understanding with formal knowledge of mathematical concepts, which could be achieved by incorporating real-life scenarios or provoking their imagination (Kent, 2000). Therefore, proficiency in delivering mathematics through diverse real-life scenarios is deemed a fundamental goal of mathematics education (Graumann, 2011; Wijaya et al., 2015). This responds to the NCTM (2000) recommendations, wherein mathematical connections were identified as one among process standards, highlighting the importance of incorporating mathematics into daily experiences. It further explains why the Programme for International Student Assessment (PISA) (OECD, 2003) stresses evaluating students' achievement based on their ability to solve math problems situated in authentic contexts.

The above argument reflects the extent to which teachers should be able to create contextual situations that relate mathematical concepts learned at school to daily experiences. It also denotes an essential indicator for effective mathematics instruction since teachers' perception of real-life connections affects their implementation of the school curriculum (Lee, 2012). Also, and from a broader perspective, teachers' ability to approach mathematical concepts through relevant and meaningful situations that pupils experience in their lives responds to the theory of Realistic Mathematics Education (Freudenthal, 1971), wherein mathematics is considered a human activity invented by individuals (Cheng, 2013). It



emphasizes that teaching and learning should be built considering principles of activity (students' active engagement), reality (start from reality), level (progression through different levels), interrelationship (math as a whole), interaction (students' interaction to one another), and guidance (teachers should guide their students) (Inci et al., 2023). Based on this, Altay et al. (2020) regarded the necessity for prospective teachers to have substantial knowledge of contexts through which mathematical concepts could be taught, particularly to address the lack of such authentic contextual tasks in school textbooks (Paredes et al., 2020). Given that, in the present study, the concept of contextual situation aligns with PISA's recognition of context-based tasks (OECD, 2003). It refers to problems presented within real-world or fictional settings that contain personal, occupational, scientific, and public information and could be imagined by students.

Regarding contextual situations, Rejeki et al. (2021) analyzed the contextual activities encountered in mathematics textbooks for vocational high school students, focusing on the three criteria of context (no context, camouflage, and essential), information (matching, missing, and superfluous), and cognitive demand (connection, reproduction, and reflection). Similarly, Altay et al. (2020) explored real-life connections within the grade 6 Turkish mathematics textbook. As a result, these studies emphasized the importance of providing rich context that is relevant and engaging for students, capturing their attention to the situation, and fostering meaningful learning experiences. Conversely, the weak context was described as the forced context that is often inappropriate or merely includes persons' names. Moreover, Paredes et al. (2020) proposed four categories of realism, cognitive domain, authenticity, and openness to assess the quality of the mathematical tasks created by the pre-service mathematics teachers.

A principal difference between the above-reported investigations and the present study is that these investigations intended to assess the quality of the contextual tasks that either appeared in textbooks or were proposed by prospective teachers using specific criteria. On the contrary, in the present study, no prior criteria were determined to explore how the treatment influenced the undergraduates' capability to create contextual situations. In other words, what they offered was analyzed inductively before and after the treatment to capture the changes. Yet, the five interpretations of rational numbers (Behr et al., 1983) were essentially considered in this analysis (see the Methodology).

As reported in the literature, pupils learn rational numbers more effectively through operating multiple presentations and diverse interpretations. Thus, it is crucial to ensure that all aspects of rational numbers are prioritized not only in textbooks but also in lessons' implementation to foster pupils' understanding (Govender, 2021). A possible approach to achieve this is exposing pre-service teachers to a comprehensive range of rational number subconstructs in their curriculum rather than being limited to a narrow understanding of this concept (Tobias, 2009). This is emphasized by Doğan and Tertemiz (2020), wherein undergraduates should maintain adequate knowledge of various interpretations of rational numbers and what examples could fit each, particularly contextual examples through which concepts and procedures of rational numbers could be delivered to pupils.

According to Vergnaud (1983, p. 160), the rational number describes "an equivalence class of ordered pairs of whole numbers." However, from an educational perspective, Kieren's (1976) and Behr et al.'s (1983) studies exposed that rational numbers encompass more than merely equivalence classes. It could be interpreted in five different ways: part-whole, operator,

quotient, ratio, and measure (Elbehary & Aboseira, 2022; Yang & Wang, 2022). In detail, (1) the rational number as *part-whole* is the commonly utilized interpretation and the simpler one to understand. It defines the result of dividing a given whole into equal parts or subsets. In this context, the denominator is the number of parts that constitute the whole, while the numerator is the number of equal parts that are taken together. (2) the rational number as an *operator* determines enlarging or reducing the size of a number through the multiplication operation. (3) the rational number as a *quotient* defines the result of a division operation. (4) the rational number as a *ratio* reflects a proportional relationship between two quantities. (5) the rational number could be interpreted as a *measure* of a discrete or continuous quantity or a part-whole segment of a line.

### **Contextual Mathematics Education and REACT Strategies**

Contextual teaching-learning is grounded in the theory of constructivism: cognitive and social constructivism. It is rooted in Piaget's work, which emphasizes the active involvement of students in constructing their own knowledge. Furthermore, it is also influenced by Vygotsky's perspective, which highlights the importance of social interaction in the process of knowledge development. Based on this, it has been argued that contextual teaching-learning encourages learners to explore new concepts by connecting them to their own experiences, thus enabling them to construct their knowledge instead of simply receiving it passively (Selvianiresa & Prabawanto, 2017). Such construction could occur in diverse social, cultural, and physical settings through collaborative work, enabling students to exchange ideas and perspectives, admit others' viewpoints, provide mutual support, and collectively promote conceptual knowledge (Cabbar & Senel, 2020).

Contextual teaching-learning is declared in the literature under multiple terms such as contextual teaching, context-based learning, or teaching contextually. It defines an approach that involves connecting the content with the context in meaningful ways (Berns & Erickson, 2001). Komalasari (2010, as cited in Suastika & Wahyuningtyas, 2018) used the term contextual teaching and learning to describe processes of establishing a link between the subject matter taught to students in the classroom and their daily experiences within their families, schools, communities, and society at large, with an ultimate goal of uncovering the relevance of this subject to real-life situations.

In mathematics education, contextual teaching-learning targets employing contextual tasks through which mathematical concepts could be introduced within real-life circumstances or scenarios relevant to learners' interests and personal experiences (Akperov et al., 2023). Boaler (1993) asserted that to prepare students for the mathematical needs they would daily encounter, it is required to shift away from abstract mathematics towards mathematics in a context that mirrors the demands of real-world situations. In other words, contextualizing mathematics education desired to ascertain that this mathematics has much to do with our lives. Hence, the significance of implementing contextual teaching approaches to effectively teach mathematics by relating it to authentic contexts was acknowledged in various studies (Adamu, 2018; Akperov et al., 2023; Jazuli et al., 2017; Taley, 2022).

For example, and in relation to the conceptual knowledge targeted in this study, Jazuli et al. (2017) argued that the difficulty of understanding mathematical concepts situated in realistic

contexts may arise from the limitations of traditional learning strategies. Considering this, Jazuli et al. (2017) conducted an experimental study to examine the improvement in conceptual understanding and problem-solving skills while teaching contextually; accordingly, a positive impact of this approach was reported. That is consistent with Byrnes and Wasik's (1991, as cited in Khashan, 2014) viewpoint on teaching for conceptual understanding; it often introduces the content through applied problems to discuss relations and principles, which contradicts the traditional approach that mostly ends by such applied problems as an application of learned principles and algorithms.

In addition to the cognitive learning outcomes, Adamu's (2018) study revealed positive attitudes towards mathematics for senior secondary school students in Kaduna State, Nigeria, through implementing the contextual REACT strategies to teach plane geometry. The researcher recommended training teachers through seminars, workshops, and conferences on executing the contextual approaches in mathematics. Similar results were obtained when Jelatu et al. (2018) supported the implementation of REACT strategies by GeoGebra to enhance the understanding of geometry concepts among junior high school students in East Nusa Tenggara, Indonesia.

For the present study, contextual teaching-learning defines a constructive educational approach that enables undergraduate mathematics students to establish connections between concepts and procedures of rational numbers and pupils' daily situations. This is considered an endeavor to qualify teacher candidates to teach rational numbers effectively by equipping them with the appropriate knowledge and skills.

Based on the Center for Occupational Research and Development (CORD) report (Crawford, 2001), REACT determines these five contextual teaching strategies: Relating, Experiencing, Applying, Cooperating, and Transferring, which were deduced from research on how individuals acquire knowledge and observations of how best teachers transmit knowledge for understanding. These strategies originated from the theory of constructivism, specifically social constructivism, which emphasizes the active involvement of students in their learning. That is, students are encouraged to use their experiences as a foundation to construct knowledge through social interactions (Vygotsky, 1978, as cited in Mishra, 2014). The implementation of REACT strategies in classrooms entails applying the subject matter being taught, engaging pupils in building new knowledge, situating this knowledge in real-life contexts, collaborating with peers to resolve issues, and establishing a connection between the acquired knowledge and the experience that pupils will encounter in the future (Abebe et al., 2023; Jelatu et al., 2018).

REACT refers to [R] Relating; it reflects placing mathematical concepts in the context of students' daily experiences (i.e., learning through real-life experiences) and previous knowledge. The relating strategy remains the most influential contextual teaching strategy wherein there is research evidence on learning enhancement when teachers use students' prior conceptions, knowledge, and beliefs to start the instruction (Crawford, 2001). [E] Experiencing defines learning by doing or by exploring mathematics; it encompasses processes of exploration, discovery, and invention that are essential to be practiced for knowledge construction. These processes could be done through, for example, manipulation (i.e., objects that could concretely represent abstract concepts) and problem-solving activities. [A] Applying expresses the application of new concepts to meaningful contexts, wherein students are given

opportunities to apply what they have learned. [C] Cooperating refers to processes of sharing and communicating ideas, which resembles real-life skills needed to function in a team. It incorporates collaborative discussions and comparisons of individual ideas to gain a deeper understanding of the taught concepts (Clarke & Roche, 2009), which aligns with one of the present study objectives. [T] Transferring describes expanding what has already been learned to authentic contexts that have not been dealt with in class (novel situations). It confirms that knowledge, skills, and attitudes should not only be memorized but also transferred to relevant situations.

### ***The Treatment Adapted to this Study***

At first, for the design of an effective learning environment to train undergraduate mathematics students to implement REACT strategies while teaching rational numbers, they should clearly understand the following principles (characteristics):

- All processes in the contextual teaching-learning environment should be consistent with the philosophy of constructivism (social constructivism).
- The proposed activities (or situations) should relate the intended mathematical concepts and procedures to pupils' prior knowledge, daily experiences, real-life situations, and other school content areas (if possible) so that pupils can realize the applicability of mathematics.
- Teachers should actively engage pupils in inquiry-based activities to trigger their curiosity towards questioning and fulfilling learning objectives by themselves.
- Scaffolding techniques are essential to facilitate the process of knowledge building. This may involve asking for support from more experienced colleagues to provide insight and feedback or preparing a series of modified questions that students can use to synthesize their knowledge.
- Authentic assessment defines an integral aspect of contextual teaching-learning through which learning outcomes could be mapped. This includes tracking pupils' worksheets (knowledge) and observations (behaviors) continuously utilizing self- and peer-reflection, besides the teacher assessment.

After an extensive literature review on the enactment of REACT strategies (Abebe et al., 2023; Clarke & Roche, 2009; Komarudin et al., 2022; Musyadad & Avip, 2020; Nurzannah et al., 2021; Sari & Darhim, 2020; Selvianiresa & Prabawanto, 2017), the following model was developed and hypothesized as a treatment to strengthen undergraduates' knowledge of rational numbers and their capability to create contextual situations (see Figure 1).

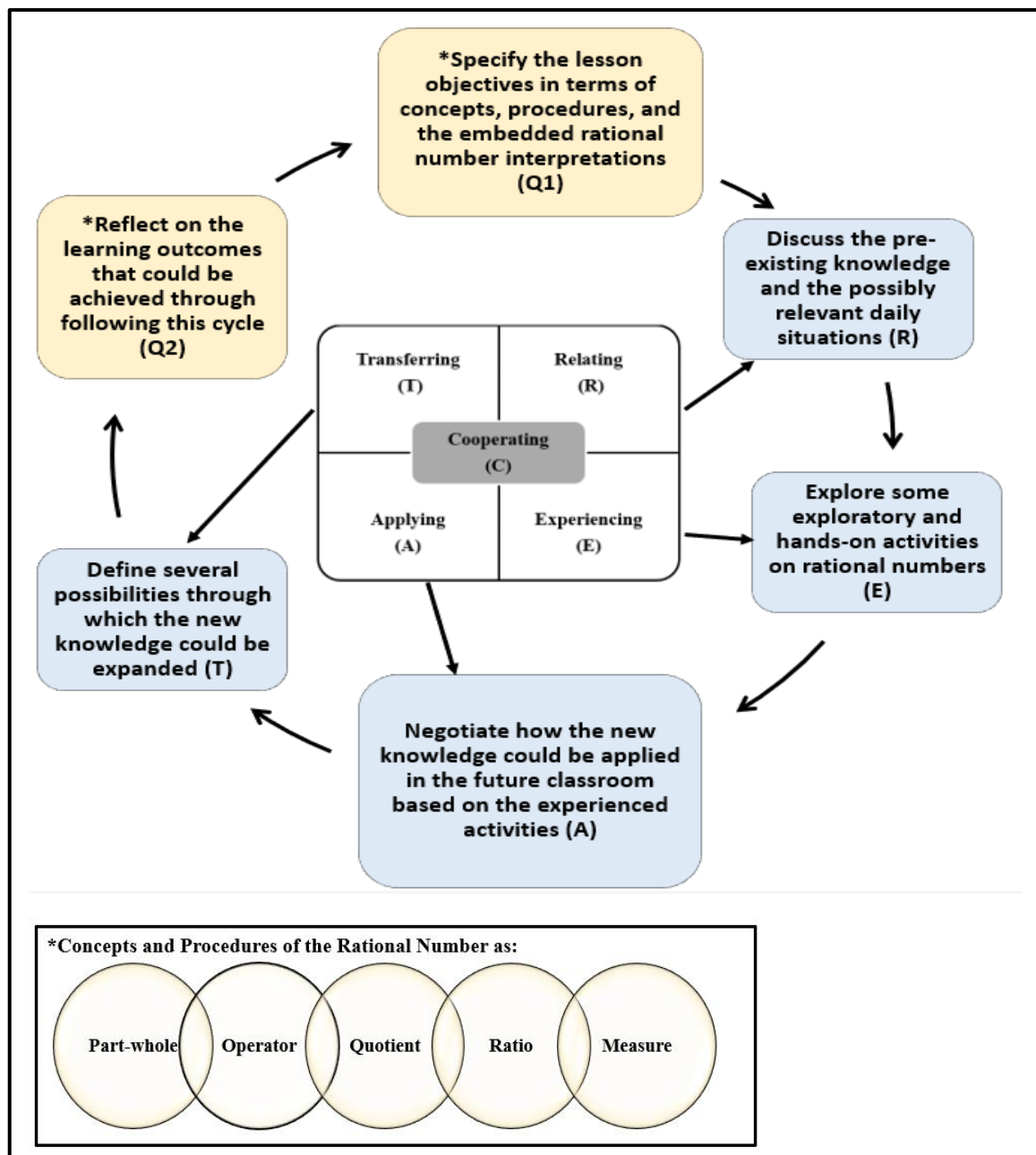


Figure 1. The Treatment Adapted to This Study

Also, Table 1 summarizes the instructor's practices while adopting the treatment that is hypothesized in this study and presented in Figure 1.

Table 1. The Instructor's Practices to Train Undergraduates to Implement REACT Strategies

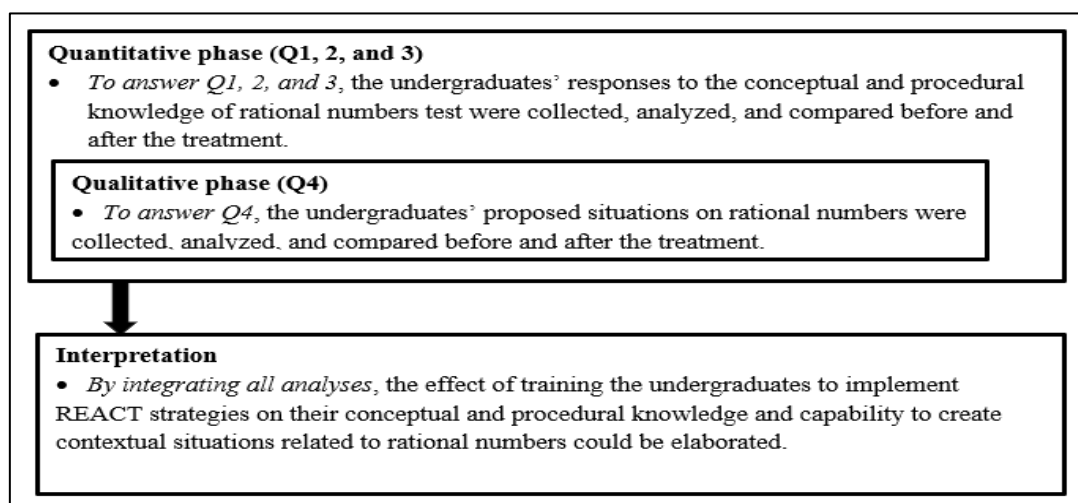
REACT strategies	Roles of the trainer: The instructor should trigger undergraduates to
<b>R: Relating</b>	<ul style="list-style-type: none"> <li>▪ propose provoking introductory questions that can be answered by nearly all pupils based on their personal experiences or existing knowledge.</li> <li>▪ build on pupils' expected answers that represent the prior knowledge to deepen their understanding by clarifying the connection between what they already know and the new concepts to learn.</li> </ul>
<b>E: Experiencing</b>	<ul style="list-style-type: none"> <li>▪ plan some discovery and exploration activities (e.g., problem-solving, manipulation) that could be carried out in the classroom. Accordingly, demonstrate how such activities are contextualized in a manner that enables pupils to realize the connection between the intended mathematical concepts and their daily encountered situations embedded in the context.</li> </ul>

	<ul style="list-style-type: none"> <li>▪ illustrate how pupils would be energetically engaged in dealing with the planned activities; more specifically, how pupils explore the mathematical concepts while solving the activities.</li> <li>▪ formulate some simplifying questions relevant to the planned activities so that pupils can rely on them to unpack the intended concepts.</li> </ul>
<b>A: Applying</b>	<ul style="list-style-type: none"> <li>▪ suggest additional relevant situations through which the new concept could be applied.</li> <li>▪ explain similarities and differences among these new situations.</li> </ul>
<b>C: Cooperating</b>	<ul style="list-style-type: none"> <li>▪ argue the various perspectives through which the intended concepts and procedures could be presented.</li> <li>▪ decide together the appropriate real-life context and the activity wherein the concepts and procedures would be placed.</li> <li>▪ negotiate why one context stays more suitable than another.</li> <li>▪ judge the contexts raised among the groups based on their appropriateness to the concepts and procedures intended.</li> <li>▪ explain how to scaffold pupils who might have difficulties while working collaboratively.</li> <li>▪ consult together their criteria on how productive groups of pupils could be formed, observed, and evaluated.</li> </ul>
<b>T: Transferring</b>	<ul style="list-style-type: none"> <li>▪ use the internet to search for new relevant situations through which pupils' knowledge could be expanded.</li> <li>▪ reflect on the suitability of these situations to pupils' levels of understanding, school curriculum, and societal circumstances.</li> </ul>

## METHODS

### *Research Design:*

The study adopted the *embedded design* as a mixed-method research technique due to its suitability to the objectives. As reported by Creswell (2012), upon this design, “the researcher collects both quantitative and qualitative data during a single study (e.g., an experiment or a correlational study), the two datasets are analyzed separately, and they address different research questions” (Creswell, 2012, p. 545). Figure 2 depicts the data collection and analysis processes in relation to the study questions. As displayed, the quantitative phase of the study aimed to investigate the impact of the training on strengthening undergraduates' conceptual and procedural knowledge of rational numbers and the correlation between the two. Besides, the qualitative phase focused on exploring how this training enhanced the capability of the undergraduates to create contextual situations related to rational numbers.



**Figure 2.** Research Design

It is necessary to acknowledge that the study solely involved one experimental group and did not incorporate a randomized control group. This limitation arose because the experiment was conducted within the existent teacher education program, which restricted access to an equivalent control group.

### ***Population and Sample:***

The population of this study consisted of 73 undergraduate mathematics students who were pursuing their second year in the preparation program during the first term of the academic year 2022-2023 at the Faculty of Education, Tanta University, Egypt. Out of this population, a convenient sample of 30 undergraduates (13 males and 17 females) aged approximately 19-20 were selected. This selection was based on participants' willingness to partake in the study and their beliefs on the significance of learning rational numbers as an essential topic in middle school mathematics.

In detail, among the 73 undergraduates enrolled, roughly 62 were attending the classes consistently. Accordingly, those 62 undergraduates were allocated to two groups considering their preferences of school topics they desired to learn deeply. That is, when the researcher invited the participants to determine what area they would like to be prepared to teach to Grade 7 pupils, their responses diverged between the Rational Numbers and Algebra units. In other words, although 62 undergraduates were exposed to the treatment, only 30 specified the study sample. Such a process was done purposefully to guarantee the faculty ethics about equating learning experiences provided to undergraduates.

### ***Instrument:***

To achieve the objectives of this study, (I) a conceptual and procedural knowledge test and (II) a survey of contextual situations of rational numbers were prepared considering the following steps:

- *Define conceptual knowledge, procedural knowledge, and contextual situations of rational numbers operationally.*

Before reporting the operational definitions of conceptual and procedural knowledge, it is crucial to mention that until recently, there has been continuous discussion and multiple perspectives on how they could be defined. Such debate on knowledge definitions made it problematic to select appropriate tasks that might measure each individually (Kieran, 2013). As declared by Crooks and Alibali (2014), there is a lack of agreement among scholars regarding the precise definitions of conceptual and procedural knowledge as well as the appropriate questions to measure each accurately. To overcome this challenge, the researcher was committed to the following operational definitions:

***Conceptual knowledge of rational numbers*** defines learners' capacity to integrate concepts and procedures to perform tasks related to rational numbers. It is specified by responding to questions on levels of understanding, applying, or analyzing according to the Revised Bloom's Taxonomy (Anderson et al., 2001). Hence, the researcher prepared conceptual knowledge test items specifically for this purpose, and the score obtained by each participant determined his level.

**Procedural knowledge of rational numbers** defines learners' capacity to perform a sequence of algorithms and techniques or to apply a specific mathematical rule to solve a regular task related to operations on rational numbers. Hence, the researcher prepared procedural knowledge test items specifically for this purpose, and the score obtained by each participant determined his level.

**Contextual situations of rational numbers** refer to any imagined problem, situation, or scenario presented within real-world or fictional settings to illustrate the concept of rational numbers through each of these five interpretations: part-whole, operator, quotient, ratio, and measure.

- *Determine the items to measure participants' conceptual and procedural knowledge of rational numbers, and capability to create contextual situations*

To do so, the following procedures were conducted:

At first, the Rational Numbers unit, studied in Grade 7, was analyzed, and the intended learning outcomes were determined and classified as conceptual and procedural aspects (Mathematics for the first preparatory grade, 2020).

Additionally, various studies relevant to the present study in terms of content and objectives were revised (Altay et al., 2020; Kainulainen et al., 2017; Khashan, 2014; Kolar et al., 2018; Lenz & Wittmann, 2021; Paredes et al., 2020; Putra, 2019; Nahdi & Jatisunda, 2020; Rejeki et al., 2021; Salifu, 2021; Wijaya et al., 2015); this enabled adapting multiple appropriate items to the variables measured in this research and assisted in exploring the contextual situations proposed by the participants. The researcher translated the materials sourced from these English studies into Arabic and adjusted some of them to align with the objectives of the Rational Numbers unit. Hence, an initial version of the study tools was prepared to be piloted and to examine its validity and reliability.

To guarantee the content validity of tools, two lecturers and two professors specializing in mathematics education from the same faculty where this study was conducted undertook the task of revising them. Those experts have solid teaching experience in delivering courses on teaching methods and curriculum development to undergraduates across various years. Notably, one of the key concerns highlighted by the experts was the possibility of some items to assess conceptual or procedural knowledge, contingent upon the approaches used to tackle them.

For example, Q6 (see Table 3) was classified as a procedural knowledge item since it was expected to operate the same denominator and compare the numerators to solve it. Still, some respondents might think of relationships between every two numbers, which reflects a conceptual understanding. Like judging that the negative number  $\frac{-4}{7}$  is smaller than the positive number  $\frac{3}{5}$  because all positive numbers are greater than all negative numbers. To overcome the dilemma, which stems from the impracticality of separating conceptual knowledge from procedural knowledge, the approaches adopted by the participants were detailed for the data analysis (see the Results). Also, based on the experts' opinions, Q11 and 12 were revised and modified to hold both aspects of conceptual and procedural knowledge.

Also, the test reliability was confirmed through a pilot study that engaged 35 undergraduates- none of the participants. They were asked to answer the conceptual and procedural knowledge test, and their responses were divided into two halves, each containing conceptual and procedural knowledge items. Accordingly, the split-half reliability method



(parallel forms split) was utilized to determine the test internal consistency, and the resulting Pearson correlation coefficient between the scores of the two halves was approximately 0.67, denoting a satisfactory level of reliability. Moreover, after considering the average time taken to complete the test questions, it was determined that the test would be one hour long.

Additionally, the survey of contextual situations of rational numbers was revised in terms of content clarity, and it was recommended to trace changes in participants' proposed situations before and after the treatment to figure out their progress. In other words, one way to explore how the training affected the undergraduates' capacity to propose contextual situations is to examine whether these situations become more inclusive of rational number subconstructs. Even though this survey was not tested beforehand due to its unconventional nature to undergraduates, an estimated time of approximately 1 hour and 15 minutes was anticipated for its completion. Besides, the initial form of the survey was applied two weeks from the duration of the treatment without asking the participants to consider the different interpretations of rational numbers (see the Results). Accordingly, the final versions of the study tools were prepared.

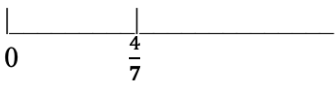


The conceptual and procedural knowledge of rational numbers test consisted of twelve questions. Seven questions were intended to measure participants' conceptual knowledge, three to test their procedural knowledge, and two questions encompassed both (see Tables 2 and 3). Table 3 summarizes the grading benchmarks depending upon the steps conducted by the participant, wherein a score of 0 indicates either no answer or an incorrect response. Thus, the maximum test score is 35 (18 marks for conceptual knowledge and 17 for procedural knowledge), and the minimum is zero.

Also, Figure 3 shows the survey of contextual situations of rational numbers after adjusting it based on experts' recommendations.

**Table 2.** Components of Conceptual and Procedural Knowledge of Rational Number Test

Items to measure conceptual knowledge	Items to measure procedural knowledge	Total Marks
Q1, Q2, Q3, Q4, Q5, Q8, Q9, Q11(b), Q12 (b)	Q6, Q7, Q10, Q11(a), Q12(a)	
<b>18 Marks</b>	<b>17 Marks</b>	<b>35</b>

Table 3. Conceptual and Procedural Knowledge of Rational Number Test Specifications

Questions	Knowledge embedded in the Q	Steps for solving the question	Mark
<p>[Q1]. Represent <math>1\frac{1}{7}</math> on the number line below:</p> 	<p><b>Conceptual knowledge:</b> Represent the measure concept (Visualization).</p>	<p>– Divide the number line appropriately based on the unit fraction (1 mark)</p> <p>– Determine the number <math>1\frac{1}{7}</math> correctly (1 mark).</p>	2
<p>[Q2]. Determine the rational number lies in the one-third of the distance between <math>\frac{-5}{6}</math> and <math>-1\frac{1}{2}</math> from the larger.</p>	<p><b>Conceptual knowledge:</b> Apply the measure concept to solve problems (Application).</p>	<p>– Determine the distance between both numbers (1 mark).</p> <p>– Calculate one-third of this distance (1 mark).</p> <p>– Subtract one-third of the distance from the larger number (1 mark).</p>	3
<p>[Q3]. If water flows through the pipe at <math>2\frac{1}{2}</math> liters per minute, how long does it take to fill 3 tanks, each of which is 20 liters?</p>	<p><b>Conceptual knowledge:</b> Apply the ratio concept to solve problems.</p>	<p>– Calculate the minutes required to fill one tank (1 mark).</p> <p>– Calculate the minutes needed to fill 3 tanks, each of 20 liters (1 mark).</p>	2
<p>[Q4]. Shade <math>\frac{3}{5}</math> of the apparent trapezoid:</p> 	<p><b>Conceptual knowledge:</b> Represent the part-whole concept (visualization)</p>	<p>– Divide the trapezoid appropriately (1 mark)</p> <p>– Determine the shaded parts correctly (1 mark).</p>	2
<p>[Q5]. If the apparent dots represent <math>\frac{3}{7}</math> of a specific unit, how many dots should be in <math>\frac{2}{3}</math> of this unit?</p> 	<p><b>Conceptual knowledge:</b> Apply the part-whole and operator concepts to solve problems.</p>	<p>– Determine the whole number of objects correctly (1 mark).</p> <p>– Calculate the required fraction <math>\frac{2}{3}</math> of the resultant whole number (1 mark).</p>	2
<p>[Q6]. Compare each of the following rational numbers: &gt; or &lt; or =</p> <ul style="list-style-type: none"> <li>▪ <math>\frac{2}{3}</math>      <math>\frac{3}{4}</math></li> <li>▪ <math>\frac{-3}{4}</math>      <math>-1\frac{1}{6}</math></li> <li>▪ <math>\frac{-4}{7}</math>      <math>\frac{3}{5}</math></li> <li>▪ <math>\frac{11}{12}</math>      0.9</li> <li>▪ <math>\frac{6}{5}</math>      <math>\frac{3}{4}</math></li> <li>▪ 20%      <math>\frac{5}{8}</math></li> </ul>	<p><b>Procedural knowledge:</b> Compare/ order different representations of rational numbers.</p>	<p>Determine the greater/smallest number appropriately (1 mark for each question).</p>	6
<p>[Q7]. Fill the missing blanks:</p> <ul style="list-style-type: none"> <li>▪ <math>\frac{3}{8} = \frac{\dots}{12}</math></li> <li>▪ <math>\frac{-4}{14} = \frac{\dots}{21}</math></li> <li>▪ <math>\frac{\dots}{5} = \frac{21}{8}</math></li> </ul>	<p><b>Procedural knowledge:</b> Specify equivalent rational numbers</p>	<p>Determine the missing values so that each two rational numbers would be equal (1 mark for each question).</p>	3

<p><b>[Q10].</b> If: <math>x = \frac{3}{2}</math>, <math>y = -\frac{1}{4}</math>, <math>z = 2</math>, find in the simplest form the numerical value of each of the following</p> <ul style="list-style-type: none"> <li>▪ <math>\frac{x-z}{y}</math></li> <li>▪ <math>\frac{1}{xyz}</math></li> <li>▪ The product of <math>\frac{1}{2} * \frac{2}{3} * \frac{3}{4} * \frac{4}{5} * \dots * \frac{99}{100}</math></li> <li>▪ What is the product when the last rational number is <math>\frac{n-1}{n}</math>?</li> </ul>	<p><b>Procedural knowledge:</b> Solve operations on rational numbers.</p>	<p>Apply the four operations correctly to get the result (1 mark for each question).</p>	<p>4</p>
<p><b>[Q11].</b> You asked a seventh-grade pupil to calculate <math>\frac{1}{2} + \frac{2}{3}</math> and <math>\frac{4}{7} - \frac{1}{3}</math></p> <p>(a) How would you, yourself, solve each question?</p> <p>(b) If you find your pupil adds and subtracts these given numbers this way: <math>\frac{1}{2} + \frac{2}{3} = \frac{3}{5}</math> and <math>\frac{4}{7} - \frac{1}{3} = \frac{3}{4}</math>, how do you interpret the pupil's method? Alternatively, what misconception did your pupil have? And how could you, as a teacher, help this pupil to overcome such a misconception?</p>	<p><b>Procedural knowledge:</b> Solve operations on rational numbers.</p> <p><b>Conceptual knowledge:</b> Speculate pupils' misconceptions (relationship between rational numbers and whole numbers)</p>	<p>– Apply operations of rational numbers correctly (4 marks).</p> <p>– Describe pupils' thinking to determine what misconceptions were held by them (4 marks).</p>	<p>8</p>
<p><b>[Q12].</b> You asked a seventh-grade pupil to calculate <math>8 * 0.25</math> and <math>8 \div 0.25</math>,</p> <p>(a) How would you, yourself, solve each question?</p> <p>(b) Suppose that your pupil used the calculator to get the result and it was found that the value of <math>8 * 0.25</math> is smaller than 8, while it was greater than 8 for <math>8 \div 0.25</math>. Thus, your pupil got confused and thought that there was something wrong with the calculator. In your opinion, why did the pupil get confused? Alternatively, what misconception did the pupil have? Also, how could you, as a teacher, help this pupil to overcome such a misconception?</p>			
<p><b>Total mark (35): 18 and 17 marks for conceptual and procedural knowledge questions, respectively</b></p>			<p>35</p>

Based on the knowledge acquired throughout this course, propose five scenarios through which the concept of rational numbers could be delivered to grade 7 pupils. These scenarios should include interpreting the rational number as

- **Part-whole:** Comparison of a part to a whole.
- **Operator:** Product of a multiplication process.
- **Quotient:** Result of a division process.
- **Ratio:** Comparison between two quantities.
- **Measure:** Unit of measurement.

**Figure 3.** The Survey of Contextual Situations of Rational Numbers

### ***Data Collection and Data Analysis:***

The data gathered for this research included the written answers of participants to the conceptual and procedural knowledge of rational numbers test, along with the contextual situations of rational numbers proposed by them.

Descriptive and inferential statistics were operated through version 25 of the Statistical Package for Social Sciences (SPSS) to analyze the collected data, specifically, paired samples t-test and Pearson's correlation test (see the Results). Moreover, qualitative data (i.e., proposed scenarios) was gathered through the survey of contextual situations of rational numbers and categorized based on (1) the type of context and (2) constructs of rational numbers embedded in each context.

The treatment lasted for seven weeks (1 period per week) and covered these topics: (1) The concept of rational numbers and its five interpretations. (2) Comparing and ordering rational numbers. (3) Adding and subtracting rational numbers. (4) Multiplying and dividing rational numbers. (5) Properties of addition and multiplication operations in the set of rational numbers. The participants were trained to deliver these lessons to their future pupils based on the model presented in Figure 1. Additionally, two extra sessions were dedicated to applying the pre- and post-assessments.

### ***Study Delimitations:***

The present study has focused on the content of rational numbers, which define a central concept in the middle school mathematics curriculum in Egypt. This specific content was selected based on Raslan's (2018) and Abd Elmalak's (2021) acknowledgments that concepts and procedures relevant to rational numbers can be effectively contextualized. Furthermore, from a broader perspective, Lee's (2012) study revealed that most prospective teachers prefer posing contextual problems on numbers and operations.

Also, the study was limited to a sample of second-year undergraduate students at a specific governmental university in Egypt, which might affect the generalizability of its results considering samples in other contexts. Likewise, the researcher was committed to the operational definition of the study variables; alternatively, the tools were developed based on these definitions, specifically to fulfill the research objectives. Thus, different results might be obtained if different instruments were utilized.

## **RESULTS AND DISCUSSION**

To determine the effect of the training on strengthening undergraduates' conceptual knowledge (**RQ1**) and procedural knowledge of rational numbers (**RQ2**), paired samples t-tests were performed, and the results were shown in Tables 4 and 5.

**Table 4.** Descriptive Statistics of Undergraduates' Level of Conceptual and Procedural Knowledge of Rational Numbers Before and After The Treatment

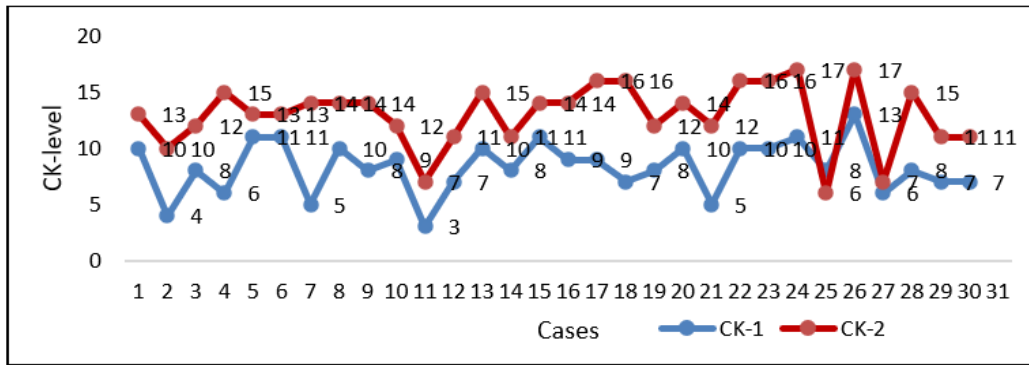
Test	N	Mean	Std. Deviation
CK-1(pre)	30	8.30	2.32
CK-2(post)		12.93	2.85
PK-1(pre)		11.67	1.88
PK-2 (post)		15.50	1.43

**Table 5.** Result of Paired Samples T-tests Undergraduates' Level of Conceptual and Procedural Knowledge of Rational Numbers Before and After The Treatment

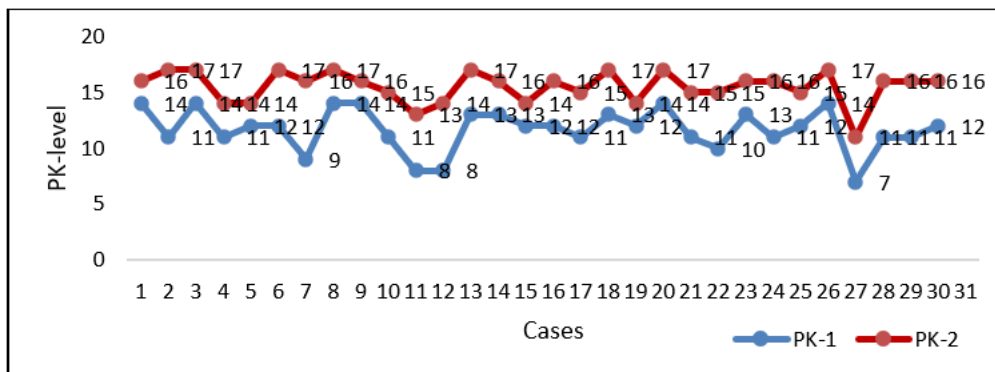
Test	Mean	Std. Deviation	t-value	df	Sig. (2-tailed)
Pair 1: CK1-CK2	-4.63	2.41	-10.51	29	.000
Pair 2: PK1-PK2	-3.83	1.32	-15.96	29	.000

The results reported in Tables 4 and 5 indicate that there is a statistically significant difference between the mean scores of the undergraduates' levels of conceptual knowledge of rational numbers before ( $M=8.32$ ,  $SD=2.32$ ) and after ( $M=12.93$ ,  $SD=2.85$ ) the treatment in favor of the post-assessment at  $t(29) = 10.51$ ,  $p=0.000 < 0.05$  at 2-tailed. Similarly, a statistically significant difference between the mean scores of the undergraduates' levels of procedural knowledge of rational numbers before ( $M=11.67$ ,  $SD=1.88$ ) and after ( $M=15.50$ ,  $SD=1.43$ ) the treatment in favor of the post-assessment at  $t(29) = 15.96$ ,  $p= 0.000 < 0.05$  at 2-tailed, was also found. Therefore, it can be argued that the training positively affected undergraduates' conceptual and procedural knowledge of rational numbers, which answers both Q1 and Q2.

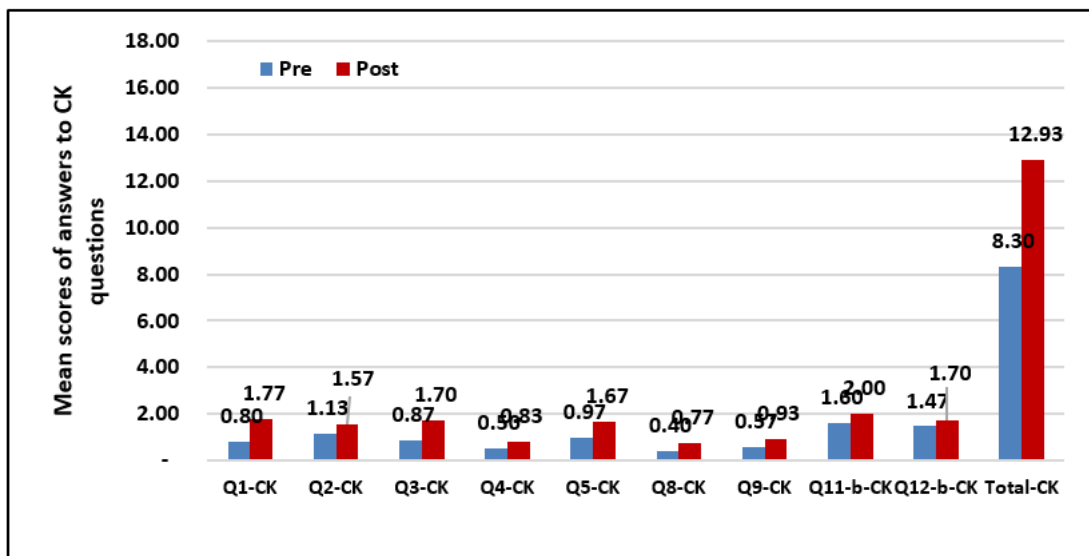
Figures 4 and 5 visualize the progress in undergraduates' conceptual knowledge and procedural knowledge of rational numbers, respectively. They demonstrate the notable improvement accomplished by each participant after the treatment compared to before it. Moreover, details of changing the mean scores of undergraduates' answers to each conceptual knowledge question are depicted in Figure 6.



**Figure 4.** Changes in Undergraduates’ Level of Conceptual Knowledge of Rational Numbers Before and After The Treatment



**Figure 5.** Changes in Undergraduates’ Level of Procedural Knowledge of Rational Numbers Before and After the Treatment



**Figure 6.** Mean Score of The Undergraduates’ Answers to Questions on Conceptual Knowledge of Rational Numbers Before and After The Treatment

In detail, Q1-CK and Q2-CK (see Table 3) aimed at assessing undergraduates’ knowledge of visualizing and applying the rational numbers concept as a measure. About Q1-CK, before the treatment, most participants began to plot number 1 on the number line, then represented  $1\frac{1}{7}$  on its right by a reasonable unprecise distance. They were careless about dividing the whole

distance into seven equal parts (i.e., the denominator of the given number). On the other side, after the treatment, the participants were able to recognize the unit fraction  $\frac{1}{7}$  as one-seventh of the distance between 0 and 1. Accordingly, they divided the number line into equal parts, each of distance  $\frac{1}{7}$ , wherein  $1\frac{1}{7}$  would come after  $\frac{7}{7}$  by moving one more step towards the right. Moreover, others first converted the mixed fraction  $1\frac{1}{7}$  to  $\frac{8}{7}$  and plotted it accurately.

Additionally, an evident improvement in responses to Q2-CK was observed after the treatment, which indicates changes in undergraduates' knowledge of the measure concept while applying it to solve problems. Initially, the participants focused on memorizing (then executing) this typical formula: [the number that lies in x of the distance between two rational numbers a and b, wherein  $a < b$  equals  $b - x(b-a)$ ]. It is commonly practiced by Egyptian teachers to operate such types of questions. Yet, several errors in the structure of this formula and, consequently, in its application were uncovered. On the contrary, after the treatment, most respondents exhibited an understanding of this formula. They first determined the distance between  $-\frac{5}{6}$  and  $-1\frac{1}{2}$ , then divided the result by 3 to get one-third of this distance to be subtracted from  $-\frac{5}{6}$  (see Figure 7).

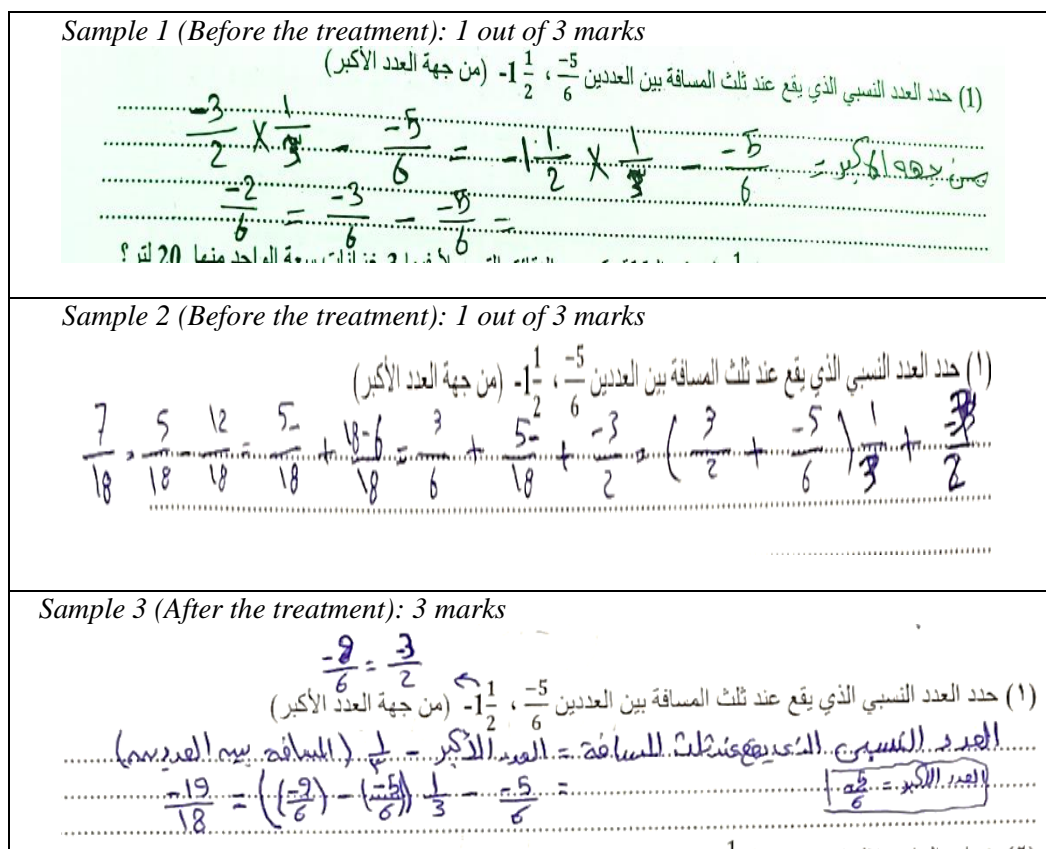


Figure 7. Samples of The Common Answer to Q2-CK Before and After The Treatment

Regardless, some mistakes remained, which appeared in the low mean score of the responses to Q2-CK even after the treatment (1.57). The undergraduates' confusion between the larger and smaller numbers when both were negatives was among the prominent mistakes. One potential cause of the continuation of such a particular mistake, even after the

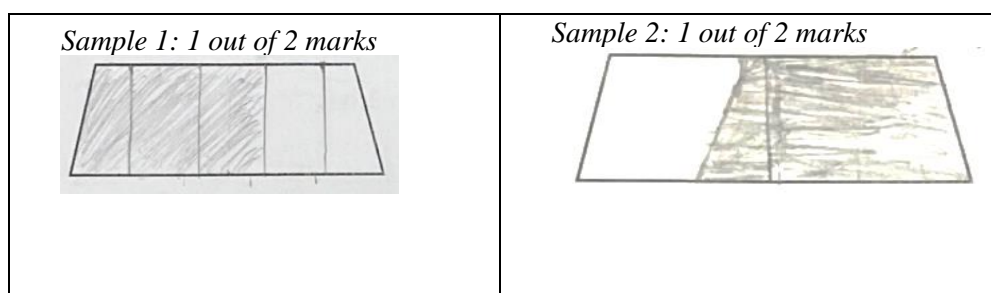
treatment, could be attributed to the primary focus on the contextual scenarios of positive data. Also, it might be related to the participants' less attention to mental visualization; alternatively, if they could imagine (mental image) the position of both numbers on the number line, they might not have maintained this mistake.

About Q3-CK, although Figure 6 displays a significant numerical change in the mean score of the responses to this question (from .87 to 1.70), the qualitative analysis of these responses did not reveal any conceptual difficulty. Before the treatment, the low score resulted from the inaccurate determination of the required. That is, instead of specifying the time to fill three tanks, the participants calculated this time for one tank of 20 liters (8 minutes); hence, they got 1 out of 2 marks. Others, who received a score of 0, decided to skip this question due to its context, arguing that it is less important than other context-free questions. After the treatment, most answers to Q3-CK were correct, wherein the undergraduates reported that 24 minutes would be needed to fill three tanks, each of 20 liters.

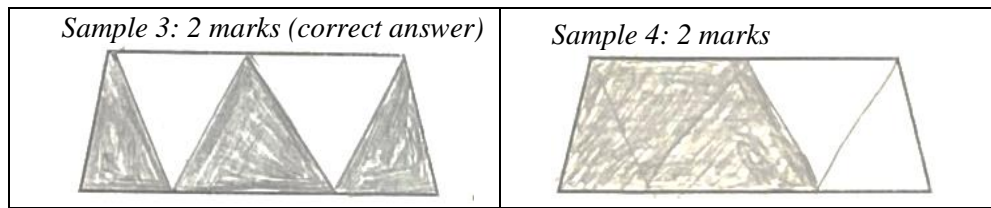
In that sense, it can be claimed that understanding the rational number as a ratio (e.g., flow rate) was adequate even with no interventions. This might be because the undergraduates get used to performing the cross-multiplication method to obtain the unknown value when two ratios are equal. This method is widely practiced in the Egyptian context across all levels of K-12 education.

Turning to Q4-CK, it attempted to clarify how undergraduates' understanding of the part-whole concept has altered after the treatment. This question carries a substantial value since the concept of part-whole designates the starting point to comprehend fractions (accordingly rational numbers) as it matches the intuitive experiences of pupils about fair sharing situations (Siemon et al., 2015). According to Figure 6, the mean score of the responses to Q4-CK increased from .50 (before) to .83 (after). This implies that the treatment had a slight impact on undergraduates' knowledge of representing the part-whole concept. Specifically, it was evident from the representations that they realized that splitting a shape into five equal regions and specifying three would result in obtaining  $\frac{3}{5}$  of this shape; nevertheless, it seems like they struggled with dividing the trapezoid into five equal parts (see Figure 8). In other words, the difficulty faced by the participants was deciding the proper way to divide the shape rather than understanding the part-whole concept.

Indeed, this finding affirms what Kolar et al. (2018) argued, wherein the shape used to represent rational numbers would affect teachers' ability to solve related tasks. Furthermore, shapes such as rectangles stayed more effective than others, with circles being intermediate and triangles being the least effective (Kolar et al., 2018). It is also consistent with Thurtell et al.'s (2019) study results about the challenges encountered by pre-service teachers when it comes to visualizing fraction concepts and related operations.







**Figure 8.** Samples of The Common Answer to Q4-CK After The Treatment

Based on the analysis of the responses to Q2-CK and Q4-CK, it could be concluded that although the treatment has a significant impact on the undergraduates' conceptual knowledge generally, a lack of their ability to visualize concepts of rational numbers was observed. Alternatively, they barely relied on the representation as a possible approach to assisting them in solving problems. Hence, this feature should be strengthened in future studies considering that the conceptual knowledge "is evidenced by multiple representations and the construction of connections between such representations." (Thurtell et al., 2019, p. 120), which is assumed to be accumulated mentally in forms of relational representations (Schneider & Stern, 2010).

The undergraduates' knowledge of rational numbers as operators and quotients was assessed (before and after the treatment) through three questions, namely Q5-CK, Q8-CK, and Q9-CK. While the operator indicates a multiplication operation, the quotient focuses on the division (Siemon et al., 2015).

The changes observed in undergraduates' answers to Q5-CK (.97 to 1.67) and Q8-CK (.40 to .77) were apparent (see Figure 6). Before the treatment, several responses to Q5-CK were incomplete and limited to translating the word unit to  $\frac{7}{7}$  or simply applying wrong algorithms (see Figure 9), which indicates insufficient understanding of the context of the problem and much focus on the procedures. On the contrary, after the treatment, most answers were accurate, wherein the participants reported that the unit would contain 21 dots; thus, 14 dots represent  $\frac{2}{3}$  of this unit.

About Q8-CK, most answers to this question involved the cross-multiplication rule, like Q3-CK. Accordingly, after the treatment, the respondents highlighted that if  $3\frac{3}{4}$  rolls were enough to decorate one room, 13 rolls would cover about 3.5 ( $13 \div 3\frac{3}{4}$ ) rooms. Yet, it is worth noting that although the participants relied on applying the cross-multiplication rule even before the treatment, understanding the data embedded in the context was apparent after it. This was exposed when some respondents were invited to justify their answers, which was a part of validating the collected data and getting more insights into it. In the interview before the intervention, they replicated same algorithms to illustrate the result. Contrarily, after the treatment, they argued that if 3.75 rolls were sufficient to decorate a room, and considering that 13 rolls equate to about three times 3.75, thus the answer would be close to 3. Such reasoning signifies a transition from procedural knowledge to conceptual understanding.

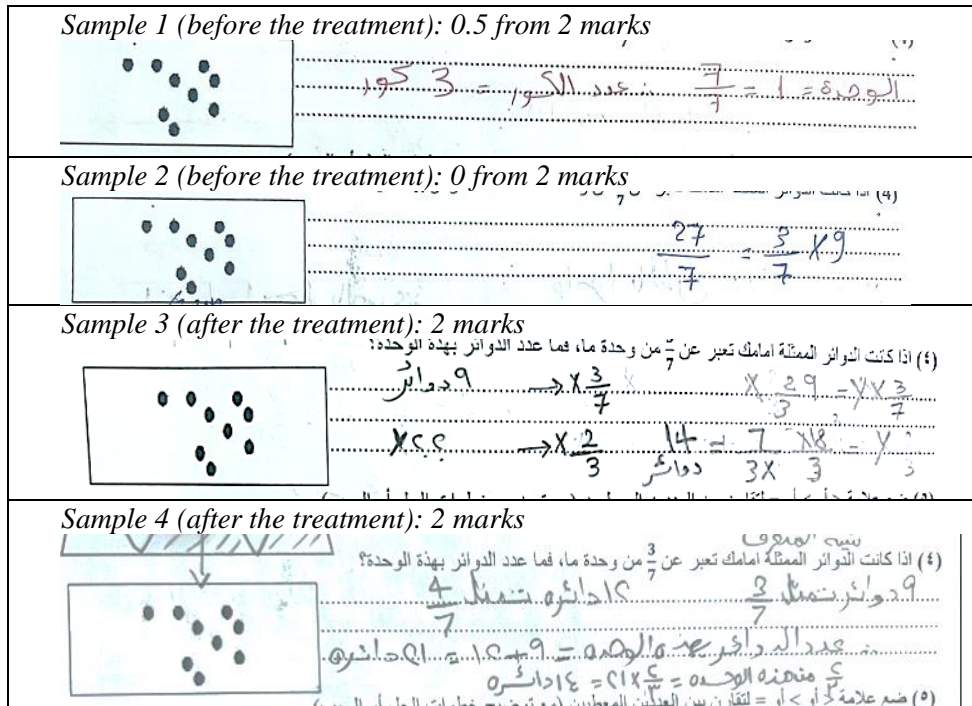


Figure 9. Samples of The Common Answer to Q5-CK Before and After The Treatment

About Q9-CK, the mean score of the undergraduates' responses to this question was raised from .57 to .93. Most respondents reported that  $\frac{9}{8}$  would be changed to  $\frac{9}{8 \times 12}$  if we increased the denominator by three folds and divided the numerator by 4. Yet, few of them discussed that  $\frac{9}{8 \times 12}$  determines  $\frac{1}{12}$  of  $\frac{9}{8}$ . Also, the multiplier “three folds” was commonly misinterpreted, even after the treatment (see Figure 10).

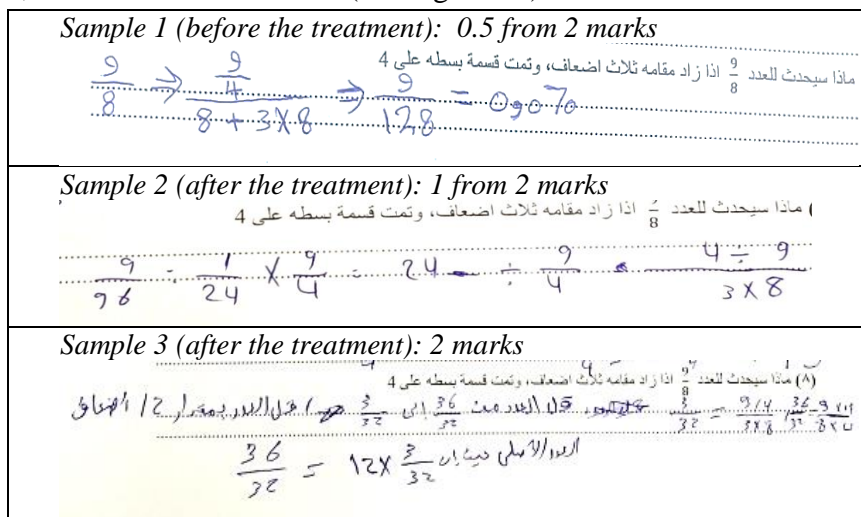


Figure 10. Samples of The Common Answer to Q9-CK Before and After The Treatment

The remaining questions on conceptual knowledge are Q11(b)-CK and Q12(b)-CK. They aimed to determine the capability of undergraduates to speculate pupils' misconceptions about the relationship between rational numbers and whole numbers (Omoruan & Osadebe, 2020; Siegler & Lortie-Forgues, 2017). It often emerges when dealing with two rational numbers of

different denominators (Putra, 2019; Rizos & Adam, 2022), and Putra et al. (2023) identified this relationship as an essential factor contributing to understanding rational numbers.

As displayed in Figure 6, the mean scores of the responses to both questions were increased from 1.60 to 2 and from 1.47 to 1.70 for Q11(b)-CK and Q12(b)-CK, respectively. On the one side, this implies that the respondents were able to anticipate pupils' misconceptions, even with no interventions. On the other side, it is evident that the treatment primarily promoted their awareness of addition and subtraction operations compared to multiplication and division. This result matches Putra's (2019) findings wherein all the pre-service Danish teachers engaged in this study correctly performed adding and subtracting rational numbers. Also, Slattery and Fitzmaurice (2014) declared that prospective teachers still make mistakes when it comes to multiplying and dividing fractions due to their belief that multiplication "increases" while division "decreases" the value.

For Q11(b)-CK, after the treatment, all participants explained that the pupil mistakenly answered  $\frac{1}{2} + \frac{2}{3}$  by adding the numerators (1+2) and denominators (2+3) to get  $\frac{3}{5}$ . Similarly,  $\frac{3}{4}$  was obtained by determining 4-1 as the numerator and 7-3 as the denominator. That is, the pupil added and subtracted the given numbers based on their position (see Figure 11). Hence, the undergraduates, in their writings, recommended orienting pupils to check whether the rational numbers have common denominators before adding or subtracting them as a possible approach to resolve such a misconception. This implicitly conveys the respondents' understanding of the holistic part-whole relationship between the numerator and denominator of a rational number, which differs from operations on whole numbers (Chinnappan & Forrester, 2014). Yet, unlike Putra's (2019) study participants, respondents of the present study did not document this relationship in their writings.

Additionally, as displayed in Figure 11, when it came to the multiplication and division operations, several respondents reported that "this pupil disbelieved the calculator since he used to get greater - than both the multiplicand and multiplier- when multiplying and a smaller- than the dividend- number when dividing two whole numbers, which was not the case in this example. The example contained a fraction." This is recognized to be a common bias connected to the multiplication and division of rational numbers (Christou & Vamvakoussi, 2021, as cited in Rizos & Adam, 2022), wherein multiplying fractions is a partitive process, leading to a product that is smaller than the original numbers (Mack, 2001, as cited in Chinnappan & Forrester, 2014). Based on this, the participants recommended training pupils to verify the calculator results by executing standard algorithms (e.g., dividing two rational numbers = multiplying the first rational number by the reciprocal of the second number). Also, pupils should pay attention to the numbers being multiplied or divided; to what set do these numbers belong?

Sample 1: an answer to Q11(b)-CK

شرح كيف قام التلميذ بحل المسألتين (ما الخطأ الذي وقع فيه)

قام التلميذ بحل المسألة الأولى كالتالي:  $\frac{1}{2} + \frac{2}{3} = \frac{1+2}{2+3} = \frac{3}{5}$  ...

قام التلميذ بحل المسألة الثانية كالتالي:  $\frac{3}{4} = \frac{4-1}{7-3} = \frac{3}{4}$  ...

الخطأ هو أنه لم يقوم بتوحيد المقام. وبإضافة المقامات ...

الخطأ هو عدم توحيد المقامات. وجمع المقامات ...

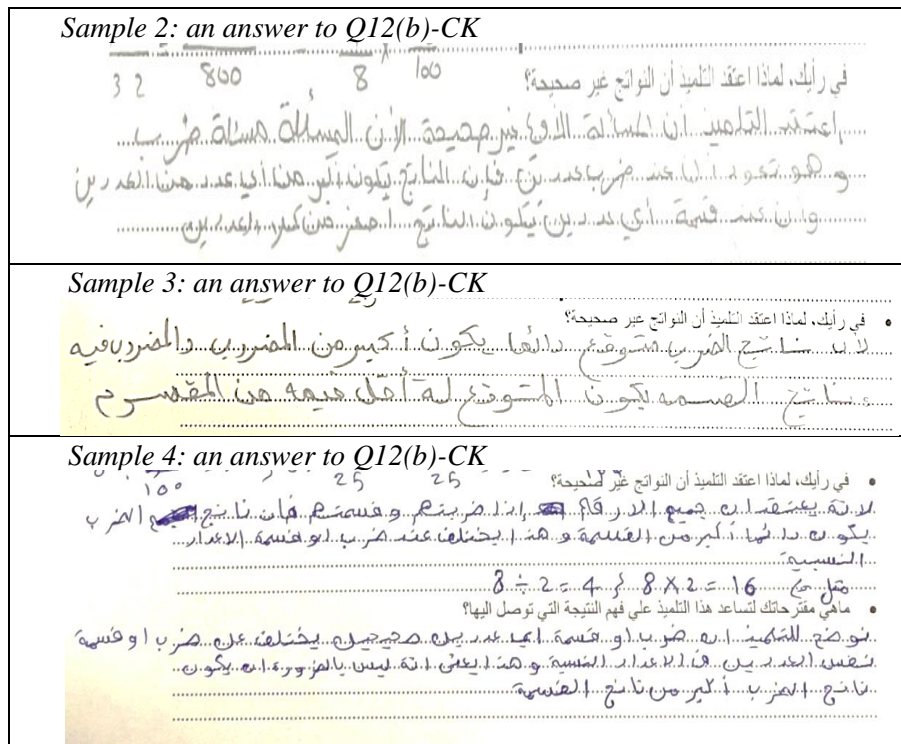


Figure 11. Samples of The Common Answer to Q11(b)-CK and Q12(b) After The Treatment

Like Figure 6, the following figure exhibits changes in the mean scores of undergraduates' answers to each question measuring procedural knowledge after exposing to the treatment:

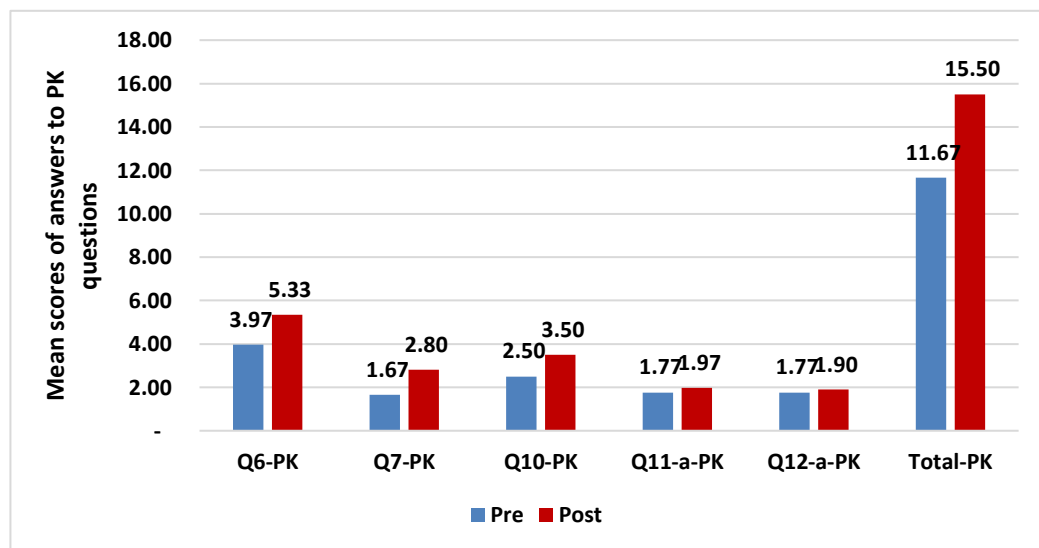


Figure 12. Mean Scores of The Undergraduates' Answers to Questions on Procedural Knowledge of Rational Numbers Before and After The Treatment

Upon comparing Figure 6 with Figure 12, it can be contended that undergraduates exhibited a higher level of procedural knowledge of rational numbers compared to their conceptual knowledge before the treatment. Quantitatively, they attained 11.67 out of 17 (68.6%) in the procedural knowledge test, while 8.30 out of 18 (46.1%) determined their initial

level of conceptual knowledge. Similarly, after the treatment, they gained adequate procedural knowledge compared to conceptual knowledge. Numerically, the mean scores of undergraduates' answers to procedural and conceptual knowledge post-tests advanced to 15.50 (91.2%) and 12.93 (71.8%).

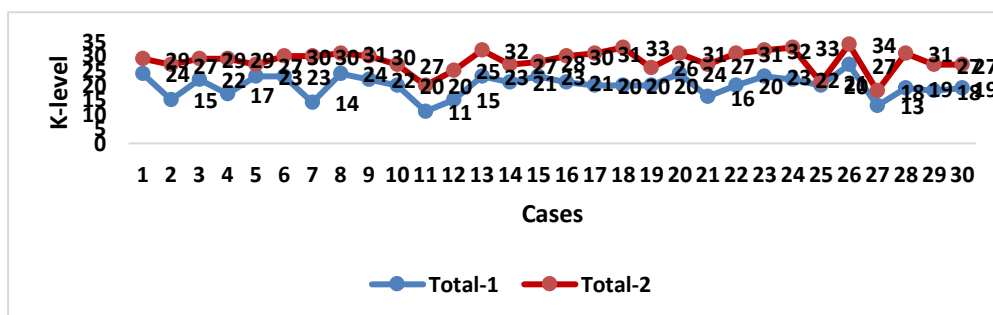
This finding aligns with the results of several studies (e.g., Kashim, 2016; Khashan, 2014; Putra et al., 2023; Salifu, 2021), which revealed that most learners possess sufficient procedural knowledge of rational numbers in comparison to their conceptual knowledge. Al-Mutawah et al. (2019) declared that the potential cause for this could be associated with exposure to similar problem types throughout their K-12 education, as exemplified by the situation in Bahrain. Also, Khashan (2014) and Kashim (2016) attributed this to emphasizing rote learning and the mechanical application of procedures while disregarding the underlying conceptual principles. Likewise, Tobias (2009, p. 33) asserted that “the knowledge that elementary and middle school teachers bring to the classroom is procedurally based and largely misunderstood.”

In addition to the above analysis and about undergraduates' responses to procedural knowledge questions after the treatment, it is crucial to highlight the mutual relationship between conceptual and procedural knowledge (see the literature review). Such interaction between both constructs was apparent in multiple post-responses. For example, while answering Q6, instead of performing the standard algorithms of determining the common denominator to compare two rational numbers (Tobias, 2009), several participants justified that  $\frac{-4}{7}$  is smaller than  $\frac{3}{5}$  because the positive number is always greater than any negative one. Also,  $\frac{6}{5} > \frac{3}{4}$  since  $\frac{6}{5} > 1$  (its numerator is greater than the denominator) while  $\frac{3}{4} < 1$ . Similarly, to compare 20% with  $\frac{5}{8}$ , some participants reported that  $20\% < \frac{5}{8}$  since the former is .20 while the latter is about .625. They converted both numbers to decimals, which implies a conceptual strategy (Lemonidis & Pilianidis, 2020).

In general, and according to the entire previous analysis, one can argue that training undergraduates to implement REACT strategies supported their knowledge of rational numbers (see Table 6 and Figure 13). Moreover, the magnitude of the treatment's impact was assessed by computing Cohen's d for the paired samples t-test. Cohen's  $d = (\mu_1 - \mu_2) / \sigma = 8.47 / 3.12 = 2.71$ ; this indicates a significantly large effect size (Cohen, 1988).

**Table 6.** Result of Paired Samples T-tests Undergraduates' Level of Knowledge of Rational Numbers Before and After The Treatment

Test	Mean	Std.Deviation	t-value	df	Sig. (2-tailed)
Total1-Total2	-8.47	3.12	-14.89	29	.000



**Figure 13.** Changes in undergraduates' level knowledge of rational numbers before and after the treatment

The following research question to be answered is RQ3, which aims to examine the potential correlation between conceptual and procedural knowledge of rational numbers among the participants. For this, the Pearson correlation coefficient was computed, and the results are detailed in Table 7. It reveals a statistically significant “moderate” positive correlation between the two variables,  $r(28) = .48, p = .007 > .05$ .

**Table 7.** Results of correlation test between undergraduates’ conceptual knowledge and procedural knowledge of rational numbers

		CK2	PK2
CK2	Pearson correlation	1	.481
	Sig.		.007
	N	30	30

In essence, and based on the statistical results, it can be argued that individuals who possess solid conceptual knowledge are reasonably anticipated to maintain an adequate level of procedural knowledge when it comes to rational numbers. Such an argument corresponds to what was discussed earlier regarding the participants who responded to procedural knowledge questions through a conceptual lens instead of using familiar algorithms. To a certain extent, this result mismatches what was revealed by Khashan's (2014) and Salifu's (2021) analyses regarding the weak positive nonsignificant correlation between conceptual and procedural knowledge of rational numbers among mathematics teachers. As reported, "a teacher who has a good procedural knowledge does not necessarily have good conceptual knowledge, and vice versa". (Khashan, 2014, p. 195). The discrepancy in findings between these studies and the present study could be attributed to the effect of the treatment.

Khashan’s (2014) and Salifu’s studies (2021) determined the actual status of knowledge of rational numbers at a specific moment among in- and pre-service mathematics teachers, respectively, with no interventions employed. In contrast, the present study examined the correlation between conceptual and procedural knowledge of rational numbers among undergraduates after subjecting them to a targeted treatment. This confirms what was argued by Khashan (2014), in which the gap between teaching to master mathematical procedures and teaching for conceptual understanding, which might be a cause for such weak correlation, could be overcome by context learning environments. Alternatively stated, one potential approach to achieve a balance between learners' conceptual and procedural knowledge of rational numbers is teaching them through contextual strategies.

Moving to the last research question; it focuses on exploring how the training affected undergraduates’ capability to create contextual situations of rational numbers. In line with this objective, the researcher asked the participants to propose scenarios two times throughout the treatment. The first time was after two weeks from the treatment duration, while the second happened at the end. As outlined in the methodology, during the initial stage, the participants were encouraged to suggest simple situations that they believed to represent the concept of rational numbers effectively to their prospective pupils. Conversely, at the end of the training, they were asked to write five situations that aligned with each interpretation of rational numbers that they acquired through the training. The results are detailed in Tables 8 and 9.

Overall, initially, the scenarios proposed by the undergraduates varied across four distinct categories: buy/sell (44%), religious (23%), fair share distribution (23%), and construction projects (10%). Moreover, the quotient concept was predominant throughout these scenarios (43%). As detailed in Table 8, when the undergraduates first thought of the contextual situations relevant to rational numbers, the daily social scenarios of buy/sell (44%) intuitively came to their minds to express the practicability of rational numbers. The prevalence of buy/sell context aligns with Lee's (2012) research findings. According to Lee's (2012) research, approximately 50% of pre-service teachers suggested contextual problems related to money or time. Also, around 32% of these problems incorporated money-related scenarios such as purchasing goods and selling items, which resembles the buy/sell situations proposed by the present study participants.

**Table 8.** The contextual situations proposed by the undergraduates at the beginning of the treatment.

	Part-whole	Operator	Quotient	Ratio	Measure	
<b>Religious</b>	0	5	0	2	0	<b>7 (23%)</b>
<b>Buy/sell</b>	0	0	6	3	4	<b>13 (44%)</b>
<b>Fair share distribution</b>	0	0	7	0	0	<b>7 (23%)</b>
<b>Construction projects</b>	0	0	0	0	3	<b>3 (10%)</b>
	<b>0</b>	<b>5 (17%)</b>	<b>13 (43%)</b>	<b>5 (17%)</b>	<b>7 (23%)</b>	<b>30 (100%)</b>

Initially, the context of buy/ sell was used to convey quotient, measure, and ratio concepts (see Table 8). For instance, the participants proposed scenarios such as “If I would like to purchase  $\frac{1}{3}$  kilo of an item, given the price of one kilo, I would simply divide this price by 3.”; “What is the total weight of the tomatoes, carrots, cucumbers, and green peppers if we purchased half a kilogram of each?”; and “If a father bought 4 kilos of apples and 2 kilos of oranges, the ratio between both quantities would be 4: 2”. These were the commonly offered situations of buy/sell that maintain concepts of quotient, measure, and ratio, respectively.

Situations related to religion and fair share distributions stayed in second place in terms of prevalence of context. While the former transmitted concepts of operator (e.g., “According to Islamic rules, a wife is entitled to a quarter share of her husband's estate upon his passing if she has no children) and ratio (e.g., “When we divide the inheritance between boys and girls according to Islamic rules, the male gets the share of the two females.”); the latter employed the quotient concept (e.g., “If the father distributed 600 pounds among his three children, each kid would obtain  $\frac{600}{3}$  pounds.”). Additionally, 10% of the participants suggested situations related to construction and building to express the measure concept, such as the following: If two house buildings are 5.2 and 4.3 meters from the beginning of a street, and we would like to place a lamp at a one-quarter distance between them, at what distance should the lamp be located from the beginning?

Table 9 implies a comprehensive summary of the findings obtained from analyzing the proposed situations at the end of the treatment. It is worth noting that the participants were encouraged to present five circumstances that match the interpretations of the rational numbers. However, out of the 150 (30 participants \* 5 situations) situations proposed, 11 lacked meaning and were consequently excluded from the analysis. Hence, Table 9 exhibits a total of 139 situations.

**Table 9.** The contextual situations proposed by the undergraduates at the end of the treatment.

	Part- whole	Oper ator	Quotie nt	Ratio	Measu re	Part- whole/q uotient	Quotie nt/mea sure	
<b>Geometric</b>	0	0	0	0	0	0	15	<b>15 (11%)</b>
<b>Distances/ travelling/ duration</b>	0	0	0	0	12	0	0	<b>12 (9%)</b>
<b>Construction projects</b>	0	0	0	0	18	0	0	<b>18 (13%)</b>
<b>Fair share distribution</b>	4	0	11	0	0	26	0	<b>41 (29%)</b>
<b>Ratio distribution</b>	0	0	0	15	0	0	0	<b>15 (11%)</b>
<b>Components/ recipes</b>	0	0	0	15	0	0	0	<b>15 (11%)</b>
<b>Scientific</b>	0	2	0	0	0	0	0	<b>2 (1%)</b>
<b>Rates of consumption (salaries, expenses)</b>	0	11	0	0	0	0	0	<b>11 (8%)</b>
<b>Rates of consumption (individuals &amp; family utilities)</b>	0	6	0	0	0	0	0	<b>6 (4%)</b>
<b>Rates of consumption (technologies)</b>	0	4	0	0	0	0	0	<b>4 (3%)</b>
	<b>4 (3%)</b>	<b>23(17 %)</b>	<b>11 (8%)</b>	<b>30 (21%)</b>	<b>30 (21%)</b>	<b>26 (19%)</b>	<b>15 (11%)</b>	<b>139 (100%)</b>

Surprisingly, after the treatment, the participants' responses exposed eight contexts through which the concept of rational numbers could be incorporated. These were arranged in a descending order based on their prevalence as follows: Fair share distribution (29%); Rates of consumption (15%); Construction projects (13%); Geometric (11%); Ratio distribution (11%); Components/ recipes (11%); Distances/ traveling/ duration (9%); and Scientific (1%). Moreover, in addition to the five interpretations of rational numbers that emerged across the proposed scenarios, the participants indicated the possibility that some scenarios could encompass two interpretations, namely Part-whole/ quotient, as well as Quotient/measure (see Table 9).

Two of those eight contexts (i.e., fair share distribution and construction projects) resembled what the participants initially proposed (see Table 8); still, six new contexts emerged. Generally, most of the resultant scenarios are consistent with Kadir et al.'s (2020) findings wherein the prospective teachers used contexts of distributions, weights of objects, lots of objects, and lengths measurement to represent a fractional problem contextually. In detail, about the new contexts, scenarios of rates of consumption (15%) were proposed by the participants to capture the operator concept essentially as expressed in the following instances:

... When a company pays a monthly electricity bill of 10000 pounds, one-fourth of this amount will be paid weekly, as calculated by multiplying 10000 by  $\frac{1}{4}$ , resulting in 2500 pounds.

**(Expenses, Operator)**

... If a family of four kids consumes 40kg of rice per month, their weekly consumption will be 10kg, which equals one-fourth of the monthly amount. **(Family utility, Operator)**

... If we knew that Apple company produces 2000 iPhones monthly, thus their weekly production would be  $\frac{1}{4} * 2000 = 500$  iPhones. **(Technologies, Operator)**

Also, geometric (11%), ratio distribution (11%), and components/ recipes (11%) contexts maintained a consistent level of popularity across responses. About the geometric context, although the participants proposed geometric-related scenarios to exhibit the quotient concept



basically, they affirmed that these could define the measure concept too (e.g., lengths and perimeters of polygons). Accordingly, these geometric-related scenarios were classified under the Quotient/measure, as displayed in Table 9. On the other hand, both ratio distribution and components/recipe-related scenarios were employed to exemplify the ratio concept. Some of the participants' proposals were as follows:

... If a square-shaped mirror has a 48 cm perimeter, then each side of this mirror will measure  $48/4 = 12\text{cm}$ . (*Geometric, Quotient/measure*)

... If a boy got two apples and his sister obtained five, the ratio between what both had would be 2 to 5. (*Ratio distribution, Ratio*)

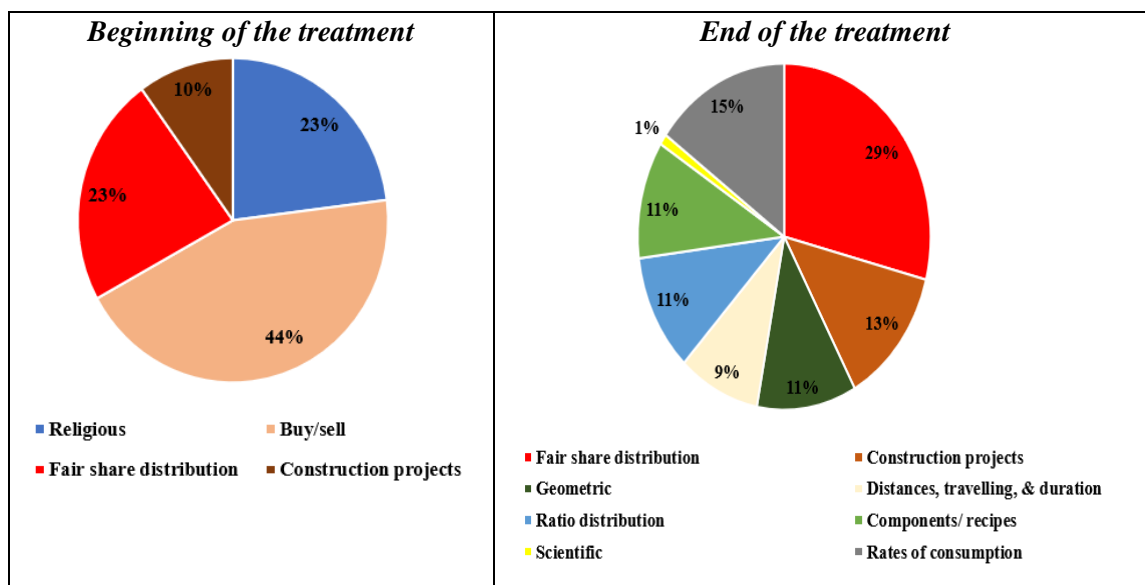
... To make a chocolate cake, we need a spoonful of cocoa powder and four spoons of milk powder. Therefore, the ratio of cocoa powder to milk powder is 1:4. (*Recipes, Ratio*)

The last two categories of responses were the distances/traveling/duration (9%) and scientific (1%) contexts, used to characterize measure and operator concepts, respectively. Some of the related scenarios suggested by the participants were as follows:

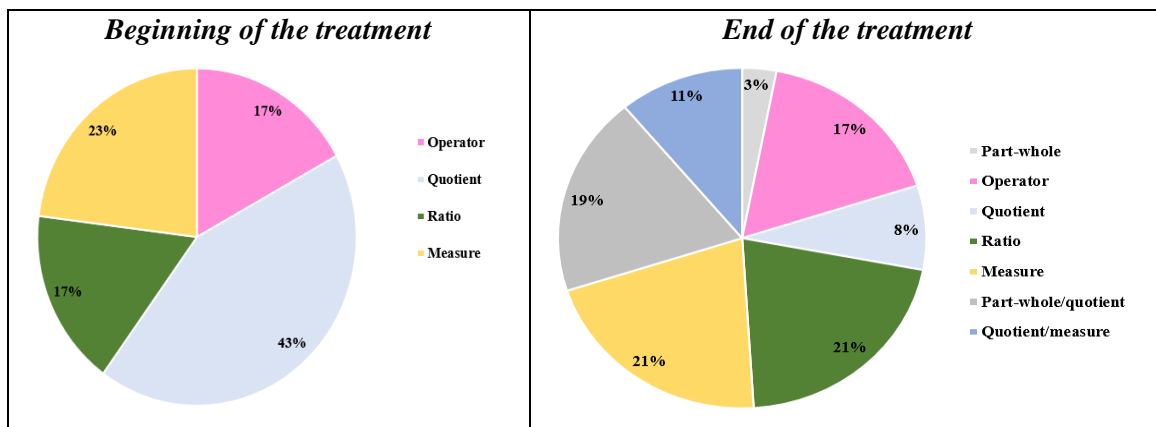
... If a car came to a stop after traveling a 10km distance between Tanta city and Almahala city, which are 30km apart, then the fraction  $10/30 = 1/3$  represents one-third of the distance where the car stopped. (*Traveling, Measure*)

... If you knew that the weight of an object on the moon equals  $1/6$  this weight on the earth, and someone's weight was 48kg on the earth, then his weight on the moon would be  $1/6 * 48 = 8\text{ kg}$ . (*Scientific, Operator*)

The above analysis explains how the training affected the undergraduates' capability to create contextual situations of rational numbers. This appeared in (a) the variability of proposed contexts and (b) the inclusion of all rational number interpretations, as depicted in Figures 14 and 15.



**Figure 14.** Variability of contextual situations proposed by the undergraduates before and after the treatment



**Figure 14.** The interpretations of rational numbers embedded in the undergraduates' proposed situations before and after the treatment.

Such progression of undergraduates' capability to suggest different contextual scenarios linked to rational numbers responded to matters made by Paredes et al. (2020), Altay et al. (2020), and Putra et al. (2023) regarding the significance of equipping prospective mathematics teachers with the skills to generate contextual situations throughout their university education.

Overall, the treatment adopted in this study highlighted how prospective teachers trained to explore concepts and procedures of rational numbers and contextualize relationships among these. It further engaged them in meaningful learning with real-life experiences through which possible connections between mathematics and daily experiences could be seen, rather than the repetition of algorithms that are mostly practiced in traditional classes. Notably, executing REACT strategies while learning mathematics supports learners in employing mathematical concepts to solve unfamiliar problems and draw conclusions. It also encourages them to actively partake in learning processes and allows them to present their ideas and concepts from different viewpoints (Jelatu et al., 2018). This refers specifically to the C strategy, wherein the practiced collaboration among learners through the active manipulation, discussion, and justification of mathematical ideas from multiple perspectives within the same group and among different groups leads to a more profound understanding of mathematics (Musyadad & Avip, 2020; Supandi et al., 2016).

Additionally, engaging prospective mathematics teachers in contextual education would help them design a good lesson that activates pupils' awareness of the connection between their prior experiences and new mathematical concepts. This highlights the importance of similar training in growing teachers' awareness, wherein students may possess prior knowledge that applies to a new learning scenario; still, they may disregard its significance (Crawford, 2001). Generally, in the present study, the undergraduates experienced a constructivist learning model that challenges the conventional one they mainly encounter. This new model, exemplified by the treatment, contradicts their perception that only teachers are responsible for imparting knowledge.

This explanation matches those of other studies. For example, Selvianiresa and Prabawanto (2017) reported that implementing contextual teaching and learning promotes learners to establish connections between the knowledge they acquire and their personal experiences, resulting in a deeper understanding and greater significance of their newfound knowledge. Furthermore, the research conducted by Abebe et al. (2023) affirmed that engaging

learners in lessons that are designed based on real-life scenarios significantly contributes to developing their conceptual knowledge.

Yet, it is necessary to interpret the findings of this study within the specific context in which it was conducted. It should be noted that implementing the same intervention may not yield the same benefits in other countries that prioritize contextual education in their teacher training programs. Additionally, it is worth mentioning that the study was limited to a group of undergraduate students who had not yet taken any pedagogical courses. As a result, their initial proposal of contextual situations that could be introduced to future classes may have been relatively superficial, which would not case if the study had included in-service or expert teachers as participants.

## **CONCLUSION AND RECOMMENDATIONS**

The present study acknowledged the priority of prospective teachers to possess a deep understanding of the mathematics they are going to teach and to have the capability to design the appropriate tasks to be implemented effectively in future classrooms. In other words, attaining the desired learning outcomes in school mathematics is contingent upon improving students' understanding, which is greatly influenced by the quality of instruction they receive (Ball, 1990). Thus, concerns about teacher education hold notable significance. This stands even more crucial in light of Kolar et al.'s (2018) assertion that the formal knowledge acquired through teacher education may not always be sufficient for effectively teaching essential mathematical concepts in classrooms.

On the one hand, the study emphasized the need to specify the undergraduates' conceptual and procedural knowledge of rational numbers, as this knowledge reflects aspects of their abilities to teach future pupils. On the other hand, it attempted to strengthen this knowledge by training them to execute the contextual-based strategies REACT, which, in turn, would affect their capacity to create contextual situations of rational numbers. Furthermore, it awakens their awareness of the connection between the mathematical concepts and procedures and the daily activities that might be faced by their pupils, which ultimately would positively affect pupils' understanding.

Considering the study results, it can be argued that implementing REACT strategies positively influenced the undergraduates' conceptual and procedural knowledge of rational numbers. It also triggered their awareness of the variability of contextual situations that could be employed to approach the different interpretations of rational numbers (i.e., part-whole, operator, quotient, ratio, and measure). In that sense, the study responds to the challenge of preparing prospective teachers to teach rational numbers effectively, which is especially crucial due to the absence of contextual activities in most school textbooks that often stress mathematical procedures (Hurrell, 2021; Wijaya et al., 2015).

Practically, this study suggested introducing REACT strategies to educators as a possible approach to enhance undergraduates' knowledge of school mathematics by connecting abstract concepts to practical situations. Hence, upcoming studies might investigate how implementing such strategies could be practiced in instructing other contents like Algebra and Geometry, which constitute primary areas of the middle school mathematics curriculum. Moreover, further inquiries might focus on evaluating the quality of the contextual scenarios proposed by the prospective teachers across various domains of mathematics and the extent to which these

scenarios align with the ones presented in the official textbooks. Also, whether this quality differs by the years of experience; put differently, exploring differences in contextual situations between preservice and in-service mathematics teachers could be a potential area of research.

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