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Exploring Students' Difficulties in Solving Nonhomogeneous 2nd Order Ordinary Differential Equations with Initial Value Problems

Yunika Lestaria Ningsih[1], Anggria Septiani Mulbasari[2]
[1,2]Universitas PGRI Palembang
[1]Correspondence Adress; yunikalestari@univpgri-palembang.ac.id

Abstract

This research aims to explore students' difficulties in resolving Nonhomogeneous 2nd Order Ordinary Differential Equations with initial value problems. The method that can be used to solve this equation is the undetermined coefficient and the Laplace transformation. This research is used descriptive method. The subjects of this study were 73 students in the second year of the Mathematics Education. Data is collected through tests and interviews. Data were analyzed descriptive qualitative. The results of data analysis show that in undetermined coefficient method, students difficult in determining the particular solution of non-homogeneous second-order ordinary differential equations. This is due to student errors in the first step especially in determining the characteristics equation. Whereas, for the Laplace transformation method, students most difficulties are in the step of solving the subsidiary equation. This is due to the weakness of students in completing arithmetic operations in the form of fractions and partial fractions.

Keywords: Ordinary differential equations; undetermined coefficient method; Laplace transformation.

INTRODUCTION

Ordinary Differential Equations (ODE) with Initial Value Problems (IVP) are the sub-topics of the Differential Equation (DE) course. This course must be attended by students of the Mathematics Education at the Universitas PGRI Palembang. Boyce & DiPrima (2001), states that DE is an equation in which there are one or several derivatives. DE is one of the advanced mathematical concepts that are widely used in applied mathematics applied in the fields of physics, chemistry, biology, engineering, social and psychology (Boyce & DiPrima, 2001; Machin, Diaz, & Trigo, 2012; Khotimah & Masduki, 2016 ; Vajravelu, 2018).

One of the objectives learning of DE is students is able to complete DE with initial value problems. But this goal is difficult to achieve. DE with this initial value problem can be solved in several ways, including the auxiliary equation (or characteristics equation) method and the Laplace transformation method. Many students experience difficulties in learning DE. This statement was reinforced by Prawoto, Hartono, & Fardah (2018) who stated that many students scored below the average for DE courses, especially on the topic of Non-Homogeneous 2nd Order ODE. Carstensen & Bernhard state that many students have difficulty understanding Laplace transformation (Holmberg & Bernhard, 2008). Furthermore, Ningsih & Rohana (2018) student understanding on the topic of ODE is still at the lowest stage of APOS theory.

According to Rasmussen (2001) students find it difficult to understand the concept of completion or solution in DE because they are accustomed to understanding a solution in the form of a certain number not a solution in the form of a function. Furthermore, Valcarce & Diaz (Ningsih & Rohana, 2018) states that in order to be able to determine DE solutions students must understand some basic concepts such as exponential, logarithmic, derivative and integral

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Al-Jabar: Jurnal Pendidikan Matematika
Vol. 10, No. 2, 2019, Hal 233 - 242

functions. Students who cannot understand the concept will certainly have difficulty in determining the solution of DE.

The difficulties of students in solving mathematical problems including completing DE are caused by students' lack of understanding both conceptual understanding and procedural understanding. This difficulty can be seen from the mistakes made by students in solving mathematical problems. According to Hiebert (Kusuma & Masduki, 2016) students who have a thorough understanding of a mathematical concept will be able to evaluate their mistakes. This means that the higher the student's understanding, the lower the level of error.

The analysis of student difficulties in learning mathematics in higher education has been carried out by researchers. The analysis included analysis of student difficulties in Calculus I learning (Rahmawati, 2017), analysis of student difficulties in learning Advanced Calculus (Apriandi & Krisdiana, 2016), analysis of student difficulties in solving DE problems (Naisunis, Taneo, & Daniel , 2018) and analysis of student difficulties in learning the Second Order Non-Homogeneous ODE (Prawoto, Hartono, & Fardah, 2018).

Therefore, this research aims to explore students' difficulties in resolving Nonhomogeneous 2nd Order Ordinary Differential Equations with initial value problems. The DE topics examined in this study are limited to Undetermined Coefficient method and Laplace transformation.

THE RESEARCH METHODS

This research is a descriptive study, which aims to describe the difficulties of the 5th semester students of the Mathematics Education at the Universitas PGRI Palembang Academic Year 2018/2019 in completing the 2nd Order Non-Homogeneous ODE with the Undetermined Coefficient method and the Laplace transformation. The research subjects were 73 students in the 5th semester of the Mathematics Education at the Universitas PGRI Palembang Academic Year 2018/2019. Data collection techniques in this study were tests and interviews. The test given is in the form of an essay about the solution to the Non-Homogeneous 2nd Order ODE. Tests given to students can be seen in Figure 1.

> Misalkan diketahui MNA : $y'' + y = x$, $y(0) = 1$, $y'(0) = -2$. Buktikan apakah $y(x) = x + cosx - 3sinx$ adalah solusi dari MNA dengan menggunakan metode *Undetermined Coefficient* dan metode transformasi Laplace

Figure 1. Test of non-homogeneous 2nd order ODE

Nonhomogeneous second order ODE has a solution $y(x) = y_h(x) + y_p(x)$, with $y_h(x)$ is a solution for homogeneous ODE and $y_p(x)$ is a trial solution that does not contain any arbitrary constants. In this question the students are asked to complete the second order ODE nonhomogeneous with the undetermined coefficient and Laplace transformation method. The solution to the problem according to (Kreyszig, Kreyszig, & Norminton, 2011) is described as follows, for the undetermined coefficient method: this method consists of three stages, namely

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a) determining a general solution of homogeneous ODE, at this stage students must be able to make ODE in the form of operator D, determine the characteristic equation of ODE, and determine the roots of the characteristic equation, b) determine the solution y_p nonhomogeneous ODE, at this stage students must be able to determine the undetermined coefficient function that corresponds to ODE, reduce the function and substitute derivative functions to ODE, and c) substitution of initial value problems to get a particular solution.

The Laplace transformation method consists of 3 stages, namely a) making a subsidiary equation, students must be able to change nonhomogeneous ODE in equations containing Laplace transforms, b) determine the solution of the subsidiary equation using partial fractions, and c) determine the inverse of the transformation Laplace.

The data analysis technique in this study was used descriptive qualitative based on Sugiyono (2008). The data analysis techniques in this study are 1) organizing students' difficulties based on the results of tests and interviews, 2) describing data on student difficulties in the category (type) difficulties, 3) determining the difficulties of students and 4) making conclusions and telling about presenting data to others.

THE RESULTS OF THE RESEARCH AND THE DISCUSSION

The test was held on January 10, 2019. The test was attended by 73 students in the 5th semester of the mathematics education Universitas PGRI Palembang Academic Year 2018/2019. Data on student difficulties in solving non-homogeneous second-order ODE problems with IVP can be seen in Table 1.

Tabel 1. Description of Student Difficulties in Solving Nonhomogeneous 2nd Order ODE with IVP

Method	Type of difficulty	Total	Percentage
Undetermined Coefficient	Stage a: determining a general solution of homogeneous ODE	45	61,64%
	Stage b: determine the solution y_p	55	70,34%
	Stage c: substitution of initial value problems to get a particular solution	56	76,71%
Laplace Transformation	Stage a: making a subsidiary equation	48	65,75%
	Stage b: determine the solution of the subsidiary equation using partial fractions	66	90,41%
	Stage c: determine the inverse of the Laplace transformation	66	90,41%

Based on Table 1, it is known that for method 1 students who have difficulty completing stage a are 45 people or 61.64%. To complete this stage students must determine the form of ODE using operator D, determine the characteristics of the characteristics, solve the characteristic equation and determine the general solution of ODE that matches those roots.

The initial form ODE $y'' + y = x$ is a nonhomogeneous second order ODE that can be solved by the undetermined coefficient method. The first step is to assume homogeneous ODE $y'' + y = 0$, then change the initial ODE to the D operator as follows $D^2y + y = 0$. Many

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Al-Jabar: Jurnal Pendidikan Matematika
Vol. 10, No. 2, 2019, Hal 232 - 242

students cannot use D operator correctly. Students' difficulties in forming ODE with operator D show that the students' ability of understanding concepts and symbols about derivatives is still weak. This statement is in line with Orton (1983) that student scores for derivative symbols are still low at 1.14 (in the range 0-4). Examples of student answers to these difficulties can be seen in Figure 2.

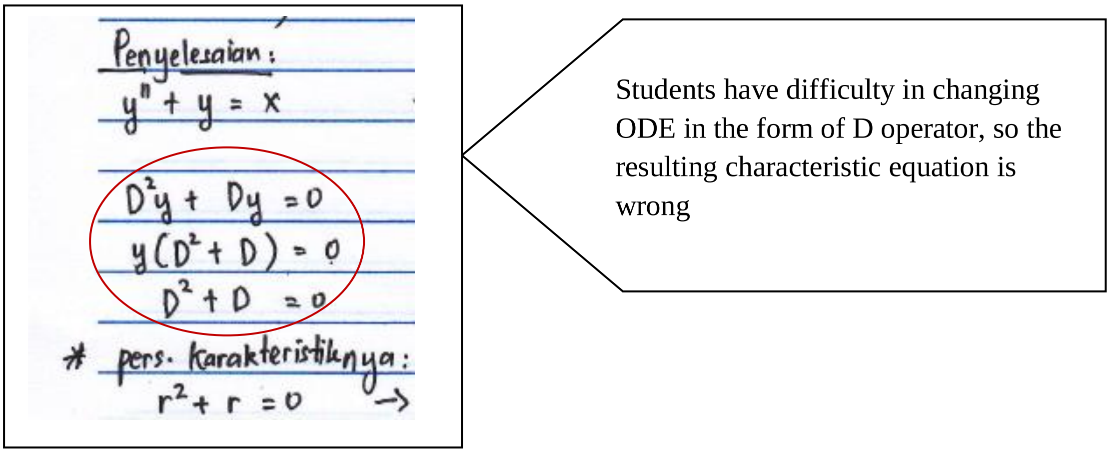

Figure 2. Difficulty in Forming ODE with D Operator.

The next step is factoring y, which is $y(D^2 + 1) = 0$ so that the characteristic equation for ODE can be obtained $r^2 + 1 = 0$. The root of the characteristic equation obtained is a complex number i, so the general solution for homogeneous ODE is $y_h(x) = C_1 \cos x + C_2 \sin x$. After analysis, there are students who have difficulty in performing algebraic operations; students are wrong in determining the roots of the equation. Students' answers to these difficulties can be seen in Figure 3.

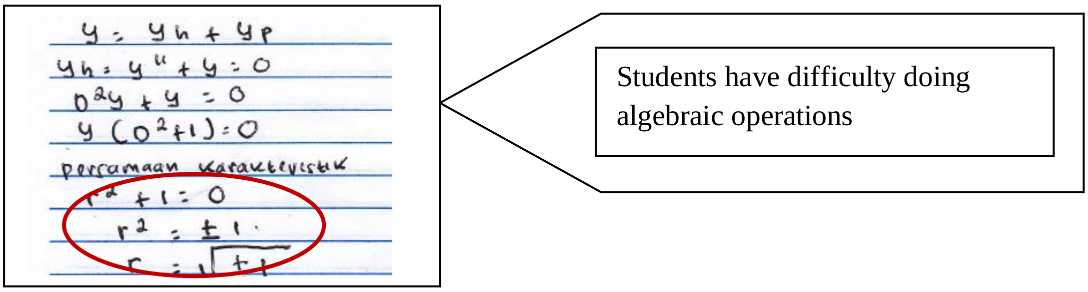

Figure 3. Subject AS Answer in Undetermined Coefficient Method Stage a.

In addition, there are also students who are correct in determining the roots of equations but wrong in choosing the form of a general solution to ODE. The general solution chosen by students is incompatible with the roots of complex numbers. This shows that students' understanding of ODE solutions with the roots of characteristic equations is still lacking. This statement is in accordance with Budiyono & Guspriati (2009). Students' error in determining the general solution can be seen in Figure 4. Students who are not able to complete stage a) in this method cannot continue the steps in working on the questions in the next stage.

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Al-Jabar: Jurnal Pendidikan Matematika
Vol. 10, No. 2, 2019, Hal 232 - 242

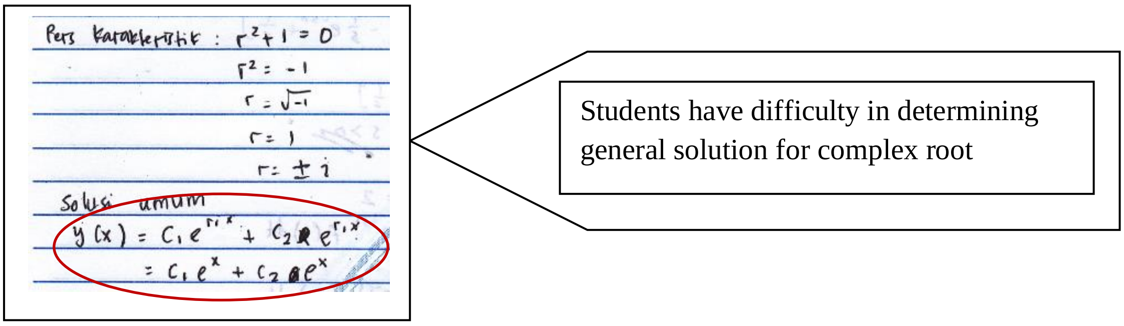

Figure 4. Subject RN Answer in Undetermined Coefficient Stage a

Next, to complete method 1 stage b students must be able to determine the y_p solution. At this stage the number of students experiencing difficulties was 55 people or 70.34%. To determine y_p, it must be adjusted to the function contained in the right hand side of ODE. Because the function on the right side is *x* (linear function) then $y_p = Ax + B$. The next step is to specify y_p''. $y_p = Ax + B$ then $y_p' = A$, and $y_p'' = 0$. Then substitute the values of y_p'' and y_p to the initial equation. So it gets: $y'' + y = x \leftrightarrow y_p'' + y_p = x \leftrightarrow 0 + Ax + B = x$. From the final equation it is known that $A = 1$ and $B = 0$. So we get y_p = x, and the general solution of nonhomogeneous ODE is $y(x) = y_h + y_p = C_1 \cos x + C_2 \sin x + x$.

The difficulty of students at this stage is that they are not determining the y_p solution, but directly substitute the IVP for the homogeneous solution. Students are still mistaken in determining general solutions and particular solutions for second-order non-homogeneous ODE, this shows that students' understanding of particular solutions is still low. Examples of student answers at this stage can be seen in Figure 5.

Students who succeed through stages a and b are able to determine a homogeneous solution, able to determine the y_p solution and determine the general solution of nonhomogeneous of ODE, but the student has difficulty in reducing the solution equation, so the student's answer to this stage is wrong. Students who arrive at this stage are students who are classified in sufficient understanding in determining non-homogeneous 2nd order ODE solutions with the undetermined coefficient method. Examples of student answers at this stage can be seen in Figure 6. After an interview, it is known that the student feels insecure and unsure of the final answer he wrote.

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Vol. 10, No. 2, 2019, Hal 232 - 242

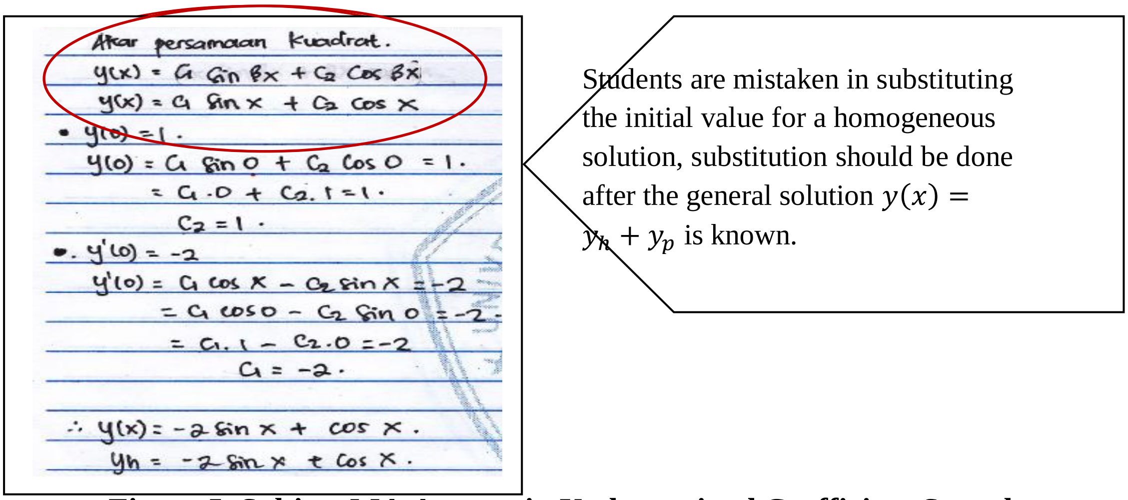


Figure 5. Subject MA Answer in Undetermined Coefficient Stage b

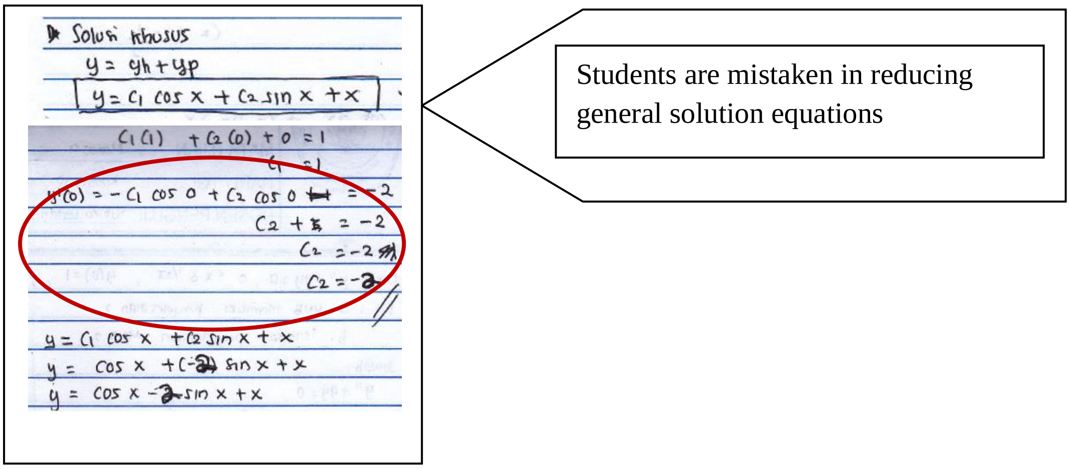


Figure 6. Subject SM Answer in Undetermined Coefficient Stage c.

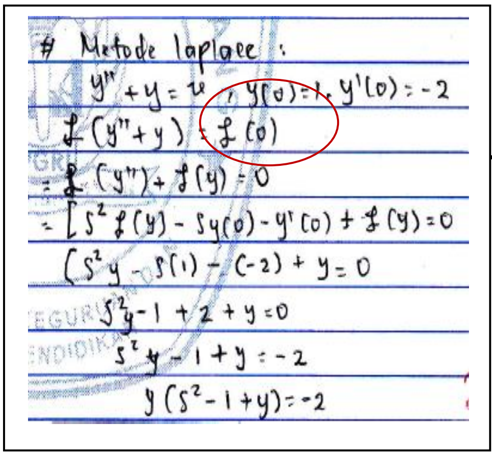
Completion of nonhomogeneous 2nd order ODE using Laplace transformation is different from the previous method. In the Laplace transformation, calculation to find a particular solution is carried out simultaneously, and the initial value is directly substituted. Based on Table 1, students who experienced difficulties in method 2 stage a), are 48 people or 65.75%.

Method 2 stage a), students are asked to be able to make a subsidiary equation. The steps to determine the subsidiary equation are as follows: Initial ODE $y'' + y = x$ is changed to $y'' + y = t$, because in the Laplace transformation the independent variable used is t. Next, apply the Laplace transforms in two segments, so that: $\mathcal{L}(y'' + y) = \mathcal{L}(t) \leftrightarrow \mathcal{L}(y'') + \mathcal{L}(y) = \mathcal{L}(t)$ Use the Laplace transform derivative rule to get the subsidiary equation: $(s^2Y - s + 2) + Y = \frac{1}{s^2})$.

The difficulty of students at this stage is students are still working on method 2 in the same way as method 1. Students make the right segment of ODE to be 0 (homogeneous ODE) so that the subsidiary equation obtained is wrong. In addition, the difficulties experienced by

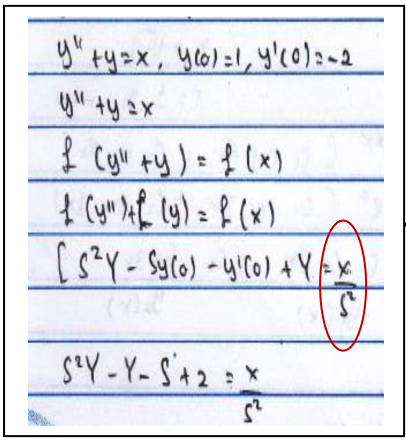
Al-Jabar: Jurnal Pendidikan Matematika
Vol. 10, No. 2, 2019, Hal 232 - 242

students in determining the subsidiary equation can be seen in the student's error in performing the Laplace transformation on the right-hand portion of the nonhomogeneous ODE. The results of the interview show that at this stage students are confused by the steps that must be taken in completing nonhomogeneous 2nd order ODE with the Laplace transformation method. The student focus is fixed on the Laplace transformation on the left side of the ODE. This difficulty indicates that students' understanding of determining the subsidiary equation in Laplace transformation is still low. Students' answer at this stage can be seen in Figure 7 and Figure 8.



Students have difficulty in determining the subsidiary equation

Figure 7. Subject RH Answer in Laplace Transformation Stage a.



Students are wrong in determining Laplace's transformation

Figure 8. Subject NR Answer in Laplace Transformation Stage a.

Method 2 stage b, asked students to solve the subsidiary equation. At this stage, students must have high competence of algebraic or arithmetic operations. The subsidiary equation can be solved using the partial fraction method. The steps are as follows:

$$(s^2Y - s + 2) + \; Y = \frac{1}{s^2} \leftrightarrow Y(s^2+1) = \frac{1}{s^2} + s - 2 \leftrightarrow Y = \frac{1}{s^2(s^2+1)} + \frac{s-2}{(s^2+1)} \leftrightarrow$$

$Y = \frac{1}{s^2(s^2+1)} + \frac{s}{(s^2+1)} - \frac{2}{(s^2+1)}$. Based on the final step, it is known that there is one fraction that is included in the partial fraction and the true fraction must be determined, $\frac{1}{s^2(s^2+1)}$.

Based on Table 1, students who experienced difficulties in stage b numbered 66 people or equal to 90.41%. Students who can answer stage a correctly, have difficulty in completing the subsidiary equation. This difficulty appears in dividing the equation with (s^2+1). This error is included in algebraic or arithmetic operations. The example of students' answer at this stage can be seen in Figure 9

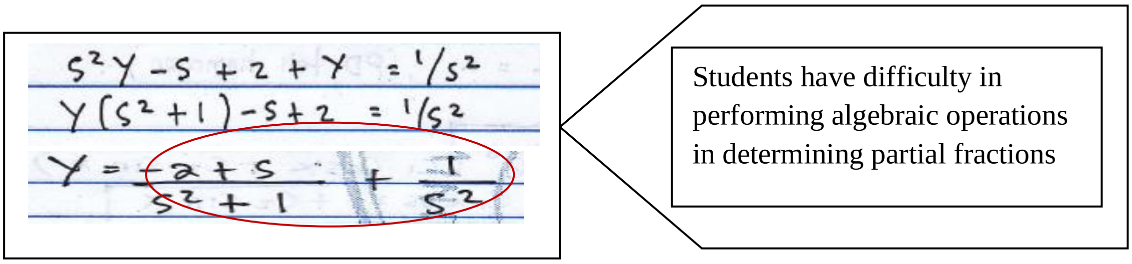


Figure 9. Subject VO Answer in Laplace Transformation Stage b.

After further interviews about this stage with the subject, it is known that the difficulties of students in arithmetic are caused by students doing calculations in a hurry and not checking their final answers. The weakness of students in arithmetic is in accordance with the statement stated by Macromah, Purnomo, Febriyanti, & Rahmawati (2017). This difficulty make students unable to continue the steps in stage c.

Furthermore, for stage c method 2 students are asked to be able to determine the inverse of the Laplace transformation. After the partial fraction is completed, the final equation is obtained, $Y = \frac{1}{s^2} - \frac{1}{(s^2+1)} + \frac{s}{(s^2+1)} - \frac{2}{(s^2+1)} \leftrightarrow Y = \frac{1}{s^2} - \frac{3}{(s^2+1)} + \frac{s}{(s^2+1)}$. The value of $y(t)$ or the solution of the final equation can be determined using the inverse of Laplace transformation as follows: $y(t) = \mathcal{L}^{-1}\left(\frac{1}{s^2}\right) - \mathcal{L}^{-1}\left(\frac{3}{(s^2+1)}\right) + \mathcal{L}^{-1}\left(\frac{s}{(s^2+1)}\right)$, so that the result is $y(t) = t - 3cost + sint$ or $y(x) = x - 3cosx + sinx$.

Based on Table 1, it is known that the number of students experiencing difficulties at this stage is 66 people or equal to 90.41%. Students who have difficulty completing partial fractions cannot determine particular solutions correctly. So that the particular solution is wrong.

CONCLUSION AND SUGGESTION

Based on the results of the study it can be concluded that in completing non-homogeneous 2nd order ODE with the IVP using the undetermined coefficient method, students experience the greatest difficulty in determining the particular solution of nonhomogeneous second-order ODE. This is due to student errors in the first step especially in determining the characteristic equation. Whereas, for the Laplace transformation method, students most difficulties are in the step of solving the subsidiary equation. This is due to the weakness of students in completing arithmetic operations in the form of fractions and partial fractions.

Suggestions that can be conveyed related to the results of this study are ODE lecturers should pay more attention to the basic abilities used as a requirement in nonhomogeneous 2^{nd} order ODE learning with initial value problems. The ability of algebraic operations, substitution and derivatives is an example of the ability that must be mastered by students in understanding the material of the 2^{nd} order ODE. Therefore, it is expected that in the future further studies can be carried out to reduce students' difficulties in solving nonhomogeneous 2^{nd} order ODE.

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