How students' obstacle in solving mathematical tasks deal with linear equation in one variabel

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Abstract

Background: Algebra is one of the most important topics in school mathematics. However, the facts in the field found that there were still many errors made by students in solving problems on linear equations in one variable.

Aim: This research aims to describe and analyze students' learning obstacles in one-variable linear equation material.

Method: This research used a qualitative research approach with a phenomenological design. This research used a purposive sampling technique, the research subjects were 28 class VIII students of Junior High School in Palu City. Data collection was carried out by triangulating data through description tests, in-depth interviews, and documentation studies.

Result: Learning obstacles found in the one-variable linear equation material are categorized into 3 types, namely ontogenic obstacles, epistemological obstacles, and didactical obstacles. Ontogenic obstacles are found because there is a leap in students' thinking from an arithmetic mindset to an algebraic mindset. Epistemological obstacles are found due to limited context for students which causes errors in working on questions. Some epistemological obstacles are the concept of linear equations, algebraic operations, and applications to equations. The didactical obstacle was found because the teacher's teaching was procedural so the formation of the concept of one-variable linear equations and inequalities in students did not go well.

Conclusion: The importance of initial strengthening is related to students' understanding of coefficients, variables, and constants, providing strengthening of the arithmetic thinking process to algebraic thinking, as well as techniques and methods in delivering material and the teaching materials used.

INTRODUCTION

Mathematics is a field of study that must be studied from elementary school to university level. With mathematics, students are encouraged to think and reason, both in solving problems related to everyday life and mathematical problems themselves. Content standards contained in mathematics learning include numbers and operations, algebra, geometry, measurement, data analysis and probability (Ferrini, 2000). Furthermore, algebra can be defined as a branch of mathematics that studies quantities, relationships and certain structures. Algebra also examines how a quantity is generalized into certain symbols, whether in the form of letters, the relationship between symbols and manipulating certain symbols (Utami et al., 2020)

Algebra is a fundamental topic to study in mathematics learning. An important function of algebra is as an instrument in carrying out generalization processes and also finding solutions to various problems. There are many problems in life whose solutions should be found using algebra, namely by modeling the problem into mathematical symbols. For example, if a trader wants to estimate the maximum profit from his trading, then he can use the language of symbols.
in algebra to express several variables that influence that profit. Another use of algebra is that it can help solve complex problems more easily. For example in arithmetic problems. When someone is asked to add the first 100 natural numbers using ordinary addition, it will take a long time. However, if the problem is solved using the arithmetic series addition formula, namely $S_n = n \frac{1}{2} (a + U_n)$, then this will become easier.

Based on this, algebra is the most fundamentally important material for students to learn and understand. This is because, either implicitly or explicitly, algebra is material that students can use in life activities and can make problems easier. Another thing was also revealed that algebra is an important aspect that is mastered by students because algebra determines the success of students' achievements in more advanced classes. Ferrini (2000) states that all students must study algebra, because algebra competency is very important for use in life, either in work or as preparation for higher education. In line with this, Kriegler (2008) stated that algebra is the main gateway in efforts to understand more complex mathematics. Jupri et al (2014) also explained that algebra also plays an important role in understanding other areas of mathematics such as analytical geometry, calculus and statistics. In line with this, Booker (2009) stated that the important role of algebra is as an instrument in finding solutions to mathematical problems and other fields including science, business, economics, trade, computing and other problems in everyday life.

There are several algebra learning topics taught in junior high schools, including material on Linear Equations in One Variable. Linear equations in one variable are one of the challenging materials in mathematics. Until now, there are a large number of errors made by students when solving problems related to the material of linear equations in one variable. According to Savitri et al (2018) many students make mistakes when trying to answer questions on linear equations in one variable. Jumiati et al (2020) in their research explained that students when working on linear equations with one variable still made several mistakes, including those related to concepts and principles. The meaning of conceptual errors is when students do not understand the concept of variables and errors when interpreting the form of a story problem into a mathematical model. Meanwhile, the principles error that is generally made is that students do not use the general form of equations and do not use variables. This finding is also supported by research Rahmania & Rahmawati (2016) which explains student errors when solving problems related to linear equations with one variable related to errors when interpreting questions into mathematical models, student errors in understanding the concept of variables and errors in understanding the concept of substitution. Other studies use the term difficulty rather than the term error. Because according to Jupri et al (2014) errors are a manifestation of difficulty. In his research, he found that there were five main difficulties experienced by students when studying algebra, especially linear equations, namely difficulties in applying arithmetic operations, difficulties in understanding the meaning of variables, difficulties in understanding algebraic statements, difficulties in understanding the meaning of the “=” sign and difficulties in translating phenomena. real into mathematical models and vice versa (Jupri et al., 2014).

This was also revealed in the results of the 2012 Program for International Student Assessment (PISA) that the mathematics ability of Indonesian junior high school students was ranked 64th out of 65 countries. One of the questions tested on PISA is algebra material (including linear equations in one variable). The low ability of students in algebra material,
especially linear equations with one variable, can also be seen from the results of the Trends in International Mathematics and Science Study (TIMSS) in 2011 which stated that the mathematics ability of Indonesian junior high school students was ranked 38th out of 42 countries.

Revealed that the mistakes that students generally make are caused by conditions that indicate obstacles during the learning process. An obstacle is a type of difficulty when learning that is caused by external factors including didactic design (Didi, 2023). The learning obstacles experienced by students in studying subject matter are categorized by Brousseau & Balacheff (2002) into three types, namely: (1) ontogenic obstacle, namely a mismatch between the learning provided and the student’s level of thinking, giving rise to difficulties in the process of understanding the material. If the level received by students is too low then students will not experience a true learning process, conversely if the level received by students is too high, then students will experience difficulties and even not like mathematics because it is difficult. (2) epistemological obstacle, namely difficulties in the learning process that occur as a result of the limited context that students know. In this case, students only receive a partial understanding of the concept, so that when faced with a different context, students experience difficulty in using it. (3) didactical obstacle, namely difficulties that occur as a result of the teacher's learning.

Research on learning obstacles has also been widely studied on different materials, with the same types of learning obstacles. Like the research conducted by Maarif et al (2020) regarding the epistemological obstacles experienced by students when solving problems on the topic of systems of linear equations in two variables. Cesaria & Herman (2019) researched students' learning barriers when studying geometry, especially flat-sided geometric figures. Students' ontogenic barriers when studying triangles and quadrilaterals. In this study, we delve deeply into the various learning obstacles encountered by junior high school students when solving single-variable linear equations. This research distinguishes itself from previous studies by employing a triadic analysis of ontogenetic, epistemological, and didactic barriers, a methodology that has not been widely applied in the local Indonesian context. Our findings not only identify common errors but also offer practical recommendations to address specific difficulties faced by students. These recommendations are expected to enhance the effectiveness of mathematics instruction at the junior high school level and improve Indonesia's international rankings in student mathematics proficiency. Consequently, this study aims to describe the learning obstacles junior high school students face with single-variable linear equations.

**METHODS**

This research applies a qualitative approach through a phenomenological research design. This research design was chosen with the aim of understanding, describing and interpreting junior high school students’ experiences of certain phenomena (learning obstacles) when students learn about linear equations with one variable in an in-depth and natural way with the aim of solving them immediately, and the analysis process can be carried out based on relevant learning theory perspectives. Qualitative research is a research method that is generally used to explore and understand the meaning of social or humanitarian problems through various
processes such as asking questions, collecting data, analyzing data and interpreting data (Ishtiaq, 2019). Qualitative research is not regulated by experimentation, which means that the results obtained are based on what the research subjects experienced, thus qualitative research does not only emphasize the process of generalization, but places great importance on aspects of meaning.

Furthermore, phenomenological design according to Creswell & John W (1998) is a study to reveal the meaning of phenomena that occur in several individuals based on what they experience. The research design used refers to the Didactical Design Research (DDR) stage. The formal stages of DDR consist of a process of analysis of the didactic situation before learning is carried out, analysis of the metapedadidactic, and analysis of the retrospective (Suryadi, 2010).

This research was carried out at Al Fahmi Integrated Islamic Middle School, Palu City, Central Sulawesi Province. As for the students involved as potential subjects in this research, there were 28 class VIII students who would be selected as subjects for in-depth interviews, provided they had studied the material on linear equations in one variable as the main source for obtaining data related to learning obstacles faced by students when studying linear equations in one variable. This research also involved teachers who taught mathematics at the school, focusing on one-variable linear equations.

The data obtained in this research was collected using data triangulation techniques. According to Creswell & John W (1998), data triangulation technique is a data collection technique by combining information from various sources and techniques to meet the credibility of the data. The data collection techniques in this research are written tests, interviews and documentation. In qualitative research activities, researchers are also used as the main instrument where descriptive data is obtained, for example in the form of written words or verbal information from the observed behavior of research subjects (Lexi, 2017). This is because the researchers themselves have the most direct understanding of the reality in the field and the efforts to overcome everything that is happening. Thus, when this research is carried out, researchers are the parties involved in planning, implementing, collecting data, analyzing data, interpreting data, and reporting research results (Creswell & John W., 1998). Apart from researchers, there are supporting instruments to help researchers collect data, namely test instruments in the form of description questions to identify students’ learning obstacles in the one-variable linear equation material, non-test instruments in the form of interview guides and documents that are data sources in the form of learning documents. who are studying linear equations in one variable, namely lesson plans, student mathematics books, power point files, and student notes. The data analysis technique used in this research follows the path proposed by Matthew et al (2019), which includes data reduction, data presentation and drawing conclusions.

RESULTS AND DISCUSSION
Identification of learning obstacles in this research was obtained from the results of diagnostic test analysis and interviews with students. The test results are grouped based on the types of errors made by students in answering questions based on test indicators. The following results of the analysis of diagnostic tests on students when working on questions can be seen in Table 1 below.
Table 1. Diagnostic Analysis Results

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Description</th>
<th>Total</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question No. 1</td>
<td>Students are expected to be able to create mathematical equations that contain variables</td>
<td>Students cannot make mathematical statements that contain variables</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students can create mathematical statements that contain variables</td>
<td>8</td>
</tr>
<tr>
<td>Question No. 2</td>
<td>Students are expected to be able to determine variable values from an equation</td>
<td>Students cannot determine the value of a variable in a linear equation with one variable</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students can determine a variable in a one-variable linear equation</td>
<td>11</td>
</tr>
<tr>
<td>Question No. 3</td>
<td>Students are expected to be able to solve linear equation problems in one variable</td>
<td>Students cannot solve a contextual problem related to linear equations with one variable</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students can solve a contextual problem related to linear equations in one variable</td>
<td>2</td>
</tr>
</tbody>
</table>

The diagnostic test results in solving one-variable linear equation problems shown in table 1 were considered low. In question number one, 28.6% of students were able to solve the question correctly, while in question two, 39.3% of students were able to solve the question correctly and in question three, 7.1% of students were able to solve the question correctly.

In question number 1, there were 60.7% of students who were still unable to make mathematical statements that contained variables. Of the 28 students who were given test question number 1, 8 students were able to solve the question correctly, 3 students did not answer the question. Meanwhile, 17 other students did not answer the questions correctly. In question number 2, there were 71.4% of students who were still unable to determine the value of the variable from an equation. Of the 28 students who were given test question number 2, 11 students were able to solve the question correctly, 2 students did not answer the question. Meanwhile, 15 other students did not answer the questions correctly. In question number 3, there were 71.4% of students who were still unable to solve the contextual problem of linear equations with one variable. 28 students were given test question number 3, 2 students were able to solve the question correctly, 15 students did not answer the question. Meanwhile, 11 other students did not answer the questions correctly.

Based on the results of diagnostic tests and interviews, there are three types of student learning obstacles found in the process of solving problems related to one-variable linear equation material, namely ontogenic obstacles, epistemological obstacles, and didactical obstacles.

Ontogenic Obstacle

This one-variable linear equation material is studied in class VII, where the average age of students is between 12 – 13 years. According to Piaget's stages of cognitive development, students in this age range enter the formal operations stage. However, this age range is still in the student's cognitive transition from the concrete operations stage to the formal operations stage. Cognitive development according to constructivism theory is the construction of knowledge which is reflected in the stages in understanding mathematical concepts. If at the stages in the construction of student knowledge there are "jumps" in the thinking process, this is what gives rise to an ontogenic obstacle.
In this research, ontogenic obstacles were found because there was a jump in students' thinking processes from arithmetic to algebraic thinking. The series of questions given are in the form of stories so students need to convert them into algebraic equations. However, to the questions given, students answered with arithmetic even though they had studied the material regarding algebraic forms and linear equations with one variable. Learning obstacles experienced by students may occur due to the use of teaching materials that are not suitable for the characteristics of the students themselves. So sometimes when introducing variables in teaching materials, we go straight to the definitions of variables, coefficients and constants without any stages for students to be involved in the process of forming meaning from the components of the algebraic form. This creates a "jump" in students' thinking from arithmetic to algebra, there is no learning stage that bridges the change in students' mindset. These learning obstacles can be seen in questions number 1 and number 3.

Figure 1. Findings (1) Ontogenic Obstacle

The students worked on both answers by writing down all the numbers in the problem without modeling the statement into an algebraic equation. For the answer to question number 1, the student immediately wrote \( = 3 + 8 \), namely 11. The answer written was wrong and did not contain variables. After being asked directly to the student what the variables and equations meant, the student was confused by the question given. After being asked in more depth, the reason why students did not write statements into algebraic equations was because they were not yet able to think algebraically, namely modeling story problems in equation form, students did not replace the meaning of unknown values in the form of \( x \) values or other variables. Likewise for question number 3, students immediately work on it arithmetic without modeling it in algebraic form. Students immediately wrote down the numbers in the problem and followed the instructions in the problem without modeling them into algebraic equations so that the length and width they found were wrong.

Epistemic Obstacle
Epistemological Obstacle is a student learning barrier because of students' understanding of a concept, but cannot be used in other problems. Brousseau & Balacheff (2002) reveals Epistemological Obstacle as knowledge that functions well in some activity domains and is therefore settling well, but then it is not appropriate in another context where it cannot be used and causes contradictions. Based on the results of the analysis and description of the data, found several epistemological obstacles, namely algebra operation and application to the form of equation.
Findings (1) Epistemological obstacles can be seen in questions number 1 and number 2, where students have not mastered the concept of linear equations with one variable. For answer number 1, students do not understand the symbols of similarity and dissimilarity. The answer written by the student was wrong, after being asked directly to the student what the signs of similarity and dissimilarity meant, the student was confused by the question given. After being asked in more depth, the reason the student could not write correctly was because he had not been able to understand what the question meant, namely translating the meaning of 44 equals 8 more than 3 times y so the student wrote only what he knew, namely more was the same as the more than sign. Likewise for answer number 2, students do not understand the similarity symbol, namely the “=” sign. The answer written by the student is wrong, because it separates the existing equation into 2 equations. After being asked directly to the student what the “=” sign meant, the student said he did not understand the material. Students only follow the method given by the teacher so that both sides are subtracted by the same number to produce zero, so students do not understand the concept of linear equations with one variable. Just like research Nabila et al (2022) and Sulastri et al (2017) students have not mastered mathematical concepts so they cannot apply the concepts in solving problems. Every difficulty in understanding concepts that occurs to students is because students do not understand the basic theory well. According to Putra, it is stated that if students understand the concept well then the learning objectives are also achieved, meaning that if the understanding of the concept is weak then the learning objectives are not achieved (Putra et al., 2018).

Findings (2) Epistemological obstacle can also be seen in question number 2, where students still have difficulty understanding basic mathematical concepts (addition, subtraction, multiplication and division) well. For answer number 2, students already understand the concept of linear equations with one variable but still make mistakes in algebraic calculation operations, especially multiplication and division. From the interview results, students were
confused about adding coefficients and constants. As a result, students have difficulty solving this problem because students cannot differentiate between coefficients, variables and constants. The results of interviews with students also showed that these students' conceptual abilities were still low, so that problem solving was not precise because students were still confused about arithmetic operations (addition, subtraction, multiplication, division). So, it can be concluded that the subject experienced learning obstacles related to arithmetic operations. After the student was interviewed directly, he only remembered that the variables were substitutes that were free to be replaced with any value.

The results of Sospita & Masjudin (2019) which found that students were not able to operate algebraic multiplication questions correctly, had difficulty with the principle of algebraic addition, and had difficulty with algebraic subtraction, in Herawati & Kadarisma (2021), the same thing was also found that students had difficulty in operating algebraic forms, this is in line with what was experienced by junior high school students in the one-variable linear equation material that students still had difficulty using the correct operations of multiplication, addition and subtraction to solve the problem.

Figure 4. Findings (3) Epistemological Obstacle

Finding (3) Epistemological obstacle can be seen in question number 3, where students still have difficulty understanding story questions that are relevant in daily activities. Students have understood the problem, that is, they can write down information related to the problem, what they know and ask about in the problem, but students have difficulty in making mathematical models in story problems. This is because students do not understand how to assume verbal sentences into mathematical sentences or mathematical models. Students are also unable to explain again the mathematical equations they have created. This is because students do not understand the steps of mathematical modeling. Factors causing errors are students' low understanding of concepts related to the material on linear equations with one variable and difficulty identifying the variables in story problems into mathematical models so that they do not have a description of the variables to become a model and it is difficult to make assumptions about the model that will be formulated into an equation.

The results of this analysis are in line with Khasanah & Sutama's (2015) which found that the difficulties of junior high school students in mathematics story problems were that students could not change the problem sentences into mathematical models so that students were confused or made mistakes in substituting formulas, this is also in line What is experienced by junior high school students in the material on linear equations with one variable, where students have difficulty understanding the problem and difficulty in changing the question sentence into a mathematical model, this is because students do not know what a mathematical model looks like.
Didactical Obstacle

Didactical obstacle is found in several basic concepts taught by the teacher, but has a major impact in the process of forming student concepts on the material of this variable linear equation. Based on the results of the interviews of several students, most of the learning carried out by the teacher procedurally. The teacher explains the material, working on examples of questions, and assigns students to work on questions whose types are usually not far from those exemplified by the teacher. This makes students understand based on the procedures taught by the teacher. The teacher does not involve students in the process of forming material concepts so that the concept of material is not understood by students well. The following is one of the students' answers that are done procedurally in accordance with the steps they remember when exemplified by the teacher in Figure 5.

Figure 5. Findings (1) Didactical Obstacle

Didactical obstacle occurs because the dependence of the teacher uses the approach only to one didactic option. This obstacle is seen from the learning approach, metaphor and analogy and how to present the concepts of the material taught. Based on the results of the analysis and description of the data, found some obstacle didatical, namely: Representation and approach given by the teacher in the form of a teacher center, learning does not emphasize the meaning and symbol of the equation material.

a. Representation and approach given by the teacher in the form of a teacher center from the findings of student answers, some difficulties experienced by students when given a problem related to finding a set of solutions. Students are only fixated on the completion process that the teacher gives such as the results of students' answers and interviews. In this answer students' mistakes in the operation process can be seen in the simplification to find the value of the set of variables X and from the results of the interview with the student gives the answer that all of that is explained by the teacher. From here students focus on the settlement that the teacher conveys is not in the completion process according to students themselves.

b. Learning does not emphasize the meaning and symbol of all students' answers presented in the results of the description and analysis of students' answers, some of the difficulties experienced by students including making mistakes on the completion and operation of algebra. Students do algebra settlement on the coefficient and constants or in the form of models and applications of one variable linear equation, this is because the teacher does not emphasize the meaning and symbol on the material of one variable linear equation so that students find it difficult to solve the problems given.
CONCLUSIONS
Three types of learning barriers were identified that students face when solving single-variable linear equations: ontogenetic, epistemological, and didactic barriers. Ontogenetic barriers arise because students struggle to transition from arithmetic thinking to algebraic thinking, as algebraic concepts are introduced abstractly without a clear foundation. Epistemological barriers encompass several aspects: firstly, a low mastery of the single-variable linear equation concept, making it difficult for students to apply it; secondly, inadequate arithmetic operation skills, leading to frequent errors as students cannot distinguish between coefficients, variables, and constants; thirdly, difficulties in modeling contextual problems into equations due to a lack of clear understanding of the involved variables. Didactic barriers are evident from the overly procedural and monotonous teaching methods used by teachers, which hinder students' comprehension of the basic concepts of single-variable linear equations. In conclusion, to enhance students' understanding, there needs to be an improvement in teaching approaches that better address conceptual transitions, foundational theory comprehension, and variation in practice problems.

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AUTHOR CONTRIBUTIONS STATEMENT
All authors contributed to design research, process question analysis, data analysis, manuscript writing, and final manuscript approval.

REFERENCES


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