Proportional Thinking Characteristics of Students In Solving Direct Proportion Problems in Junior High School

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Abstract: One of the most common obstacles students face when solving proportion problems is their tendency to use the addition concept rather than the multiplication concept. This study aims to analyze and characterize the proportional thinking characteristics of students who solve direct proportion problems. The research employs a qualitative, exploratory-descriptive methodology. The subjects were three eighth-grade students. Data was collected in two stages using supplementary instruments, including direct proportion questions and interviews. In the initial stage, students answered questions orally while researchers observed their activities and took notes, which were confirmed during interviews. In the second stage, task-based interviews were used to investigate the students' proportional thinking processes. Data analysis involved examining the students' work and interview transcripts, which were then coded to identify noteworthy findings. The study identified three characteristics of proportional thinking: (1) additive different thinking; (2) multiplication continuation thinking; and (3) multiplication proximity thinking. The foundation for solving direct proportion problems lies in students' understanding of the relationship between two quantities in a ratio. Teachers should prioritize the explicit disclosure of students' cognitive processes related to the subject matter. Additional studies are recommended to further investigate students' cognitive processes.

INTRODUCTION

The primary objective of teaching mathematics in schools, besides fostering comprehension of concepts, is to develop mathematical thinking (Nur & Sari, 2021; Basri et al., 2023). Mathematical thinking refers to the capacity to solve problems and comprehend mathematical concepts logically, leading to the formation of conclusions or judgments (Lohman & Lakin, 2009). Students acquire not only the ability to passively receive and transcribe information but also actively engage in cognitive processes, making learning a mental endeavor intrinsically linked to the thinking process (Walle, 2007). Studying mathematics cultivates students' cognitive abilities, particularly in problem-solving, as students encounter problems requiring critical thinking to determine solutions (Solso et al., 2008).

Examining students' cognitive capacities for solving mathematical problems is crucial. Bayazit (2013) asserts that the resolution of mathematical problems necessitates a thorough examination employing cognitive abilities. Proportional thinking is a cognitive skill that aids students in resolving mathematical problems (Nur &
Proportional thinking holds significant importance in learning mathematics. According to Pitta-Pantazi & Christou (2011), proportional thinking varies, requiring students to master various types of understanding, processes, and contexts. Additionally, Johar & Yusniarti (2018), Norton (2006), and Sappaile (2007) noted that complex physical systems can be comprehended through proportional reasoning as a qualitative structure, based on proportions and ratios.

Proportional thinking characterizes the concepts and mental processes needed to understand ratios and proportions. According to Dole et al. (2012) and Steinthorsdottir & Sriraman (2009), a ratio is a comparison of two or more quantities of the same type, often expressed as a fraction a/b. Meanwhile, proportion is a statement of equality between two ratios, such as a/b = c/d. Sen & Guler (2017) exemplify that if the ratio of female to male students in a class is 2:3, and there are 30 students, there would be 12 female students and 18 male students, demonstrating a proportion.

Students must comprehend proportional thinking to solve multiplication problems involving proportions or ratios. The fundamental concepts of ratios and proportions are significant in fields like mathematics (Chaim et al., 2012). Ratios and proportions are intertwined with numerous mathematical concepts, including multiplication, division, fractions, and decimals (Dole et al., 2012; Lobato et al., 2010; Nugraha et al., 2016; Permatasari et al., 2017; Susilowati et al., 2022). Many topics in mathematics education, such as scales, drawings, fractions, probabilities, percentages, measurements, field geometry, and algebra, require knowledge of ratios and proportions.

When solving proportion problems, students often struggle to articulate their reasoning. Research by Jitendra et al. (2009) found that one main issue is students' tendency to use the addition concept rather than the multiplication concept, leading to incorrect answers. Furthermore, Nur (2022) reported that many students struggle to differentiate between direct and indirect proportions, often confusing the two.

Incorrect problem identification by students necessitates multiple solution attempts to reach correct conclusions (Nur, 2022). Errors in solving proportion problems can also stem from the concept not being well-embedded in students' memories. Walle (2007) identified characteristics of students capable of proportional thinking: (1) understanding covariation and the relationship between two varying quantities; (2) recognizing proportional and non-proportional relationships; (3) developing strategies to solve proportion problems; (4) understanding the ratio as a distinct entity expressing the relationship between quantities.


Based on previous studies and their findings, this research focuses on the proportional reasoning characteristics of students solving direct proportion problems. Unlike previous research, which utilized a descriptive qualitative methodology, this study employs a qualitative, exploratory-descriptive approach with eighth graders as subjects,
investigating direct proportion problems and students' proportional cognitive processes.

Investigating how students solve direct proportion problems is critical. According to Bayazit (2013), solving proportion problems necessitates employing cognitive processes. Identifying students' thought processes helps pinpoint their problem-solving errors. Assigning direct proportion problems can reveal students' thought process characteristics. This research aims to contribute to: (1) developing theories on thinking processes, particularly proportional thinking in solving direct proportion problems; (2) providing material considerations for teachers designing mathematics lessons related to direct proportions; (3) serving as reference material for future researchers on proportional thinking processes in solving direct proportion problems.

The characteristics of proportional thinking play an essential role in solving direct proportion problems. Describing these characteristics helps understand students' thought processes in solving such problems. Thus, this study aims to analyze and describe the "Proportional Thinking Characteristics of Students in Solving Direct Proportion Problems in Junior High School".

**METHOD**

This study aimed to identify the characteristics of students' proportional reasoning when solving problems involving direct proportions. The focus of this study was a comprehensive analysis of the characteristics of student thought using a qualitative exploratory descriptive methodology. A descriptive approach was utilized to analyze the data, assessing students' thinking abilities through their verbal and written expressions of thought processes.

According to Creswell (2014), qualitative research is characterized by: (1) natural environment, where researchers collect data in locations where subjects experience the issues or problems being studied; (2) researchers as key instruments, collecting data through documentation, behavioral observation, or interviews; (3) various data sources, collecting data from interviews, observations, and documentation, then reviewing and categorizing them; (4) inductive data analysis, developing patterns and categories from abstract information units; (5) evolving design, with a dynamically developing research process; and (6) overall view, creating a complex picture of the problem under study.

The research subjects were 25 seventh-grade students from SMP Islam 1 Kota Ternate, selected based on their prior exposure to direct proportion material during elementary school education. The selection criteria included students' potential in solving math problems and their communication skills, both orally and in writing, as observed directly by the researchers and their mathematics teachers. Students were given test questions related to direct proportion problems to evaluate their proportional thinking processes. Interviews were conducted after students completed the problems to delve deeper into their thinking processes. Data was extracted through interviews with questions tailored to the respondents' unique characteristics and circumstances.

Out of the 25 students, 3 students provided complete answers to the direct proportion problems and were selected as research subjects. These 3 subjects represented different categories of proportional thinking: 1 exhibited additive different thinking, 1 exhibited multiplication continuation thinking, and 1 exhibited multiplication proximity thinking. The results of subject selection for each characteristic are shown in Table 1.
Table 1. Characteristics of Research Subjects.

<table>
<thead>
<tr>
<th>Student Total</th>
<th>The Number of Subjects of Each Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Incomplete answer</td>
</tr>
<tr>
<td>22 students</td>
<td>3 students</td>
</tr>
</tbody>
</table>

Subjects in the additive different thinking category chosen for presentation and analysis were taken from one student, referred to as Subject S1. The subject in the multiplication continuation thinking category chosen for presentation and analysis was taken from one student, referred to as Subject S2. Subject S3, chosen for presentation and analysis in the multiplication proximity thinking category, was taken from one student. Research subjects were continuously selected until data saturation was achieved. The consistent pattern observed among the subjects in each category defined data saturation.

The researchers played a pivotal role in this study, serving as planners, data collectors, data analysts, interpreters, and ultimately reporters of the research findings. To facilitate their research tasks, the researchers used test-question instruments and interview guidelines. Data on students' proportional thinking characteristics in solving direct proportion problems were collected using the test question instrument. Additionally, interview guidelines were utilized to gather crucial supplementary information. Data collection integrated structured and unstructured interviews. In structured interviews, the interviewer independently selected the issues and inquiries to obtain responses to specific allegations. Unstructured interviews involved questions that were not premeditated but tailored to the specific circumstances and attributes of the interviewee.

The test question instrument consisted of one essay question. Researchers developed the test question instrument in consultation with experts in mathematics and mathematics education. The instrument aimed to formulate questions that enable students to work on non-routine problems requiring proportional thinking rather than merely recalling previously solved questions. The problem was open-ended, requiring divergent thinking in both answers and methods, and necessitated an understanding of several previously encountered mathematical concepts and properties.

The research followed procedures encompassing preparation, data collection, data analysis, and conclusion drawing. During the preparation stage, researchers refined the proposal, prepared validated instruments, set up data collection devices (handheld camera, laptop, headphones), trained students involved in data collection, and organized the equipment. The data collection stage involved administering direct proportion tests to students without intervention from the researchers, who focused on observing and taking notes for confirmation during interviews.

In the data analysis stage, activities included transcribing data from direct proportion test results and interviews, reducing data, coding, and describing students' thinking characteristics in solving direct proportion problems. The conclusion-drawing stage followed data analysis, summarizing the research results regarding the characteristics of students' thinking in solving direct proportion problems. The flow of research procedures is presented in Figure 1.
The data analysis in this study followed several structured procedures. Initially, all written test results conducted by the research subjects were examined. This was followed by an analysis of data obtained from interviews and video recordings. The collected data was then transcribed to ensure accuracy and comprehensiveness. Subsequently, the written test results and interview data were compiled and categorized in coded form, enabling systematic analysis. The next step involved creating a diagram of the thinking structure for each subject, which provided a visual representation of their cognitive processes. Each subject's thinking structure was then analyzed in detail. During this stage, particular attention was given to identifying distinct and intriguing findings that were relevant to the problem under investigation. Finally, conclusions were drawn regarding the proportional thinking characteristics of students when solving direct proportion problems, summarizing the key insights and findings of the study.

RESULT AND DISCUSSION
The characteristics of proportional thinking demonstrated by students in solving direct proportion problems can be categorized into three distinct types: (1) additive different thinking, (2) multiplication continuation thinking, and (3) multiplication proximity thinking. Additive different thinking occurs when students approach cases by adding two quantities with different variations. Multiplication continuation thinking is observed when students solve cases by multiplying two whole quantities with variations. Multiplication proximity thinking arises when students address cases by multiplying two separate quantities with variations. These characteristics of students' thinking are summarized in Table 2.

<table>
<thead>
<tr>
<th>Characteristics of Thinking</th>
<th>Description</th>
<th>Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additive different thinking</td>
<td>The characteristic of additive different thinking occurs when students observe cases directly by adding up two quantities that have different variations.</td>
<td>S1</td>
</tr>
<tr>
<td>Multiplication continuation thinking</td>
<td>The characteristics of multiplication continuation thinking occur when students observe cases by multiplying two whole quantities that have variations together.</td>
<td>S2</td>
</tr>
</tbody>
</table>
The first finding, the characteristics of additive different thinking, was observed in Subject S1, who demonstrated additive different thinking in solving direct proportion problems. The second finding, the characteristics of multiplication continuation thinking, was observed in Subject S2, who exhibited multiplication continuation thinking in solving direct proportion problems. The third finding, the characteristics of multiplication proximity thinking, was observed in Subject S3, who displayed multiplication proximity thinking in solving direct proportion problems. The characteristics of students’ thinking are illustrated in Figure 2.

**Figure 2.** Students’ Proportional Thinking Characteristics.

### Data Exposure and Analysis of Additive Different Thinking

The data presented on the characteristics of additive different thinking pertained to Subject S1. The data for additive different thinking was derived from interviews, field notes, and subject answer sheets in solving direct proportion problems. Additive different thinking characteristics were observed when students approached cases by adding two quantities with different variations. The characteristics of additive different thinking in Subject S1 are illustrated in Figure 3.

**Figure 3.** Additive Different Thinking Characteristics of Subjects S1.

When the direct proportion question was given, Subject S1 observed and read the questions carefully. Subject S1 wrote that Ann spun 3 rounds, and Rachel spun 12 rounds. If Ann has spun 6 rounds, then Rachel must have spun 24 rounds. Because:

- \( A = 12 + 12 = 24 \)
- \( R = 3 + 3 = 9 \)
- So, it’s a fair comparison.

The first finding, the characteristics of thinking multiplication proximity occurs when students observe cases by multiplying two separate quantities that have variations together. The characteristics of students’ thinking are illustrated in Figure 2.

### Characteristics of Thinking

<table>
<thead>
<tr>
<th>Characteristics of Thinking</th>
<th>Description</th>
<th>Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication proximity</td>
<td>The characteristic of thinking multiplication proximity occurs when students observe cases by multiplying two separate quantities that have variations together.</td>
<td>S3</td>
</tr>
</tbody>
</table>
Subject S1 understood that the problem could be solved using the concept of addition. Specifically, 12 rounds added to 12 rounds results in 24 rounds, and 3 rounds added to 3 rounds results in 6 rounds. Therefore, Subject S1’s approach to solving the direct proportion problem can be understood from the following interview results.

Table 3. Description of Interview Subject Results S1.

<table>
<thead>
<tr>
<th>No</th>
<th>Researcher Response</th>
<th>Subject Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>From this question, what do you understand.</td>
<td>In my opinion, it is a matter of direct proportion.</td>
</tr>
<tr>
<td>2</td>
<td>In your opinion, what is known from the problem</td>
<td>What is known is that Ann rolls 3 rounds and Rachel does 12 rounds</td>
</tr>
<tr>
<td>3</td>
<td>Then, what is asked of that question</td>
<td>It is asked how many rounds Rachel get</td>
</tr>
<tr>
<td>4</td>
<td>Why did you have to write 12 + 12 = 24</td>
<td>Yes sir, because in the first round Rachel got 12 rounds, of course in the 2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>round it was the same so I added 12 rounds with 12 rounds to get 24 rounds</td>
</tr>
<tr>
<td>5</td>
<td>Well, then why was it written 3 + 3 = 6</td>
<td>Because you would get 3 rounds, of course in the following round it would be the</td>
</tr>
<tr>
<td></td>
<td></td>
<td>same, that was, 3 rounds adding up to 3 rounds equals 6 rounds</td>
</tr>
<tr>
<td>6</td>
<td>Who do you think between Ann and Rachel which have the speed</td>
<td>Ann sir, uh no sir Rachel sir! (Previously subject S1 answered incorrectly)</td>
</tr>
</tbody>
</table>

Based on the interview results, Subject S1 added 12 rounds plus 12 rounds and 3 rounds plus 3 rounds based on conjecture without verification. The results obtained from these conjectures revealed a spontaneous emergence of the numbers 12 and 3. This activity of adding information, despite the form of solving direct proportions not involving the addition of numbers, was used by Subject S1 as a strategy for problem-solving. Although the resulting completion was 24 and 6 rounds, this indicated a misunderstanding of the concept of proportion by Subject S1. The results of Subject S1’s work were mixed up; problems that should have been solved using multiplication were instead solved using addition.

Data Exposure and Analysis of Multiplication Continuation Thinking

The data presented on the characteristics of multiplication continuation thinking pertained to Subject S2. The data for multiplication continuation thinking was derived from interviews, field notes, and subject answer sheets in solving direct proportion problems. The characteristics of multiplication continuation thinking occur when students approach cases by multiplying two whole quantities with variations. The characteristics of multiplication continuation thinking observed in Subject S2 are illustrated in Figure 4.

Figure 4. Multiplication Continuation Thinking Characteristics of Subject S2.
The occurrence of multiplication continuation thinking characteristics is described based on the results of Subject S2's work. When given the direct proportion questions, Subject S2 observed and read the questions carefully. After reading the questions, Subject S2 wrote that Rachel had spun 12 times. In the first situation, Ann had only spun 3 times, while in the second situation, Ann had spun 6 times and Rachel had spun 24 times. Subject S2 did not initially write down what was known and what was being asked in the problem, nor did they consider the direct proportion formula used to solve the problem. Therefore, Subject S2's approach to solving the direct proportion problem can be understood from the following interview results.

Table 4. Description of Interview Subject Results S2.

<table>
<thead>
<tr>
<th>No</th>
<th>In your opinion, what is known and asked in the question.</th>
<th>It is known that Rachel spinned 12 times while Ann spinned 3 times. Meanwhile, what was asked was that Anna spinned 6 times how many rounds did Rachel go through.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Well, your previous answer stated that the question is in the form of direct proportions, try to explain the formula for direct proportions.</td>
<td>If I'm not mistaken, the direct proportion formula is ( \frac{a}{b} = \frac{c}{d} ).</td>
</tr>
<tr>
<td>2</td>
<td>Why didn’t you use the direct proportion formula.</td>
<td>Without using the formula I already knew the answer sir!</td>
</tr>
<tr>
<td>3</td>
<td>How did you know the answer.</td>
<td>Right, Rachel spinned 12 rounds, Ann just 3 rounds. Then if Ann spinned 6 rounds it meant Rachel had spinned 24 rounds.</td>
</tr>
<tr>
<td>4</td>
<td>Were they both spinning at the same speed.</td>
<td>Yes, sir, they did at the same speed.</td>
</tr>
<tr>
<td>5</td>
<td>OK, what was the next step.</td>
<td>The next step I wrote down, for example, when Rachel was in round 4, then Ann had just had round 1. So 4 divided by 1 multiplied by 6, you get 24.</td>
</tr>
</tbody>
</table>

Based on the interview results, Subject S2 correctly understood that the given problem involved direct proportions. Subject S2 identified that Rachel spun 12 rounds and Ann spun 3 rounds. The problem asked how many rounds Rachel would spin if Ann spun 6 rounds. Subject S2 formulated a problem-solving plan by writing \( \frac{12}{3} = \frac{24}{6} \), without considering the formula that could be used, and did not provide a complete explanation of the steps to find the answers. During the interview, Subject S2 stated that Rachel spun 12 rounds while Ann spun 3 rounds. Then, if Ann spun 6 rounds, Rachel would have spun 24 rounds. In the next step, Subject S2 explained that when Rachel completed her 4th round, Ann had just finished her 1st round, so 4 rounds divided by 1 round, then multiplied by 6 rounds equals 24 rounds. Therefore, Subject S2 understood the concept of direct proportion well and arrived at the correct conclusion that if Ann spun 6 times, Rachel would have spun 24 times.

Data Exposure and Analysis of Multiplication Proximity Thinking

The data on the multiplication proximity thinking characteristic was presented for Subject S3. The data for multiplication proximity thinking was derived from interviews, field notes, and subject answer sheets in solving direct proportion problems. The characteristic of multiplication proximity thinking occurs when students approach cases by multiplying two separate quantities with
variations. The comparative thinking characteristics observed in Subject S3 are illustrated in Figure 5.

![Image: Figure 5. Multiplication Proximity Thinking Characteristics of subject S3.]

The description of the occurrence of the multiplication proximity thinking characteristic is based on the results of the work done by Subject S3. When given the direct proportion questions, Subject S3 observed and read the questions carefully. For a clearer understanding of the answers provided by Subject S3 in solving the direct proportion problem, refer to the following interview results.

<table>
<thead>
<tr>
<th>No</th>
<th>Researcher Response</th>
<th>Subject Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>What do you think about this problem.</td>
<td>Rachel spun 12 rounds and Ann did 3 rounds.</td>
</tr>
<tr>
<td>2</td>
<td>What was asked.</td>
<td>Meanwhile, what was asked was how many rounds Rachel did.</td>
</tr>
<tr>
<td>3</td>
<td>Why did you write 12 multiplied by 6 then divided again by 3.</td>
<td>Because in the first situation Rachel did 12 rounds, I multiplied in the second situation the 6 rounds that Anna did, then divided by the 3 rounds that Anna did in the first situation.</td>
</tr>
<tr>
<td>4</td>
<td>Then what was the next step.</td>
<td>Yes, because 12 rounds multiplied by 6 rounds equals 72 rounds then divided by 3 rounds you got 24 rounds.</td>
</tr>
<tr>
<td>5</td>
<td>Was there any other way you could understand to solve this problem.</td>
<td>This is the only way I understand, sir!</td>
</tr>
</tbody>
</table>

Based on the interview results, Subject S3 understood that the given problems were related to direct proportions. Subject S3 stated that the known information was Rachel spinning 12 rounds and Ann spinning 3 rounds. Furthermore, Subject S3 identified that the question asked how many times Rachel would spin if Ann spun 6 times. Subject S3 explained that the first situation involved multiplying Rachel's 12 rounds by Ann's 6 rounds in the second situation, then dividing by the 3 rounds Ann spun in the first situation. Subject S3 applied this method to determine the number of times Rachel would spin if Ann spun 6 times. Specifically, 12 rounds multiplied by 6 rounds equals 72 rounds, which is then divided by 3 rounds, resulting in 24 rounds. Although Subject S3 did not use the direct proportion formula in her solution, she understood the concept of direct proportion well and correctly concluded that if Ann spun 6 times, Rachel would spin 24 times.

Based on the research findings, three characteristics of students' thinking were identified: (1) additive different thinking, (2) multiplication continuation thinking, and (3) multiplication proximity thinking. These characteristics were analyzed based on the work results and interview responses of Subject S1, Subject S2, and Subject S3. Additive different thinking occurs when a student, like Subject S1, approaches a problem by directly adding two quantities with different variations. When presented with the direct proportion problem, Subject S1 observed and read the problem, then noted the known and
asked elements. Subject S1 solved the problem without considering appropriate formulas or strategies. According to Nur and Sari (2022), using a formula as a starting point is essential for determining the strategy to solve a mathematical problem. Furthermore, Santrock (2008) stated that students need the ability to create strategies or new ideas regarding the problems they face. New ideas that appear suddenly are based on intuitive thinking from given facts (Hidajat et al., 2019). Through this intuitive thinking, Subject S1 can develop new problem-solving methods. Subject S1 believed that solving the direct proportion problem required addition: 12 rounds plus 12 rounds equals 24 rounds, and 3 rounds plus 3 rounds equals 6 rounds. Although the result was 24 rounds, this indicated a misunderstanding of the concept of proportion. Problems that should have been solved using multiplication were solved using addition. As Sumarto et al. (2013) stated, proportions are often taught formally in class, but teaching proportions through addition is meaningless for students, as they merely memorize procedures without understanding them.

Multiplication continuation thinking was observed in Subject S2, who approached the problem by multiplying two whole quantities with variations. Subject S2 read the problem, identified the known and asked elements, and wrote 12 divided by 3 and 24 divided by 6 without considering the direct proportion formula. According to Weber (2001), students' ability to solve a problem depends on their ability to identify the known and asked elements. After this initial step, Subject S2 determined a problem-solving strategy, explaining that solving the direct proportion problem was easier using multiplication. Subject S2 wrote that 3 multiplied by 4 equals 12 and 6 multiplied by 4 equals 24, although the number 4 was not written on the answer sheet. This approach is part of a problem-solving strategy involving the addition of information (Abdillah, 2017). Subject S2 concluded that if Ann spun 6 times, Rachel would spin 24 times, demonstrating a good understanding of direct proportion.

Multiplication proximity thinking occurs when a student, like Subject S3, approaches cases by multiplying two separate quantities with variations. When given the direct proportion problem, Subject S3 clearly understood that the problem involved direct proportions. Subject S3 read the questions carefully and thoroughly to ensure a proper understanding. According to Nur (2022), students who read questions carefully and thoroughly demonstrate attention to detail and an understanding of the problem. Subject S3 identified that Rachel spun 12 rounds and Ann spun 3 rounds in the first situation, and in the second situation, Ann spun 6 times. The question asked how many rounds Rachel would spin in the second situation. To determine this, Subject S3 chose the appropriate method or strategy for solving it, preferring the distribution method.

Based on the interviews, Subject S3 explained that in the first situation, Rachel's 12 rounds were multiplied by Ann's 6 rounds in the second situation, then divided by Ann's 3 rounds in the first situation. Subject S3 applied this method to determine how many times Rachel would spin if Ann spun 6 times. The calculation was: 12 rounds multiplied by 6 rounds equals 72 rounds, then divided by 3 rounds equals 24 rounds. Although Subject S3 used a different solving method, the results were correct. To determine the number of rounds Rachel spun, Subject S3 used division, thus correctly concluding that Rachel spun 24 rounds in the second
situation. This indicates that the concept was well embedded in Subject S3's memory.

The proportional thinking characteristics demonstrated by Subjects S1, S2, and S3 align with the characteristics of additive different thinking, multiplication continuation thinking, and multiplication proximity thinking in solving direct proportion problems. As Walle (2007) stated, students who can think proportionally have several characteristics: (1) understanding covariation and the relationship between two quantities with a common variation, (2) recognizing the relationship between multiplicative and additive concepts, (3) developing multiple strategies to solve direct proportion problems, and (4) understanding the ratio as a separate entity representing the relationship between quantities. Therefore, in solving direct proportion problems, teachers should explain the use of multiplication and division involving proportion situations. According to Sapti (2015), using multiplication and division in solving direct proportion problems is efficient and widely applicable.

Based on the thought processes of Subjects S1, S2, and S3, the characteristics of proportional thinking in direct proportion problem-solving are as follows: (1) additive different thinking occurs when students add two quantities with different variations; (2) multiplication continuation thinking occurs when students multiply two whole quantities with variations; and (3) multiplication proximity thinking occurs when students multiply two separate quantities with variations.

Students' understanding of the relationship between two quantities in a ratio forms the basis for solving direct proportion problems. It is essential that students comprehend the requirements for using ratios in the learning sequence before tackling direct proportion problems. Therefore, future researchers should explore different levels of proportional thinking and consider the number of subjects involved in their studies. In this research, three subjects demonstrated proportional thinking, suggesting that further investigation with a larger sample size is necessary.

CONCLUSION

Based on the problem formulation in this study, three characteristics of students' proportional thinking in solving direct proportion problems were identified: (1) additive different thinking, (2) multiplication continuation thinking, and (3) multiplication proximity thinking. Additive different thinking occurs when students directly add two quantities with different variations. Multiplication continuation thinking occurs when students multiply two whole quantities with variations. Multiplication proximity thinking occurs when students multiply two separate quantities with variations. Students' understanding of the relationship between two quantities in a ratio forms the basis for solving direct proportion problems. It is essential that students comprehend the requirements for using ratios in the learning sequence before tackling direct proportion problems. Therefore, future researchers should explore different levels of proportional thinking and consider the number of subjects involved in their studies. In this research, three subjects demonstrated proportional thinking, suggesting that further investigation with a larger sample size is necessary.
REFERENCES


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Susilowati, N. E., Muslim, Efendi, R., & Samsudin, A. (2022). What is the most impressive treatment to foster students’ creative thinking skills? A meta-analysis and bibliometric review. *Tadris: Jurnal Keguruan dan Ilmu Tarbiyah, 7*(2), 201-219. https://doi.org/10.24042/tadris.v7i2.12690
